21. Experimental Tests of Gravitational Theory

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21.1 General Relativity

Einstein's theory of General Relativity (GR), the current "standard" theory of gravitation, describes gravity as a universal deformation of the Minkowski metric:

$$g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h_{\mu\nu}(x^\lambda) \ , \ \text{where } \eta_{\mu\nu} = \text{diag}(-1,+1,+1,+1) \ . \tag{21.1}$$

GR is classically defined by two postulates, embodied in the total action defining the theory:

$$S_{\text{tot}}[g_{\mu\nu}, \psi, A_\mu, H] = c^{-1} \int d^4x (\mathcal{L}_{\text{Ein}} + \mathcal{L}_{\text{SM}}) \, . \tag{21.2}$$

The first postulate that states that the Lagrangian density describing the propagation and self-interaction of the gravitational field is

$$\mathcal{L}_{\text{Ein}}[g_{\alpha\beta}] = \frac{c^4}{16\pi G} \sqrt{g} g^{\mu\nu} R_{\mu\nu}(g_{\alpha\beta}) \ , \tag{21.3}$$

where G denotes Newton's constant, $g = -\det(g_{\mu\nu})$, $g^{\mu\nu}$ is the matrix inverse of $g_{\mu\nu}$, and where the Ricci tensor $R_{\mu\nu} \equiv R^\alpha{}_{\mu\alpha\nu}$ is the only independent trace of the curvature tensor

$$R^\alpha{}_{\mu\beta\nu} = \partial_\beta \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\beta} + \Gamma^\alpha_{\sigma\beta}\Gamma^\sigma_{\mu\nu} - \Gamma^\alpha_{\sigma\nu}\Gamma^\sigma_{\mu\beta} \ , \tag{21.4}$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}) \, . \tag{21.5}$$

The second postulate states that $g_{\mu\nu}$ (and its associated connection) couples universally, and minimally, to all the bosonic (respectively fermionic) fields of the Standard Model by replacing everywhere the Minkowski metric $\eta_{\mu\nu}$ (respectively the flat Minkowski connection). Schematically (suppressing matrix indices and labels for the various gauge fields and fermions and for the Higgs doublet),

$$\begin{aligned}\mathcal{L}_{\text{SM}}[\psi, A_\mu, H, g_{\mu\nu}] = &-\frac{1}{4}\sum \sqrt{g} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} - \sum \sqrt{g}\,\overline{\psi}\,\gamma^\mu (D_\mu + \frac{1}{4}\omega_{ij\mu}\gamma^{ij})\psi \\ &- \tfrac{1}{2}\sqrt{g} g^{\mu\nu}\overline{D_\mu H} D_\nu H - \sqrt{g}\,V(H) - \sum \lambda \sqrt{g}\,\overline{\psi}\,H\psi \ .\end{aligned} \tag{21.6}$$

Here $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_A f^a{}_{bc} A^b_\mu A^c_\nu$ and the (representation-dependent) gauge-field covariant derivative $D_\mu = \partial_\mu + g_A A^a_\mu T^{\text{rep}}_a$ are defined as in Special Relativity, while the derivative of spin-$\frac{1}{2}$ fermions also includes a coupling to the gravitational "spin-connection" $\omega_{ij\mu} = -\omega_{ji\mu}$, via its contraction with $\gamma^{ij} = \frac{1}{2}(\gamma^i\gamma^j - \gamma^j\gamma^i)$, where $i,j = 0,1,2,3$ and $\gamma^i = e^i{}_\mu\gamma^\mu$ are usual (numerical) Dirac matrices satisfying $\gamma^i\gamma^j + \gamma^j\gamma^i = 2\eta^{ij}$. The connection components $\omega_{ij\mu}$ are defined in terms of the local orthonormal frame (vierbein) $e^i{}_\mu$ (such that $g_{\mu\nu} = \eta_{ij}e^i{}_\mu e^j{}_\nu$) used to describe the components of the various fermions ψ, and of its inverse $e_i{}^\mu$ (such that $e_i{}^\mu e^j{}_\mu = \delta^j_i$), by $\omega_{ij\mu} = \frac{1}{2}\left(C_{i[jk]} + C_{j[ki]} - C_{k[ij]}\right)e^k_\mu$ where $C_{i[jk]} = \eta_{is}C^s{}_{[jk]}$, with $C^i{}_{[jk]} \equiv \left(\partial_\mu e^i{}_\nu - \partial_\nu e^i{}_\mu\right)e_j{}^\mu e_k{}^\nu$. From the total action follow Einstein's field equations,

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \ . \tag{21.7}$$

Here $R = g^{\mu\nu}R_{\mu\nu}$ is the scalar curvature, and $T_{\mu\nu} \equiv g_{\mu\alpha}g_{\nu\beta}T^{\alpha\beta}$ where $T^{\mu\nu} = (2/\sqrt{g})\delta\mathcal{L}_{\text{SM}}/\delta g_{\mu\nu}$ is the (symmetric) energy-momentum tensor of the Standard Model matter. The theory is invariant

under arbitrary coordinate transformations: $x'^\mu = f^\mu(x^\nu)$ (as well as under arbitrary local SO(3,1) rotations of the vierbein, $e'^i_\mu = \Lambda^i_j(x)e'^j_\mu$). To solve the field equations Eq. (21.7), one needs to fix the coordinate gauge freedom, *e.g.*, the "harmonic gauge" (which is the analogue of the Lorenz gauge, $\partial_\mu A^\mu = 0$, in electromagnetism) corresponds to imposing the condition $\partial_\nu(\sqrt{g}g^{\mu\nu}) = 0$.

In this *Review*, we only consider the classical limit of gravitation (*i.e.* classical matter and classical gravity). Quantum gravitational effects are expected (when considered at low energy) to correct the classical action Eq. (21.2) by additional terms involving quadratic and higher powers of the curvature tensor. This suggests that the validity of classical gravity extends (at most) down to length scales of order the Planck length $L_P = \sqrt{\hbar G/c^3} \simeq 1.62 \times 10^{-33}$ cm, *i.e.*, up to energy scales of order the Planck energy $E_P = \sqrt{\hbar c^5/G} \simeq 1.22 \times 10^{19}$ GeV. Considering quantum matter in a classical gravitational background also poses interesting challenges, notably the possibility that the zero-point fluctuations of the matter fields generate a nonvanishing vacuum energy density ρ_{vac}, corresponding to a term $-\sqrt{g}\,\rho_{\text{vac}}$ in $\mathcal{L}_{\text{SM}}$ [1]. This is equivalent to adding a "cosmological constant" term $+\Lambda\, g_{\mu\nu}$ on the left-hand side of Einstein's equations, Eq. (21.7), with $\Lambda = 8\pi G\,\rho_{\text{vac}}/c^4$. Recent cosmological observations (see the following *Reviews*) suggest a positive value of Λ corresponding to $\rho_{\text{vac}} \approx (2.3 \times 10^{-3}\text{eV})^4$. Such a small value has a negligible effect on the non-cosmological tests discussed below.

21.2 Key features and predictions of GR

The definition of GR recalled above makes predictions both about the coupling of gravity to matter, and about the structure of the gravitational field beyond its previously known Newtonian aspects.

21.2.1 *Equivalence Principle*

First, the universal nature of the coupling between $g_{\mu\nu}$ and the Standard Model matter postulated in Eq. (21.6) entails many observable consequences that go under the generic name of "Equivalence Principle".

A first aspect of the Equivalence Principle is that the outcome of a local non-gravitational experiment, referred to local standards, should not depend on where, when, and in which locally inertial frame, the experiment is performed. This means, for instance, that local experiments should neither feel the cosmological evolution of the Universe (constancy of the "constants"), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance).

A second aspect of the Equivalence Principle is that the kinetic terms, $g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ or $\overline{\psi}\gamma^i e_i{}^\mu\partial_\mu\psi$, of all the fields of Nature (including the gravitational field itself) are universally coupled to the same curved spacetime metric $g_{\mu\nu}(x) = \eta_{ij}e^i{}_\mu e^j{}_\nu$. This implies in particular that all massless fields should propagate with the same speed.

A third aspect of the Equivalence Principle is that two (electrically neutral) test bodies dropped at the same location and with the same velocity in an external gravitational field should fall in the same way, independently of their masses and compositions ("universality of free fall" or "Weak Equivalence Principle"). In addition, the study (using the nonlinear structure of GR) of the motion, in an external gravitational field, of bodies having a non-negligible, or even strong, self-gravity (such as planets, neutron stars, or black holes) has shown that the latter property of free-fall universality holds equally well for self-gravitating bodies ("Strong Equivalence Principle").

A last aspect of the Equivalence Principle concerns various universality features of the gravitational redshift of clock rates. GR predicts that, when intercomparing them by means of electromagnetic signals, two (non gravity-based) clocks located along two different spacetime worldlines should exhibit a universal difference in clock rate that depends on their worldlines, but that is independent of their nature and constitution. For instance, two clocks located at two different positions in a static external Newtonian potential $U(\boldsymbol{x}) = \sum Gm/r$ should exhibit, when intercompared by elec-

tromagnetic signals, the difference in clock rate, $\tau_1/\tau_2 = \nu_2/\nu_1 = 1 + [U(\boldsymbol{x}_1) - U(\boldsymbol{x}_2)]/c^2 + O(1/c^4)$, ("universal gravitational redshift of clock rates"). Similarly, the comparison of atomic-transition frequencies when observing on Earth a transition that took place on a far-away galaxy should involve (at lowest order in cosmological perturbations) the universal cosmological redshift factor $1 + z = a(t_{\text{reception}})/a(t_{\text{emission}})$ between the Friedmann scale factors $a(t)$ (see below).

21.2.2 *Quasi-stationary, weak-field (post-Newtonian) gravity*

When applied to quasi-stationary, weak-field gravitational fields, Einstein equations, Eq. (21.7), entail a spacetime structure which predicts deviations from Newtonian gravity of the first post-Newtonian (1PN) order, *i.e.*, fractionally smaller than Newtonian effects by a factor $O(v^2/c^2) \sim O(GM/(c^2 r))$. The 1PN-accurate solution of Eq. (21.7) reads (in harmonic gauge)

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2}{c^4}V^2 + O\left(\frac{1}{c^6}\right),$$

$$g_{0i} = -\frac{4}{c^3}V_i + O\left(\frac{1}{c^5}\right),$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2}{c^2}V\right] + O\left(\frac{1}{c^4}\right), \tag{21.8}$$

where $x^0 = ct$, $i, j = 1, 2, 3$, and where the scalar, V, and vector, V_i, (retarded) potentials are defined in terms of the sources $\sigma = \frac{T^{00} + T^{ii}}{c^2}$, $\sigma_i = \frac{T^{0i}}{c}$ by

$$V = \Box_{\text{ret}}^{-1}[-4\pi G\sigma] \; ; \; V_i = \Box_{\text{ret}}^{-1}[-4\pi G\sigma_i]. \tag{21.9}$$

In GR the gravitational interaction of N moving point masses (labeled by $A = 1, \ldots, N$) is described by a reduced (classical) action that admits a diagrammatic expansion:

$$S_{\text{reduced}} = S^{\text{free}} + S^{\text{tree-level}} + S^{\text{one-loop}} + \cdots \tag{21.10}$$

where the free (special-relativistic) action reads

$$S^{\text{free}} = -\sum_A \int m_A c \sqrt{-\eta_{\mu\nu} dx_A^\mu dx_A^\nu}$$

$$= -\sum_A \int dt\, m_A c^2 \sqrt{1 - \mathbf{v}_A^2/c^2}, \tag{21.11}$$

while the tree-level (one-graviton-exchange) interaction term reads

$$S^{\text{tree-level}} = -\frac{8\pi G}{c^4} \int d^4x\, T^{\mu\nu} \Box^{-1}(T_{\mu\nu} - \tfrac{1}{2}T\eta_{\mu\nu}) = \int dt\, L^{(2)}. \tag{21.12}$$

Corresponding to the 1PN-accurate metric of Eq. (21.8), the 1PN-accurate expansion of the latter tree-level, two-body interaction Lagrangian $L^{(2)}$ reads (with $r_{AB} \equiv |\mathbf{x}_A - \mathbf{x}_B|$, $\mathbf{n}_{AB} \equiv (\mathbf{x}_A - \mathbf{x}_B)/r_{AB}$)

$$L^{(2)} = \tfrac{1}{2} \sum_{A \neq B} \frac{G\, m_A\, m_B}{r_{AB}} \left[1 + \frac{3}{2c^2}(\boldsymbol{v}_A^2 + \boldsymbol{v}_B^2) - \frac{7}{2c^2}(\boldsymbol{v}_A \cdot \boldsymbol{v}_B) - \frac{1}{2c^2}(\boldsymbol{n}_{AB} \cdot \boldsymbol{v}_A)(\boldsymbol{n}_{AB} \cdot \boldsymbol{v}_B) + O\left(\frac{1}{c^4}\right)\right] \tag{21.13}$$

The two-body interactions, Eq. (21.13), exhibit v^2/c^2 corrections to Newton's $1/r$ potential induced by spin-2 exchange ("gravito-magnetism"). Consistency at the 1PN level, $v^2/c^2 \sim G\, m/rc^2$, requires

that one also considers the three-body interactions contained in the one-loop contribution $S^{\text{one}-\text{loop}}$, corresponding to terms induced by some of the three-graviton vertices and other non-linearities (terms $O(h^2)$ and $O(hT)$ in Eq. (21.15) below), *i.e.*, to the $O(V^2)$ term in Eq. (21.8):

$$L^{(3)} = -\frac{1}{2} \sum_{B \neq A \neq C} \frac{G^2 \, m_A \, m_B \, m_C}{r_{AB} \, r_{AC} \, c^2} + O\left(\frac{1}{c^4}\right) . \tag{21.14}$$

21.2.3 *Gravitational Waves in GR*

The linearized approximation to Einstein's field equations, Eq. (21.7), in harmonic gauge $\partial^\nu(h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}) = 0$ (with $h \equiv \eta^{\mu\nu}h_{\mu\nu}$), reads

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}) + O(h^2) + O(hT) . \tag{21.15}$$

Outside of any source (*i.e.*, when $T_{\mu\nu} = 0$), this yields $\Box h_{\mu\nu} = 0$, with $\partial^\nu(h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}) = 0$. The generic linearized solution (modulo the diffeomorphism freedom) of the latter vacuum Einstein equations can be written as (with $k^2 = k \cdot k = \eta_{\mu\nu}k^\mu k^\nu$, $k \cdot x = k_\mu x^\mu$)

$$h_{\mu\nu}(x) = \int d^4k\delta(k^2)\epsilon_{\mu\nu}(k)e^{ik\cdot x} , \tag{21.16}$$

where the polarization tensor $\epsilon_{\mu\nu}(k)$ must be transverse ($\epsilon_{\mu\nu}k^\nu = 0$) and traceless ($\eta^{\mu\nu}\epsilon_{\mu\nu} = 0$). In addition, $\epsilon_{\mu\nu}(k)$ can be freely submitted to the gauge freedom $\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + \xi_\mu k_\nu + \xi_\nu k_\mu$. This implies that gravitational waves (GW) propagate with the speed of light, and (like electromagnetic waves) have only two independent polarizations. In a frame where, say, $k^\mu = (ck, 0, 0, k)$, the two independent linear polarization tensors can be taken to have components only in the transverse 1-2 plane, of the following form: $\epsilon_{11}^+ = -\epsilon_{22}^+ = \epsilon^+$, with $\epsilon_{12}^+ = \epsilon_{21}^+ = 0$; or $\epsilon_{12}^\times = +\epsilon_{21}^\times = \epsilon^\times$, with $\epsilon_{11}^\times = \epsilon_{22}^\times = 0$. Under a little-group rotation of angle θ in the 1-2 plane, the two circular polarization amplitudes $\epsilon^{(\pm)} = \epsilon^+ \mp i\,\epsilon^\times$ vary as $\epsilon'^{(\pm)} = e^{\pm 2i\theta}\epsilon^{(\pm)}$, thereby characterizing the helicity-2 nature of GWs.

When solving the inhomogeneous equation Eq. (21.15), taking into account the nonlinear contributions $O(h^2) + O(hT)$, one finds that, to lowest order, the GW amplitude emitted at large distances by a matter distribution is given by the following "quadrupole formula"

$$h_{ij}^{\text{TT}}(T, \mathbf{X}) \approx \frac{2G}{c^4}P_{ijab}^{\text{TT}}(\mathbf{N})\frac{\ddot{Q}_{ab}(T - R/c)}{R} , \tag{21.17}$$

where $Q_{ij}(t) = \int d^3x\sigma(t, \mathbf{x})(x^i x^j - \frac{1}{3}\delta_{ij}\mathbf{x^2})$ $(a, b, i, j = 1, 2, 3)$ is the quadrupole moment of the source, $R = |\mathbf{X}|$ the distance to the source, $\mathbf{N} = \mathbf{X}/R$ the unit direction from the source to the observer, and $P_{ijab}^{\text{TT}}(\mathbf{N}) = (\delta_{ia} - N_i N_a)(\delta_{jb} - N_j N_b) - \frac{1}{2}(\delta_{ij} - N_i N_j)(\delta_{ab} - N_a N_b)$ the transverse-traceless projector onto the 2-plane orthogonal to $\mathbf{N}$.

21.2.4 *Strong gravitational fields: neutron stars and black holes*

The nonlinear structure of Einstein's equations implies many predictions for strong gravitational fields that distinguish GR from Newtonian gravity. For instance, in Newtonian gravity, there is no upper limit to the dimensionless gravitational potential U/c^2, with U satisfying Poisson's equation $\Delta U = -4\pi G\rho$, where ρ denotes the Newtonian mass density. By contrast, in GR, the dimensionless surface gravitational potential $GM/(c^2 R)$ of a spherically symmetric (perfect fluid) body cannot exceed $\frac{4}{9}$ [2].

Given an equation of state $p = f(\rho)$ modeling the interior of a (cold) spherically symmetric body (say a non-rotating neutron star), Einstein equations, Eq. (21.7), with $T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}$, and

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{2\Phi(r)}c^2dt^2 + \frac{dr^2}{1 - \frac{2GM(r)}{c^2r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \qquad (21.18)$$

yield the following Tolman-Oppenheimer-Volkoff radial equations:

$$p'(r) = -\frac{G(\rho + p/c^2)(M(r) + 4\pi r^3 p/c^2)}{r^2(1 - 2GM(r)/(c^2r))}; \qquad (21.19)$$

$$M'(r) = 4\pi r^2 \rho; \qquad (21.20)$$

$$\Phi'(r) = \frac{G(M(r) + 4\pi r^3 p/c^2)}{r^2(1 - 2GM(r)/(c^2r))}. \qquad (21.21)$$

In the exterior of the star $(r \geq R)$, the metric takes the Schwarzschild form

$$g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 + \frac{dr^2}{1 - \frac{2GM}{c^2r}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \qquad (21.22)$$

where $M \equiv M(R)$ is the total gravitational mass of the star. GR predicts, for any given $p = f(\rho)$, several (in principle) observable features of neutron stars, such as: (i) the maximum mass of a neutron star; (ii) the relation between the radius R and the total mass M; (iii) the dimensionless surface gravitational potential $GM/(c^2R)$ (linked to the surface redshift $\sqrt{-g_{00}} = \sqrt{1 - \frac{2GM}{c^2R}}$ measured by an observer at infinity); (iv) the moment of inertia; and (v) the Love number (tidal polarizability). The current uncertainty on the equation of state of a neutron star yields the GR-predicted approximate range for the maximum mass of non-rotating neutron stars $1.5\,M_\odot \lesssim M_{\max} \lesssim 2.5\,M_\odot$, and the absolute upper bound $M_{\max} < 3\,M_\odot$ [3]. The surface gravitational potential of a typical neutron star is $GM/c^2R_{\mathrm{NS}} \simeq 0.17$, which is a factor $\sim 10^8$ higher than the surface potential of the Earth, and a mere factor 3 below the black hole limit $GM/c^2R_{\mathrm{BH}} = \frac{1}{2}$ to be discussed next.

The existence of a maximum mass for a neutron star led Oppenheimer and Snyder [4] to predict that the end point of stellar evolution for sufficiently heavy stars, after exhaustion of all thermonuclear sources of energy, will be what are now called "black holes." The latter are solutions of Einstein's equations whose past structure involves a gravitationally collapsing star, but whose presently observable structure is essentially described (for non-rotating black holes) by the vacuum Schwarzschild solution Eq. (21.22). It took many years for theoretical (and mathematical) physicists to understand that the apparent singularity of the Schwarzschild solution at $r = \frac{2GM}{c^2}$ was a coordinate singularity and that the Schwarzschild spacetime was regular at the "black hole horizon", $R_{\mathrm{BH}} \equiv \frac{2GM}{c^2}$. The rotating analog of the Schwarzschild spacetime is the Kerr black hole [5].

Black holes are outstanding consequences of GR which enjoy many remarkable properties, notably: (i) presence of a one-way surface (the horizon) for all waves and particles; (ii) absence of "hair" (*i.e.*, barring a possible electric charge, their structure is fully described by only two parameters, total mass, M, and total angular momentum, $J \leq GM^2/c$); (iii) existence of a spectrum of damped quasi-normal vibrational modes; and (iv) a behavior under external perturbations similar to ordinary physical objects satisfying the laws of (dissipative) thermodynamics. Moreover, though no classical waves or particles can get out of the horizon, black holes are predicted to slowly evaporate via quantum particle creation.

21.2.5 *Cosmology*

To complete our short tour of the main predictions of GR, let us mention that GR offers the current standard framework for describing the large-scale structure of the Cosmos, from the nearly homogeneous Big Bang (and its plausible inflationary beginning) to the current inhomogeneous Universe undergoing an accelerated expansion. The spacetime structure on large (temporal and spatial) scales is well described by a solution of Einstein's equations of the form

$$ds^2 = -\left(1 + 2\Phi(t,\mathbf{x})\right)c^2 dt^2 + 2W_i(t,\mathbf{x})dt dx^i + a^2(t)\left(\left(1 - 2\Psi(t,\mathbf{x})\right)\delta_{ij} + h_{ij}(t,\mathbf{x})\right)dx^i dx^j,$$
(21.23)

where, after a suitable gauge-fixing [6], $W_i(t,\mathbf{x})$ is transverse, while $h_{ij}(t,\mathbf{x})$ is transverse and traceless. The source $T^{\mu\nu}$ must involve a certain number of postulated ingredients: an inflaton field; the matter of the Standard Model; a dark matter component; and a cosmological constant contribution $T_\Lambda^{\mu\nu} = -\rho_{\text{vac}}g^{\mu\nu}$, with $\rho_{\text{vac}} \equiv c^4\Lambda/(8\pi G)$. The scale factor $a(t)$ of the Friedmann background metric $ds_0^2 = -c^2 dt^2 + a^2(t)\delta_{ij}dx^i dx^j$ satisfies the GR-predicted Friedmann equations (with vanishing spatial curvature $k = 0$),

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}},$$
(21.24)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\text{tot}} + \frac{3}{c^2}p_{\text{tot}}\right),$$
(21.25)

while the scalar $(\Phi(t,\mathbf{x}), \Psi(t,\mathbf{x}))$, vector $(W_i(t,\mathbf{x}))$, and tensor $(h_{ij}(t,\mathbf{x}))$ inhomogeneous perturbations satisfy some GR-predicted propagation equations (coupled to matter perturbations); see [6] and the following *Reviews*. When the cosmic fluid is well approximated by a perfect fluid, Einstein's equations predict the following link between the scalar perturbations

$$\Phi(t,\mathbf{x}) = \Psi(t,\mathbf{x}).$$
(21.26)

21.3 A roadmap of parametrizations of deviations from GR, and of modified gravity

As will be discussed below, all currently performed gravitational experiments are compatible with GR. However, similarly to what is done in discussions of precision electroweak experiments, it is useful to quantify the significance of precision gravitational experiments by parameterizing possible deviations from GR. One can distinguish two main approaches to considering, and parameterizing, deviations from GR: (i) theory-agnostic phenomenological approaches; or, (ii) the study of the predictions of specific classes of alternative theories of gravity. Both types have led to useful ways of discussing tests of gravity. Both types also have their limitations. Considering them together leads to cross-fertilization.

21.3.1 *Theory-agnostic phenomenological approaches to parameterizing deviations from GR*

The theory-agnostic phenomenological approach is the oldest, and, arguably, the most robust one. It essentially consists in starting from specific observable predictions within the considered standard theory, and of deforming them by introducing some free parameters measuring either deviations from effects already present within the standard theory, or new effects absent from the standard theory. A classic example is the periastron advance of Mercury (and the other planets). When working within Newtonian gravity as a standard theory of gravity, the rate of periastron advance of Mercury, $\dot{\omega}$, is (when neglecting the quadrupole moment of the Sun) a calculable function of the masses and semi-major axes of the other planets of the solar system, say $\dot{\omega}^{\text{Newton}}(m_i, a_i)$.

However, $\dot{\omega}$ is also a directly observable quantity, so that one can parameterize the periastron advance of Mercury by writing

$$\dot{\omega}^{\mathrm{obs}} = \dot{\omega}^{\mathrm{Newton}}(m_i, a_i) + \Delta\dot{\omega}\,. \tag{21.27}$$

Using other observable data to determine some "observed" values of the m_i's and a_i's, one can then measure the anomalous periastron precession $\Delta\dot{\omega}$ and see whether it is compatible with zero, or not. As is well-known, Leverrier used such a methodology and, in 1859, measured an anomalous periastron precession of about $\Delta\dot{\omega} \simeq 38$ arcsec/century (later re-estimated at 43 arcsec/century), which was explained in 1915 as a GR prediction. Let us discuss further examples of the use of such theory-agnostic approaches for discussing deviations from GR.

21.3.2 *Parameterized post-Newtonian (PPN) formalism.*

When considering the weak-field slow-motion limit appropriate to describing gravitational experiments in the solar system, it has been traditional to parameterize possible (long-range) deviations from the GR-predicted 1PN metric by introducing extra dimensionless coefficients in the various terms of the metric of Eq. (21.8). The minimal version of the parameterized post-Newtonian (PPN) formalism (essentially due to Eddington) involves only two parameters β and γ, namely

$$g_{00} = -1 + \frac{2}{c^2}V - \frac{2\beta}{c^4}V^2 + O\left(\frac{1}{c^6}\right)\,, \tag{21.28}$$

$$g_{0i} = -\frac{2(\gamma+1)}{c^3}V_i + O\left(\frac{1}{c^5}\right)\,, \tag{21.29}$$

$$g_{ij} = \delta_{ij}\left[1 + \frac{2\gamma}{c^2}V\right] + O\left(\frac{1}{c^4}\right)\,, \tag{21.30}$$

with V and V_i defined by Eq. (21.9), with the same vectorial source $\sigma_i = \frac{T^{0i}}{c}$, but a modified scalar source

$$\sigma^{\mathrm{PPN}} = \frac{1}{c^2}\left(\left[1 + (3\gamma - 2\beta - 1)\frac{V}{c^2}\right]T^{00} + \gamma\,T^{ii}\right)\,. \tag{21.31}$$

In GR, $\beta^{\mathrm{GR}} = 1$ and $\gamma^{\mathrm{GR}} = 1$, so that deviations from GR are parameterized by $\overline{\beta} \equiv \beta - 1$ and $\overline{\gamma} \equiv \gamma - 1$. Richer versions of the PPN formalism (involving up to ten parameters) were developed in interaction with the study of classes of alternative theories of gravity [7,8]. This led to parameterizing new types of contributions to the 1PN metric that are absent in the GR framework.

When deriving the 1PN-accurate dynamics of N point masses predicted by the PPN-modified metric, Eq. (21.28), one finds that the free Lagrangian is not modified (because we are considering here a Lorentz-invariant subclass of PPN metrics), while there are modifications of both the two-body Lagrangian, $L^{(2)}$, Eq. (21.13), and the three-body one, $L^{(3)}$, Eq. (21.14). More precisely, denoting $\eta \equiv 4\overline{\beta} - \overline{\gamma}$, the Newtonian interaction energy term in Eq. (21.13) is modified into $G_{AB}m_A m_B/r_{AB}$, with a body-dependent gravitational "constant"

$$G_{AB} = G[1 + \eta(E_A^{\mathrm{grav}}/m_A c^2 + E_B^{\mathrm{grav}}/m_B c^2) + O(1/c^4)]\,, \tag{21.32}$$

where E_A^{grav} denotes the gravitational binding energy of body A. In addition, there is the additional contribution $+\overline{\gamma}(\boldsymbol{v}_A - \boldsymbol{v}_B)^2/c^2$ in the brackets on the right-hand side of $L^{(2)}$, Eq. (21.13). As for the three-body interaction term $L^{(3)}$, Eq. (21.14), it is modified by the overall factor $1 + 2\overline{\beta}$.

These results show how the introduction of the two minimal PPN deviation parameters $\overline{\beta} \equiv \beta - 1$ and $\overline{\gamma} \equiv \gamma - 1$ suffices to introduce many different observable effects. Some of them (the ones linked with $\overline{\gamma}$) concern deviations at the linearized (one-graviton-exchange) level (and affect, for instance, light deflection and time-delay effects), while the deviation parameter $\overline{\beta}$ parameterizes effects linked

to the cubic vertex of Einstein's gravity (and affects, for instance, periastron precession). Of particular interest is the fact that Eq. (21.32) shows that the combination $\eta \equiv 4\bar{\beta} - \bar{\gamma}$ parameterizes a violation of the Strong Equivalence Principle, because the gravitational interaction between self-gravitating bodies is seen to be influenced by the gravitational binding energy of each body [9]. As stated above, this effect is absent in GR (where $\eta^{\text{GR}} = 0$). This is an example where the fact of contrasting GR with some deviations from it gives physical significance to a null effect in GR (namely the universality of free fall of self-gravitating bodies).

Finally, one can extend the PPN formalism by allowing for a slow, phenomenological time variation of Newton's constant:

$$G(t) = G_0 \left[1 + \frac{\dot{G}_0}{G_0}(t - t_0) \right] . \tag{21.33}$$

Here, one assumes that there exist units in which the masses, m_i, of elementary particles stay constant, and that G is measured in such units. A possible time variation of G then corresponds to a possible common variation of the dimensionless couplings $Gm_i^2/(\hbar c)$.

21.3.3 *Parameterized post-Keplerian (PPK) formalism.*

The discovery of pulsars (*i.e.*, rotating neutron stars emitting a beam of radio noise) in gravitationally bound orbits [10, 11] has given us our first experimental handle on a regime of relativistic gravity going significantly beyond the uniformly weak-field, and quasi-stationary regime of solar-system gravity. Binary pulsars allow us to probe some radiative effects, and also some strong-gravitational-field effects. In these systems, the finite speed of propagation of the gravitational interaction between the pulsar and its companion generates damping-like terms at order $(v/c)^5$ in the equations of motion [12]. These damping forces are the local counterparts of the gravitational radiation emitted at infinity by the system ("gravitational radiation reaction"). They cause the binary orbit to shrink and its orbital period P_b to decrease. The remarkable stability of pulsar clocks has allowed one to measure the corresponding very small orbital period decay $\dot{P}_b \equiv dP_b/dt \sim -(v/c)^5 \sim -10^{-12}$–$10^{-14}$ in several binary systems, thereby giving us a direct experimental handle on the propagation properties of the gravitational field. In addition, the large surface gravitational potential of a neutron star allows one to probe the quasi-static strong-gravitational-field regime, as is discussed below.

It is possible to extract phenomenological (theory-independent) tests of gravity from binary pulsar data by using the parameterized post-Keplerian (PPK) formalism [13]. The basis of this formalism is the fact that, after correcting for the Earth's motion around the Sun and for the dispersion due to propagation in the interstellar plasma, the time of arrival of the Nth pulse t_N can be described by a generic, parameterized "timing formula" [13, 14], whose functional form is common to the whole class of tensor-scalar gravitation theories:

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p); \{p^K\}; \{p^{PK}\}] . \tag{21.34}$$

Here, T_N is the pulsar proper time corresponding to the Nth turn given by $N/2\pi = \nu_p T_N + \frac{1}{2}\dot{\nu}_p T_N^2 + \frac{1}{6}\ddot{\nu}_p T_N^3$ (with $\nu_p \equiv 1/P_p$ the spin frequency of the pulsar, *etc.*), $\{p^K\} = \{P_b, T_0, e, \omega_0, x\}$ is the set of "Keplerian" parameters (notably, orbital period P_b, eccentricity e, periastron longitude ω_0 and projected semi-major axis $x = a \sin i/c$), and $\{p^{PK}\} = \{k, \gamma_{\text{timing}}, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$ denotes the set of (separately measurable) "post-Keplerian" parameters. Most important among these are: the fractional periastron advance per orbit $k \equiv \dot{\omega} P_b/2\pi$; a dimensionful time-dilation parameter γ_{timing}; the orbital period derivative $\dot{P}_b$; and the "range" and "shape" parameters of the gravitational time delay caused by the companion, r and s.

Without assuming any specific theory of gravity, one can phenomenologically analyze the data from any binary pulsar by least-squares fitting the observed sequence of pulse arrival times to the timing formula of Eq. (21.34). This fit yields the "measured" values of the parameters $\{\nu_p, \dot{\nu}_p, \ddot{\nu}_p\}$, $\{p^K\}$, $\{p^{PK}\}$. Now, each specific relativistic theory of gravity predicts that, for instance, k, γ_{timing}, $\dot{P}_b$, r, and s (to quote parameters that have been successfully measured from some binary pulsar data) are some theory-dependent functions of the Keplerian parameters and of the (unknown) masses m_1, m_2 of the pulsar and its companion. For instance, in GR, one finds (with $M \equiv m_1 + m_2$, $n \equiv 2\pi/P_b$),

$$
\begin{aligned}
k^{\text{GR}}(m_1, m_2) =&\, 3(1 - e^2)^{-1}(GMn/c^3)^{2/3} \ , \\
\gamma^{\text{GR}}_{\text{timing}}(m_1, m_2) =&\, en^{-1}(GMn/c^3)^{2/3}m_2(m_1 + 2m_2)/M^2 \ , \\
\dot{P}^{\text{GR}}_b(m_1, m_2) =&\, -(192\pi/5)(1 - e^2)^{-7/2}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \\
& \times (GMn/c^3)^{5/3}m_1 m_2/M^2 \ , \\
r^{\text{GR}}(m_1, m_2) =&\, Gm_2/c^3 \ , \\
s^{\text{GR}}(m_1, m_2) =&\, nx(GMn/c^3)^{-1/3}M/m_2 \ .
\end{aligned}
\tag{21.35}
$$

In alternative gravity theories each of the functions $k^{\text{theory}}(m_1, m_2)$, $\gamma^{\text{theory}}_{\text{timing}}(m_1, m_2)$, $\dot{P}^{\text{theory}}_b(m_1, m_2)$, *etc.*, is modified by quasi-static strong field effects (associated with the self-gravities of the pulsar and its companion), while the particular function $\dot{P}^{\text{theory}}_b(m_1, m_2)$ is further modified by radiative effects [15–18]. If one measures $N > 2$ PPK parameters from the data of a specific binary pulsar, these N measurements determine, for each given theory, N curves (defined by the N equations $k^{\text{theory}}_i(m_1, m_2) = k^{\text{obs}}_i$) in the two-dimensional mass plane (m_1, m_2). This yields $N - 2$ tests of the specified theory, according to whether the N curves (or strips) have one point in common, as they should.

21.3.4 *Parameterized-post-Friedmannian (PPF) formalisms.*

We have recalled above that, in GR, the two functions, $\Phi(t, \mathbf{x})$, and $\Psi(t, \mathbf{x})$, parameterizing (in the "longitudinal gauge") the scalar perturbations of the background Friedmann metric are related (in absence of anisotropic stresses) by Eq. (21.26). Several authors [19–28] have defined various types of parameterized-post-Friedmannian (PPF) formalisms involving (generally space and time dependent) phenomenological parameters. The simplest versions of these formalisms involve two phenomenological parameters measuring: (i) the ratio between $\Phi(t, \mathbf{x})$, and $\Psi(t, \mathbf{x})$, say (using a parametrization which parallels the usual PPN parametrization)

$$
\Psi(t, \mathbf{x}) = \gamma_{\text{cosmo}}(t, \mathbf{x})\Phi(t, \mathbf{x}) \ ; \tag{21.36}
$$

and (ii) the effective gravitational constant entering the Poisson equation for $\Phi(t, \mathbf{x})$, say

$$
\Delta\Phi(t, \mathbf{x}) = 4\pi G_\Phi(t, \mathbf{x})\delta\rho(t, \mathbf{x}) \ . \tag{21.37}
$$

However, the peculiarities of cosmological observables limit the domain of applicability of such phenomenological approaches [26] (notably because the strong dependence of cosmological probes on epochs and scales obliges one to rely on specific parameterizations of the functions $\gamma_{\text{cosmo}}(t, \mathbf{x})$ and $G_\Phi(t, \mathbf{x})$, *e.g.*, [25, 28]). Approaches based on specific classes of modified-gravity theories allow for a more complete treatment involving, in principle, all existing cosmological observables: Big Bang nucleosynthesis, cosmic microwave background, large-scale structure, Hubble diagram,

weak lensing, etc. Discussing the current cosmological tests using either such PPF formalisms, or comparisons with the predictions of modified-gravity theories, is beyond the scope of this review. See [29] for a comprehensive recent discussion. The bottom line is that all present cosmological data have been found to be compatible with GR (within the Friedmann-Lemaître-based ΛCDM model). Beyond the quantitative limits on various parameterized theoretical models [29], one should remember the striking (strong-field-type) qualitative verification of GR embodied in the fact that relativistic cosmological models give an accurate picture of the Universe over a period during which the spatial metric has been blown up by a gigantic factor, say $(1 + z)^2 \sim 10^{19}$ between Big Bang nucleosynthesis and now.

21.3.5 *Various phenomenological tests of GR from gravitational wave (GW) data*

The observations by the US-based Laser Interferometer Gravitational-wave Observatory (LIGO), later joined by the Europe-based Virgo detector, of gravitational-wave (GW) signals [30–34], have opened up a novel testing ground for relativistic gravity. The first three observing runs (O1, O2, O3a and O3b) of the LIGO-Virgo collaboration, recently joined by the Japan-based KAGRA detector (LVK collaboration), have led to the confident detection of 93 GW signals, most of which come from binary black-hole coalescences. These observations are summarized in three GW transient catalogs: GWTC-1 [35], GWTC-2 [36] and GWTC-3 [37].

Several approaches have been used to either test consistency with GR, or to look for special types of possible deviations. Making accurate predictions for GW signals from coalescing black holes within GR took years of both analytical [38, 39] and numerical [40] work. Some works have started (both analytically [15, 41–45] and numerically [46–48]) to derive the corresponding predictions within some modified-gravity theories. Phenomenological approaches are very useful for parameterizing general, conceivable deviations from GR when analyzing the GW signals emitted by coalescing black holes or neutron stars.

A first phenomenological, global consistency test simply consists of measuring the noise-weighted correlation $\mathcal{C}$ between each detected strain signal and the corresponding best-fit GR-predicted waveform. $\mathcal{C}^{\mathrm{obs}}$ should be equal to 1, modulo statistical (and/or systematic) errors.

Various other phenomenological tests of the structure of the GR-predicted waveforms emitted by coalescing compact binaries have been suggested. One general idea [49–51] (dubbed "parameterized post-Einsteinian formalism" in [52]) is to modify the GR-predicted Fourier-domain value $\psi(f)$ of the phase of black-hole coalescence GW signals $h(f) = A(f)e^{i\psi(f)}$ by introducing GR-deviation parameters, say

$$\psi(f) = \sum_i \left[p_i^{\mathrm{GR,NS}}(m_1, m_2)(1 + \delta\hat{p}_i) + p_i^{\mathrm{GR,S}}(m_1, m_2, S_1, S_2) \right] u_i(f). \qquad (21.38)$$

Here, the $u_i(f)$'s define a basis of functions of the GW frequency f, and the superscript NS refers to the nonspinning contribution, while the superscript S refers to the spinning one. Such a GR-modification directly applies to the phenomenological representation [53] of $\psi(f)$, and can be generalized to any waveform model by adding the non-GR phase term $\sum_i p_i^{\mathrm{GR,NS}}(m_1, m_2)\delta\hat{p}_i u_i(f)$ to the corresponding GR-predicted Fourier-domain phase $\psi(f)$ [54]. For instance, the leading-order (LO), quadrupolar term in the GR phase evolution during the early inspiral corresponds to $u_0(f) = f^{-5/3}$ and $p_0^{\mathrm{GR}} = \frac{3(m_1+m_2)^2}{128 m_1 m_2}(\pi G(m_1 + m_2)/c^3)^{-5/3}$, while the next-to-leading-order (NLO) term is a $O(v^2/c^2)$ correction $p_2^{\mathrm{GR}}u_2(f)$ with $u_2(f) = f^{-1}$. In the phenomenological model [53] these terms (as well as the other inspiral contributions) are cut-off beyond the frequency $G(m_1 + m_2)f/c^3 = 0.018$.

Each dimensionless parameter $\delta\hat{p}_i$ introduces a fractional deviation from the corresponding individual phasing GR effect having the frequency dependence $u_i(f)$, and can, in principle, be extracted

by fitting the inspiral part of the observed waveform to the deformed template of Eq. (21.38). However, one must also use this deformed template for simultaneously extracting the values of m_1, m_2, S_1, and S_2. Together with signal-to-noise ratio (SNR) considerations, and parameter-correlation issues, this limits the applicability of such a test to introducing only one deformation parameter $\delta \hat{p}_i$ at a time. A particularly meaningful test [49] is to leave undeformed the LO and NLO terms $p_0^{\text{GR}} u_0(f) + p_2^{\text{GR}} u_2(f)$ and to vary the third coefficient p_3 parameterizing the next, "GW tail"-related $O(v^3/c^3)$ correction, with $u_3(f) = f^{-2/3}$. Another well-motivated test [52] is to introduce a new coefficient $\delta \hat{p}_{-2}$, which is absent in GR, and which parameterizes an $O((\frac{v}{c})^{-2})$ fractional correction to the LO, quadrupolar term, thereby allowing for a possible dipolar GW flux (indeed, dipolar GW radiation generally exists in theories containing scalar excitations). As p_{-2}^{GR} vanishes, $\delta \hat{p}_{-2}$ is added as an absolute deviation, scaled by the LO term p_0^{GR}.

The coalescence of two black holes, or of a black hole and a neutron star (or of two heavy-enough neutron stars) leads to the formation of a black hole that is initially formed in a perturbed state. The relaxation of the latter perturbed black hole into its stationary, equilibrium state leads to the emission of characteristic (rapidly decaying) ringing GW modes (a.k.a. quasi-normal modes) [55,56], whose frequencies and decay times are functions of the mass (M_f) and spin ($S_f \equiv GM_f^2 a_f/c$) of the final black hole, say

$$\omega_a = (c^3/GM_f)[2\pi \hat{f}_a^{\text{QNM}}(a_f) - i/\hat{\tau}_a^{\text{QNM}}(a_f)] \,, \tag{21.39}$$

where $a = 1, 2, \ldots$ labels the various ringing modes, starting from the least-damped one. In principle, if the SNR is large enough, one can directly test for the presence of one or several of these modes in the post-merger signal, and measure both $\text{Re}(\omega_a)$ and $\text{Im}(\omega_a)$ in a theory-independent way. These phenomenological measurements then lead to null tests of GR, from which one can extract theoretical information about eventual deviations from GR [57,58].

As recalled above, GR predicts that GWs propagate (in vacuum) at exactly the same speed as light (*i.e.*, they have the same dispersion law $g^{\mu\nu}k_\mu k_\nu = 0$ in curved spacetime). Deviations from such a universal, scale-free dispersion law can be phenomenologically parameterized in several ways. If one phenomenologically assumes that the graviton dispersion law includes a mass term, say $g^{\mu\nu}k_\mu k_\nu + m_g^2/\hbar^2 = 0$, or some more general type of frequency-dependent modification, such changes affect the phasing of the inspiral GW signal and can be directly tested [59]. When one observes *both* GWs and electromagnetic waves emitted by the same system, one can also directly test whether both types of waves propagate in the same way.

Let us now present some examples of theory-dependent discussions of experimental tests based on considering specific classes of alternative theories. The most conservative deviations from Einstein's pure spin-2 theory are defined by adding new, bosonic, light or massless, macroscopically coupled fields.

21.3.6 *Gravity tests within classes of tensor-scalar theories of gravity*

The possible existence of new gravitational-strength couplings leading to deviations from Einsteinian (and Newtonian) gravity has been suggested by many natural extensions of GR, starting with the classic Kaluza-Klein idea, and continuing up to now with the study of extended supergravity theories, and of (super-)string theory. In particular, a recurrent suggestion of such theories (which dates back to pioneering work by Jordan, and by Fierz [60]) is the existence of a scalar field φ coupled both to the scalar curvature R and to the various $F_{\mu\nu}^{a\,2}$ gauge-field actions. Such fields ("dilaton" or "moduli") generically appear in string theory and are massless at the tree-level, but could acquire a self-interaction potential $V(\varphi)$ beyond the tree-level.

The exchange of such a dilaton-like field leads to several types of observational deviations from GR. For experimental limits on the gravitational inverse-square-law (down to the micrometer range)

see Refs. [61–65]. If the potential $V(\varphi)$ is zero or negligible for the considered range, the coupling of φ to $F^a_{\mu\nu}{}^2$ leads to apparent violations of the weak equivalence principle, with rather specific composition-dependence [66]. Next, when neglecting the fractionally small composition-dependent effects, such a field approximately couples to the trace of the energy-momentum tensor $T = g_{\mu\nu}T^{\mu\nu}$. The most general such theory contains (after suitable field redefinitions) two arbitrary functions of the scalar field, namely the self-interaction potential $V(\varphi)$, and a matter-coupling function $a(\varphi)$:

$$
\begin{aligned}
\mathcal{L}_{\text{tot}}[g_{\mu\nu}, \varphi, \psi, A_\mu, H] =& \frac{c^4}{16\pi G_*}\sqrt{g}(R(g_{\mu\nu}) - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) \\
& - \sqrt{g}V(\varphi) + \mathcal{L}_{\text{SM}}[\psi, A_\mu, H, \widetilde{g}_{\mu\nu}] \, .
\end{aligned}
\tag{21.40}
$$

Here G_* is a "bare" Newton constant, and the Standard Model matter is coupled not to the "Einstein" (pure spin-2) metric $g_{\mu\nu}$, but to the conformally related ("Jordan-Fierz") metric

$$
\widetilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}. \tag{21.41}
$$

The scalar field equation

$$
\Box_g\varphi = \frac{4\pi G}{c^4}\left(-\alpha(\varphi)T + \frac{\partial V(\varphi)}{\partial\varphi}\right) , \tag{21.42}
$$

features

$$
\alpha(\varphi) \equiv \partial a(\varphi)/\partial\varphi \, , \tag{21.43}
$$

as the basic (field-dependent) coupling between φ and matter [15,67]. The best-known, special case of these theories is the one-parameter (ω) Jordan-Fierz-Brans-Dicke theory [68], with $V(\varphi) = 0$ and $a(\varphi) = \alpha_0\varphi$, leading to a field-independent coupling $\alpha(\varphi) = \alpha_0$ (with $\alpha_0{}^2 = 1/(2\omega + 3)$). More generally, if we consider the massless theories ($V(\varphi) = 0$) with arbitrary (non-linear) coupling function $a(\varphi)$, they modify Einstein's predictions in the weak-field slow-motion limit appropriate to describing gravitational experiments in the solar system (1PN approximation) only through the appearance of exactly the same two "post-Einstein" dimensionless parameters $\overline{\gamma} = \gamma - 1$ and $\overline{\beta} = \beta - 1$ that entered the minimal (Eddington) PPN formalism presented above. However, we now have the following theoretical expressions relating the latter phenomenological parameters to the coupling functions entering the tensor-scalar action Eq. (21.40):

$$
\overline{\gamma} = -2\frac{\alpha_0^2}{1 + \alpha_0^2} \, ; \tag{21.44}
$$

$$
\overline{\beta} = +\frac{1}{2}\frac{\beta_0\alpha_0^2}{(1 + \alpha_0^2)^2} \, . \tag{21.45}
$$

Here $\alpha_0 \equiv \alpha(\varphi_0)$, and $\beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$, with φ_0 denoting the vacuum expectation value (VEV) of φ around the solar system. In addition, the observable value G^{obs} of the gravitational constant is found to be field-dependent and given (at a place where $\varphi = \varphi_0$) by

$$
G^{\text{obs}} = G(\varphi_0) \equiv G_* \exp[2a(\varphi_0)](1 + \alpha_0^2) \, . \tag{21.46}
$$

This makes it clear that the parameter $\overline{\gamma}$ is the basic post-Einstein parameter, which measures the admixture of an additional field (here a spin-0 field) to the pure spin-2 GR. One also sees how the parameter $\overline{\beta}$ is linked to non-linear effects (here coupling terms $\beta_0(\varphi - \varphi_0)^2T$ in the action), and how the Nordtvedt parameter $\eta \equiv 4\overline{\beta} - \overline{\gamma}$ is related to the field-dependence of G^{obs} ($\eta = (\alpha_0/(1 + \alpha_0^2))\partial\ln G(\varphi_0)/\partial\varphi_0$).

The advantage of a theory-dependent approach, such as Eq. (21.40), over the phenomenological minimal PPN approach of Eq. (21.28), is that it allows one to consistently predict the observational deviations from GR in all possible gravity regimes: the quasi-stationary weak-field regime; the wavelike weak-field regime; the strong-field regime; the cosmological regime, etc. All such observational deviations can be consistently worked out once one chooses specific forms of the coupling function $a(\varphi)$, and of $V(\varphi)$. The simple choice of a two-parameter quadratic coupling function, say $a(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2$, has been found useful for describing many possible observable deviations from GR.

The observable consequences for binary pulsar observations of the strong-field and radiative effects linked to the coupling to φ have been explicitly worked out in Refs. [15,44] in the case where φ is massless (see Ref. [69] for the case where φ is massive). In particular, the strong-field nature of the pulsar tests is demonstrated by the fact that some tensor-scalar theories can be as close as desired to GR in the weak-field regime of the solar-system (*i.e.*, $\overline{\gamma}$ and $\overline{\beta}$ can be as small as desired, or even exactly zero), while developing (via a "spontaneous scalarization" mechanism) differences of order unity with GR in binary pulsar experiments [17,18].

21.3.7 *Attractor and screening mechanisms in modified gravity*

As will follow from the discussion of experimental data below, the comparison between the predictions of general massless tensor-scalar theories and current data shows that the basic coupling parameter α_0 must be tuned to a small value (especially when allowing for composition-dependent effects). This raises the issue of the naturalness of such small coupling parameters. It has been shown in this respect that, in many tensor-scalar theories, there is an *attractor mechanism* by which the cosmological evolution naturally drives the VEV $\varphi_0(t)$ towards a value for which the coupling parameter $\alpha_0 = \alpha(\varphi_0)$ vanishes, thereby making it natural to expect only small deviations from GR (at least for the weak-field regime) at our current cosmological epoch [70,71].

There are other theoretical mechanisms (generically called "screening mechanisms") that could explain why a theory of gravity whose theoretical content significantly differs from that of GR could naturally pass all the stringent, GR-compatible experimental limits that will be discussed below. In particular, when considering a self-interacting scalar field ($V(\varphi) \neq 0$), the interplay between the two terms on the right-hand side of Eq. (21.42) tends to drive the local VEV φ_0 of φ to a density-dependent value. In turn, this leads to a corresponding density-dependent effective mass $m_0(\varphi_0) = \sqrt{4\pi G \partial^2 V(\varphi_0)/\partial\varphi_0^2}$ of the φ field, and to density-dependent matter couplings [72]. Various choices of the functions $V(\varphi)$ and $a(\varphi)$ can then reduce the φ-induced deviations from GR in dense environments while still allowing for significant deviations in different (*e.g.*, cosmological) regimes [73–77].

Other screening mechanisms have been invoked, based on an environment dependence mediated by (first or second) derivatives of a scalar degree of freedom. Roughly speaking, such mechanisms involve a (possibly effective) scalar degree of freedom φ that satisfies a field equation that is more general than Eq. (21.42) in that the left-hand side, $\Box_g\varphi$, is replaced by a non-linear function of φ, $\partial\varphi$ and $\partial^2\varphi$. The presence of non-linear derivative self-interactions of φ can weaken the effective coupling of φ to matter. A simple toy-model showing this weakening would be to replace Eq. (21.42) by an equation of the form

$$Z(\varphi, \partial\varphi, \partial^2\varphi)\Box_g\varphi = \frac{4\pi G}{c^4}\left(-\alpha(\varphi)T + \frac{\partial V(\varphi)}{\partial\varphi}\right). \qquad (21.47)$$

Such an equation is equivalent, at a first level of approximation, to replacing the gravitational constant G entering Eq. (21.42) by $G_{\text{eff}}(\varphi_0, \partial\varphi_0, \partial^2\varphi_0) \equiv G/Z_0$, where $Z_0 \equiv Z(\varphi_0, \partial\varphi_0, \partial^2\varphi_0)$. This has a screening effect if $Z_0 \gg 1$. Indeed, the replacement $G \to G_{\text{eff}}$ diminishes the strength of the

interaction potential due to φ exchange by a factor of $1/Z_0$. In addition, the range of this interaction is also affected: $m_0(\varphi_0) = \sqrt{4\pi G \partial^2 V(\varphi_0)/\partial \varphi_0^2} \to m_{0\text{eff}}(\varphi_0, \partial\varphi_0, \partial^2\varphi_0) = \sqrt{4\pi G_{\text{eff}} \partial^2 V(\varphi_0)/\partial \varphi_0^2} = Z_0^{-1/2} m_0$.

Screening mechanisms based on such non-linear derivative self-interactions are often referred to as being "Vainshtein-like" because a similar mechanism was first invoked in Ref. [78] as a conjectural way to ensure that the extra degrees of freedom associated with a massive (rather than massless) graviton become effectively weakly coupled to matter within a large domain around gravitational sources. Here, one is considering massive deformations of the massless spin-2 metric field of GR by a very small mass, possibly of cosmological scale: $m_g \sim \hbar H_0 \sim 10^{-33}$ eV. The construction of ghost-free potential terms for a spin-2 field has turned out to be a delicate matter [79]. The phenomenology of a very-low-mass graviton is still partly uncontrolled, both because of the unknown extent to which the Vainshtein screening is really active, and because of subtle constraints linked to an eventual UV completion of the theory beyond the unusually low energy scale where it becomes strongly coupled:

$$\Lambda_{\text{strong coupling}} \sim (M_{\text{Planck}} m_0^2)^{1/3} \sim 10^{-13} \left(\frac{m_0}{\hbar H_0} \right)^{2/3} \text{eV} \,. \tag{21.48}$$

The search for modified gravity theories incorporating an extra scalar degree of freedom potentially able to yield a Vainshtein-like screening led to writing down the following general class of tensor-scalar Lagrangian [80, 81]:

$$\begin{aligned}
L_{\text{tot}}[g_{\mu\nu}, \varphi, \psi] =& G_2(\varphi, X) - G_3(\varphi, X)\Box_g\varphi + G_4(\varphi, X)R \\
&+ G_{4X}(\varphi, X)[(\Box_g\varphi)^2 - \varphi^{\mu\nu}\varphi_{\mu\nu}] \\
&+ G_5(\varphi, X)G^{\mu\nu}\varphi_{\mu\nu} - \frac{1}{6}G_{5X}(\varphi, X)[(\Box_g\varphi)^3 \\
&- 3\Box_g\varphi\varphi^{\mu\nu} + 2\varphi_{\mu\nu}\varphi^{\mu\lambda}\varphi^\nu_\lambda] + L_{\text{matter}}[g_{\mu\nu}, \psi] \,.
\end{aligned} \tag{21.49}$$

Here $g_{\mu\nu}$ denotes the matter-coupled metric, $X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$, $\varphi_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\varphi$, $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu}$, and the various coefficients $G_n(\varphi, X)$ are arbitrary functions of two variables (with $G_{nX} \equiv \partial G_n/\partial X$). The field equations derived from the Lagrangian of Eq. (21.49) are only of second order in derivatives in spite of the non-linear structure of L_{tot}. This implies that the tensor-scalar theories defined by Eq. (21.49) feature three degrees of freedom, corresponding to a massless spin-2 excitation (GW) and a spin-0 excitation. Contrary to the simpler tensor-scalar theories of Eq. (21.40), it is found that the speed of propagation of GWs implied by Eq. (21.49) is generically different from the speed of light:

$$\frac{c_{\text{GW}}^2}{c^2} = \frac{G_4 - X(\ddot\varphi G_{5X} + G_{5\varphi})}{G_4 - 2XG_{4X} - X(H\dot\varphi G_{5X} - G_{5\varphi})} \,. \tag{21.50}$$

More general modified gravity models have been proposed (see, *e.g.* Refs. [82, 83]). Apart from the simplest of them, most of these models have a rather artificial flavor, and do not lead to convincing alternative explanations either of dark matter or of dark energy. In addition, many of them do not lead (contrary to GR) to mathematically "well-posed" evolution problems [84–86]. This entails a serious challenge to deriving strong-field predictions for such models. It has been argued that many of these (dark-energy motivated) models should be viewed as effective field theory (EFT) approximations that need some sort of UV completion at an unusually low frequency scale [87]. In spite of these shortcomings, such models are conceptually interesting because they

give examples of deviations for various predictions of GR, existing independently from each other, in various regimes. For instance, some special tensor-scalar models lead to black hole solutions modified by scalar-hair [88–90]. For other types of black holes with scalar-hair, see Ref. [91]. This shows the interest of phenomenologically testing, in a democratic and agnostic way, all conceivable deviations from GR.

Let us now turn to briefly presenting current experimental results of various phenomenological tests of the main GR predictions recalled in Section 21.2 above.

21.4 Experimental tests of the Equivalence Principle (*i.e.*, of the matter-gravity coupling)

21.4.1 *Tests of the constancy of constants*

Stringent limits on a possible time variation of the basic coupling constants have been obtained by analyzing a natural fission reactor phenomenon that took place at Oklo, Gabon, two billion years ago [92, 93]. These limits are at the 1×10^{-8} level for the fractional variation of the fine-structure constant $\alpha_{\rm em}$ [93], and at the 4×10^{-9} level for the fractional variation of the ratio $m_q/\Lambda_{\rm QCD}$ between the light quark masses and $\Lambda_{\rm QCD}$ [94]. The determination of the lifetime of Rhenium 187 from isotopic measurements of some meteorites dating back to the formation of the solar system (about 4.6 Gyr ago) yields comparably strong limits [95]. Measurements of absorption lines in astronomical spectra also give stringent limits on the variability of both $\alpha_{\rm em}$ and $\mu = m_p/m_e$ at cosmological redshifts, *e.g.*,

$$\Delta\alpha_{\rm em}/\alpha_{\rm em} = (1.2 \pm 1.7_{\rm stat} \pm 0.9_{\rm sys}) \times 10^{-6} \,, \tag{21.51}$$

at redshifts $z = 1.0$–2.4 [96], and

$$|\Delta\mu/\mu| < 4 \times 10^{-7} (95\% \text{ C.L.}) \,, \tag{21.52}$$

at a redshift $z = 0.88582$ [97]. There are also significant limits on the variation of $\alpha_{\rm em}$ and $\mu = m_p/m_e$ at redshift $z \sim 10^3$ from cosmic microwave background data, *e.g.*, $\Delta\alpha_{\rm em}/\alpha_{\rm em} = (3.6 \pm 3.7) \times 10^{-3}$ [98]. Direct laboratory limits (based on monitoring the frequency ratio of several different atomic clocks) on the present time variation of $\alpha_{\rm em}$, and $\mu = m_p/m_e$ have reached the levels [99, 100]

$$d\ln(\alpha_{\rm em})/dt = (1.8 \pm 2.5) \times 10^{-19} {\rm yr}^{-1},$$
$$d\ln(\mu)/dt = (-8 \pm 36) \times 10^{-18} {\rm yr}^{-1}. \tag{21.53}$$

There are also experimental limits on a possible dependence of coupling constants on the gravitational potential [99, 100].

Experimental limits on the present time variation of the gravitational constant, Eq. (21.33), have been derived from planetary ephemerides [101], lunar laser ranging [102], and binary-pulsar data [103, 104]. The most stringent limits come from lunar-laser-ranging data [102]:

$$\frac{\dot{G}_0}{G_0} = (7.1 \pm 7.6) \times 10^{-14} \ {\rm yr}^{-1} \,. \tag{21.54}$$

21.4.2 *Tests of the isotropy of space and of Local Lorentz invariance*

The highest precision tests of the isotropy of space have been performed by looking for possible quadrupolar shifts of nuclear energy levels [105]. The (null) results can be interpreted as testing the fact that the various pieces in the matter Lagrangian, Eq. (21.6), are indeed coupled to the same external metric $g_{\mu\nu}$ to the 10^{-29} level.

Stringent tests of possible violations of local Lorentz invariance in gravitational interactions have been obtained both from solar-system data [8] and pulsar data [106, 107]. For astrophysical constraints on possible Planck-scale violations of Lorentz invariance, see Ref. [108].

21.4.3 *Tests of the universality of free fall (weak, and strong equivalence principles)*

The universality of the acceleration of free fall has been verified, for laboratory bodies, both on the ground [109, 110] (at the 10^{-13} level), and in space [111, 112] (at the 10^{-15} level):

$$(\Delta a/a)_{\text{BeTi}} = (0.3 \pm 1.8) \times 10^{-13} \,;$$
$$(\Delta a/a)_{\text{BeAl}} = (-0.7 \pm 1.3) \times 10^{-13} \,;$$
$$(\Delta a/a)_{\text{TiPt}} = (-1.5 \pm 2.3(\text{stat}) \pm 1.5(\text{syst})) \times 10^{-15} \,. \tag{21.55}$$

The universality of free fall has also been verified when comparing the fall of classical and quantum objects (at the 6×10^{-9} level [113]), or of two quantum objects (at the $(0.3 \pm 5.4) \times 10^{-7}$ [114], and $(1 \pm 1.4) \times 10^{-9}$, levels [115]).

The universality of free fall of self-gravitating bodies (strong equivalence principle) has been verified in both the weak-gravity, and the strong-gravity regimes. The gravitational accelerations of the Earth and the Moon toward the Sun have been checked to agree at the 10^{-13} level [102]

$$(\Delta a/a)_{\text{EarthMoon}} = (-3 \pm 5) \times 10^{-14}. \tag{21.56}$$

The latter result constrains the Nordtvedt PPN parameter [9] $\eta \equiv 4\overline{\beta} - \overline{\gamma}$ to the 10^{-4} level:

$$\eta = (-0.2 \pm 1.1) \times 10^{-4} \,. \tag{21.57}$$

See below for strong-field tests of the strong equivalence principle.

Finally, the universality of the gravitational redshift of clock rates has been verified at the 10^{-4} level by comparing a hydrogen-maser clock flying on a rocket up to an altitude of about $10,000$ km to a similar clock on the ground [116]. The redshift due to a height change of only 33 cm has been detected by comparing two optical clocks based on $^{27}\text{Al}^{+}$ ions [117]. The gravitational redshift has also been detected in the orbit of a star near the supermassive black hole at the center of our Galaxy [118, 119], and its universality has been verified at the 5% level [120].

21.5 Tests of quasi-stationary, weak-field gravity

All currently performed gravitational experiments in the solar system, including perihelion advances of planetary orbits, the bending and delay of electromagnetic signals passing near the Sun, and very accurate ranging data to the Moon obtained by laser echoes, are compatible with the post-Newtonian results of Eq. (21.15), Eq. (21.13), and Eq. (21.14). The "gravito-magnetic" interactions $\propto v_A v_B$ contained in Eq. (21.13) are involved in many of these experimental tests. They have been particularly tested in lunar-laser-ranging data [121], in the combined LAGEOS-LARES satellite data [122, 123], and in the dedicated Gravity Probe B mission [124].

To assess in a quantitative manner the results of the various solar-system tests of gravity it is convenient to express them in terms of the PPN parameters defined above. The best current limit on the post-Einstein parameter $\overline{\gamma} \equiv \gamma - 1$ is

$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5} \,, \tag{21.58}$$

as deduced from the additional Doppler shift experienced by radio-wave beams connecting the Earth to the Cassini spacecraft when they passed near the Sun [125].

The (cubic-vertex-related) post-Einstein parameter $\overline{\beta} \equiv \beta - 1$ is constrained at the 10^{-4} level both from a study of the global sensitivity of planetary ephemerides to post-Einstein parameters [101],

$$|\overline{\beta}| < 7 \times 10^{-5} \,, \tag{21.59}$$

and from lunar-laser-ranging data [102]

$$\overline{\beta} = (-4.5 \pm 5.6) \times 10^{-5} \ . \tag{21.60}$$

The periastron advance of the star S2 around the Galactic center massive black hole has been observed to agree with GR within 20% [126]. More stringent limits on $\overline{\gamma}$ (*i.e.* the coupling of φ to matter) are obtained in dilaton-like models where scalar couplings violate the Equivalence Principle [127].

21.6 Tests of strong-field gravity (neutron stars and black holes)
Experimental tests of strong-field gravity have been obtained in various physical systems, notably binary pulsars and coalescing binary black holes.

It is convenient to quantitatively express binary-pulsar tests of strong-field gravity by using the PPK formalism defined above. We recall that the measurement of N phenomenological PPK parameters leads to $N-2$ tests of strong-field gravity. In all, *thirteen* tests of strong-field and/or radiative gravity have been obtained in the four different (double neutron-star) binary pulsar systems PSR1913+16 [10, 11, 128], PSR1534+12 [129–131], PSR J1141−6545 [132–135], and PSR J0737−3039 A,B [136–140]. These consist of $N-2 = 5-2 = 3$ tests from PSR1913+16 ; $5-2 = 3$ tests from PSR1534+12; $4-2 = 2$ tests from PSR J1141−6545; and $7-2 = 5$ tests from PSR J0737−3039 (see, also, Ref. [141] for additional, less accurate tests of relativistic gravity). Among these tests, four of them (those involving the measurement of the PPK parameter $\dot{P}_b$) probe radiative effects, and will be discussed in the following section. The four binary pulsar systems PSR1913+16, PSR1534+12, PSR J1141−6545, and PSR J0737−3039 A,B have given nine tests of quasi-static, strong-field gravity. GR passes all these tests within the measurement accuracy. Let us only highlight here some of the most accurate strong-field tests.

In the binary pulsar PSR 1534+12 [129] one has measured *five* post-Keplerian parameters: k, γ_{timing}, r, s, and (with less accuracy) $\dot{P}_b$ [130, 131]. This yields *three* tests of relativistic gravity. Among these tests, the two involving the measurements of k, γ_{timing}, r, and s accurately probe strong field gravity, without mixing of radiative effects [130]. The most precise (10^{-3} level) of these pure strong-field tests is the one obtained by combining the measurements of k, γ_{timing}, and s; namely, [131],

$$\left[\frac{s^{\text{obs}}}{s^{\text{GR}}[k^{\text{obs}}, \gamma_{\text{timing}}^{\text{obs}}]}\right]_{1534+12} = 1.002 \pm 0.002 \ . \tag{21.61}$$

The discovery of the remarkable *double* binary pulsar PSR J0737−3039 A and B [136, 137] has led to the measurement of *seven* independent parameters [138–140]: five of them are the post-Keplerian parameters k, γ_{timing}, r, s, and $\dot{P}_b$ entering the relativistic timing formula of the fast-spinning pulsar PSR J0737−3039 A; a sixth is the ratio $R = x_B/x_A$ between the projected semi-major axis of the more slowly spinning companion pulsar PSR J0737−3039 B, and that of PSR J0737−3039 A (the theoretical prediction for the ratio $R = x_B/x_A$, considered as a function of the (inertial) masses $m_1 = m_A$ and $m_2 = m_B$, is $R^{\text{theory}} = m_1/m_2 + O((v/c)^4)$ [13, 14], independently of the gravitational theory considered). Finally, the seventh parameter $\Omega_{\text{SO,B}}$ is the angular rate of (spin-orbit) precession of PSR J0737−3039 B around the total angular momentum vector [139, 140]. These seven measurements give us *five* tests of relativistic gravity [138, 142, 143], four of which are quasi-static, strong-field tests. GR passes all those tests with flying colors [144]. The most accurate is at the 2×10^{-4} level:

$$\left[\frac{s^{\text{obs}}}{s^{\text{GR}}[k^{\text{obs}}, \gamma_{\text{timing}}{}^{\text{obs}}]}\right]_{0737-3039} = 1.00009 \pm 0.00018 \ . \tag{21.62}$$

Binary pulsar data on other types of pulsar systems can be used to test strong-field aspects of the "strong equivalence principle," namely the GR prediction that strong-self-gravity objects (such as neutron stars) should fall with the same acceleration as weak-self-gravity objects (such as white-dwarfs) in the (external) gravitational field created by other objects (such as the Galaxy, or another white dwarf). The first binary-pulsar tests of this property have been obtained in nearly circular binary systems (made of a neutron star and a white dwarf) falling in the field of the Galaxy, and have led to strong-field confirmations (at the 2×10^{-3} level) of the strong equivalence principle [104, 145–147]. The remarkable discovery of the pulsar PSR J0337+1715 in a hierarchical triple system [148] has allowed one to derive a much more accurate test of the strong equivalence principle because the inner binary (comprising a pulsar and a close white-dwarf companion) falls toward the outer white-dwarf companion with an acceleration that is 10^8 times larger than the Galactic acceleration. This leads to a 95% confidence level limit on a possible fractional difference in free-fall acceleration of the pulsar and its close companion of [149, 150]

$$|\Delta a/a| < 2.05 \times 10^{-6} \ (95\% \ \text{C.L.}) . \tag{21.63}$$

This limit yields strong constraints on tensor-scalar gravity models.

Measurements over several years of the pulse profiles of various pulsars have detected secular changes compatible with the prediction [151] that the general relativistic spin-orbit coupling should cause a secular change in the orientation of the pulsar beam with respect to the line of sight ("geodetic precession"). Such confirmations of general-relativistic spin-orbit effects were obtained in PSR 1913+16 [152], PSR B1534+12 [131], PSR J1141−6545 [153], PSR J0737−3039 [139, 140], and PSR J1906+0746 [154, 155]. In some cases (notably PSR 1913+16 and PSR J1906+0746) the secular change in the orientation of the pulsar beam is expected to lead to the disappearance of the beam (as seen on the Earth) on a human time scale (the second pulsar in the double system PSR J0737−3039 already disappeared in March 2008 and is expected to reappear around 2035 [140]).

Recently, the ultimate strong-field regime of black holes has started to be quantitatively probed via GW observations. The LIGO-Virgo(-Kagra) collaboration has detected (starting in September 2015) GW signals [35], which, besides testing the radiative structure of gravity (see next section), are in excellent qualitative and quantitative agreement with the structure and dynamics of black-hole horizons in GR. Because of the mixing of strong-field effects with radiative effects during the coalescence of two black holes, and because of the lack of detailed alternative-theory predictions for this process (see, however, Refs. [46–48]), it is not easy to set quantitative limits on possible strong-field deviations from GR, independently of radiative effects. Direct tests of the existence of black-hole horizons are scarce (see, however, Sec. VIIIB of [156] which reports the lack of any statistical evidence for GW echoes).

Let us also mention that the Event Horizon Telescope collaboration has obtained event-horizon-scale images of the supermassive black hole candidates in the center of the giant elliptical galaxy M87, and in the center of our Galaxy. These images are consistent with GR models of accreting Kerr black holes [157, 158]. For discussions of the corresponding constraints on the black hole geometry in the vicinity of the light ring see Refs. [159–161].

21.7 Tests of radiative gravity (both in binary-pulsar data and in GW data)

Experimental confirmations of the GR predictions for the radiative structure of gravity have been obtained both in binary-pulsar data and in the observation of GW signals from coalescing compact binaries (binary black holes and binary neutron stars).

Binary-pulsar observations involving the measurement of the orbital period derivative $\dot{P}_b$ give *direct* experimental tests of the reality of gravitational radiation, and, in particular, an experimental confirmation that the speed of propagation of gravity c_g is equal to the speed of light c (indeed, as

recalled above, $\dot{P}_b$ is a consequence of the propagation of the gravitational interaction between the two neutron stars [12]). Even in the presence of screening mechanisms within the binary system, the value of $\dot{P}_b$ yields a measurement of the speed of propagation of GWs at the 10^{-2} level [162]. The currently most accurate binary-pulsar tests of the radiative properties of gravity come from the binary neutron-star systems PSR1913+16 and PSR J0737−3039 A,B, as well as from several neutron-star-white-dwarf systems, notably PSR J1738+0333.

After subtracting a small ($\sim 10^{-14}$ level in $\dot{P}_b^{\rm obs} = (-2.423 \pm 0.001) \times 10^{-12}$), but significant, "Galactic" perturbing effect (linked to Galactic accelerations and to the pulsar proper motion) [163], one finds that the phenomenological test obtained by combining the measurements of the three PPK parameters $(k - \gamma_{\rm timing} - \dot{P}_b)_{1913+16}$ is passed by GR with complete success [128]:

$$\left[\frac{\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal}}{\dot{P}_b^{\rm GR}[k^{\rm obs}, \gamma_{\rm timing}^{\rm obs}]} \right]_{1913+16} = 0.9983 \pm 0.0016 \,. \tag{21.64}$$

Here $\dot{P}_b^{\rm GR}[k^{\rm obs}, \gamma_{\rm timing}^{\rm obs}]$ is the result of inserting in $\dot{P}_b^{\rm GR}(m_1, m_2)$ the values of the masses predicted by the two equations $k^{\rm obs} = k^{\rm GR}(m_1, m_2)$, and $\gamma_{\rm timing}^{\rm obs} = \gamma_{\rm timing}^{\rm GR}(m_1, m_2)$. This yields experimental evidence for the reality of gravitational radiation damping forces at the $(-1.7 \pm 1.6) \times 10^{-3}$ level.

An even better experimental test of the radiative structure of gravity (6×10^{-5} level) has been recently obtained from the combined measurement in PSR J0737−3039 A,B of the three parameters k, s, and $\dot{P}_b$ [144]:

$$\left[\frac{\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal}}{\dot{P}_b^{\rm GR}[k^{\rm obs}, s^{\rm obs}]} \right]_{0737-3039} = 0.999963 \pm 0.000063 \,. \tag{21.65}$$

In addition to the above tests, further very stringent tests of radiative gravity follow from the measurement of the orbital period decay $\dot{P}_b$ of low-eccentricity pulsar-white dwarf systems. Notably, the system PSR J1738+0333 yields an intrinsic orbital decay of [164]

$$\left[\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal} \right]_{1738+0333} = (-25.9 \pm 3.2) \times 10^{-15} \,, \tag{21.66}$$

to be compared to

$$\left[\dot{P}_b^{\rm GR} \right]_{1738+0333} = (-27.7_{-1.9}^{+1.5}) \times 10^{-15} \,. \tag{21.67}$$

The fractional agreement between the (corrected) observed period decay and the GR-predicted one seems to be quantitatively less impressive than the double-neutron-star results cited above, but the crucial point is that asymmetric binary systems (such as neutron-star-white-dwarf ones) are strong emitters of dipolar gravitational radiation in tensor-scalar theories, with $\dot{P}_b$ scaling (modulo matter-scalar couplings) like $m_1 m_2/(m_1 + m_2)^2 (v/c)^3$, instead of the parametrically smaller GR-predicted quadrupolar radiation $\dot{P}_b \sim (v/c)^5$ [7, 15]. In view of the very small absolute value of $\dot{P}_b$, this makes such systems (and notably PSR J1738+0333) very sensitive probes of tensor-scalar gravity [103, 164–168]. It is then useful to turn to a theory-dependent analysis of pulsar data. Such an analysis (see, *e.g.*, [17, 130, 164, 167, 168]) leads to excluding a large portion of the parameter space of tensor-scalar gravity allowed by solar-system tests. As a result, the basic matter-scalar coupling α_0^2 is more strongly constrained, over most of the parameter space, than the best current solar-system limits of Eq. (21.58) (namely below the 10^{-5} level) [164, 167].

We now turn to the tests of radiative gravity that can be deduced from the GW data gathered from the first three observing runs of the LIGO-Virgo-Kagra collaboration. All currently detected GW signals are consistent with GR predictions. Several phenomenological approaches were used and led to setting limits on possible deviations from GR [51, 156, 169, 170].

A theory-agnostic quantitative assessment on possible deviations from GR is given by measuring the agreement between the full observed GW signal of coalescing binary black holes, and the GR-predicted one. One of the strongest results was obtained with the first event: GW150914, which had an SNR of 24. The noise-weighted correlation between the GW150914 signal and the best-fit GR-predicted waveform was found to be $\geq 97\%$ [51, 169]. In other words, GR-violation effects that cannot be reabsorbed in a redefinition of physical parameters are limited (in a noise-weighted sense) to less than 3%.

Besides checking the agreement between the *full* observed GW signals and the corresponding best-fit full signals predicted by GR, one also tested the consistency between separate parts of the signals. A first approach [156, 169, 170] separates: (i) the lower-frequency signal emitted during the *inspiral* phase (considered up to the innermost stable circular orbit); and (ii) the higher-frequency remaining signal emitted during the *postinspiral* phase, comprising the late-inspiral, the merger, and the ringdown. Separately fitting each of these partial signals to GR-based templates then yields separate estimates of the binary's parameters, leading to separate estimates of the mass M_f and dimensionless spin parameter $a_f = J_f/(GM_f^2)$ of the final black hole that would be formed (in GR) by the coalescence of the two initial black holes. The consistency with GR then consists in testing whether the two estimates $(M_f, a_f)_{\text{insp}}$ and $(M_f, a_f)_{\text{postinsp}}$ are compatible with each other. They were found to be compatible for all events whose corresponding separate SNRs made such an analysis meaningful (see Figs. 3 and 4 in [170] and Fig.3 in [156]). Quantitatively, the (population-marginalized) fractional differences

$$\frac{\Delta M_f}{\bar{M}_f} = 2\frac{M_f^{\text{insp}} - M_f^{\text{postinsp}}}{M_f^{\text{insp}} + M_f^{\text{postinsp}}},$$

$$\frac{\Delta a_f}{\bar{a}_f} = 2\frac{a_f^{\text{insp}} - a_f^{\text{postinsp}}}{a_f^{\text{insp}} + a_f^{\text{postinsp}}}, \tag{21.68}$$

between the two estimates were found to be consistent with zero (*i.e.* with GR) [156]

$$\frac{\Delta M_f}{\bar{M}_f} = -0.02_{-0.06}^{+0.07},$$

$$\frac{\Delta a_f}{\bar{a}_f} = -0.06_{-0.07}^{+0.10}. \tag{21.69}$$

A second approach studies the consistency of the ringdown GW signal (emitted by the final, vibrating black hole) with GR predictions. See, in particular, section VIIIA, and Figs. 13 and 14, of [156]. The results are rather sensitive to various data analysis assumptions (notably the use of complete waveform models versus an analysis using only the ringdown signal). Though there is no clear sign of any deviations from GR, the current SNRs do not yield strong theory-agnostic evidence for the presence of ringing overtones.

The parametrization of Eq. (21.38) for possible deviations in the frequency dependence of the Fourier-domain phase $\psi(f)$ of the black hole coalescence GW signal was used to measure best-fit values for each fractional deviation parameter $\delta\hat{p}_i$, considered separately (the other ones being set to zero). In all cases, the posterior distribution for each $\delta\hat{p}_i$ is consistent with the GR value, *i.e.*, $\delta\hat{p}_i^{\text{GR}} = 0$ (see Fig. 6 in [156]). The current limits on $\delta\hat{p}_i$ are (roughly) of order unity, except for the two parameters highlighted above: $\delta\hat{p}_3$ (parameterizing the $O((\frac{v}{c})^3)$ fractional correction to the LO, quadrupolar term); and $\delta\hat{p}_{-2}$ (parameterizing a possible dipolar-radiation-related $O((\frac{v}{c})^{-2})$ fractional correction to the LO, quadrupolar term). Ref. [170] gives 90%-credible intervals for $\delta\hat{p}_3$ of $-0.02_{-0.10}^{+0.11}$ (when using a phenomenological model [53]), and $-0.01_{-0.11}^{+0.10}$ (when using an effective

one-body model [38]). Ref. [156] gives a 90%-credible upper bound for a possible dipolar term of $|\delta\hat{p}_{-2}| \leq 7.3 \times 10^{-4}$.

As recalled above, GR predicts that the polarization content of GWs is pure helicity-2, *i.e.* described by the two independent components of a traceless tensor transverse to the propagation direction. A (massless) scalar excitation would add a pure-trace "breathing mode" in the plane transverse to the propagation direction. A phenomenological approach to generic metric theories of gravity would allow for up to six polarizations for a GW [171], namely two tensor, two vector and two scalar modes. The LVK collaboration tested possible polarization deviations from GR in various ways [33,156,169]. Recent results are given in section VII of [156]. Though pure-scalar, pure-vector and vector-scalar hypotheses are significantly disfavored, any mixed hypothesis involving tensor modes (i.e., tensor-scalar, tensor-vector, and tensor-vector-scalar) cannot be ruled out conclusively.

GR also predicts that GWs are non dispersive, and propagate at the same speed as light. One can phenomenologically modify the GR-predicted GW phase evolution by adding the putative effect of an anomalous dispersion relation of the form $E^2 = p^2c^2 + Ap^\alpha c^\alpha$. GW data have been used to set bounds on the anomalous coefficient A for various values of the exponent α. The case $\alpha = 0$ is equivalent to assuming that gravitons disperse as a massive particle [59]. The current (90%-credibility) phenomenological GW limit on the graviton mass is $m_g \leq 1.27 \times 10^{-23}\text{eV}/c^2$ [156]. This is 2.5 times more stringent than the Solar System bound of $3.16 \times 10^{-23}\text{eV}/c^2$ [65].

Finally, a very constraining bound on the speed of propagation of gravity c_{GW} was derived from the observed time delay of 1.7 s between GW170817 and the associated γ-ray burst. Namely, the fractional difference between c_{GW} and $c_{\text{light}} \equiv c$ is constrained to be [172]

$$-3 \times 10^{-15} < \frac{c_{\text{GW}} - c}{c} < +7 \times 10^{-16}. \tag{21.70}$$

When comparing the latter bound to the prediction Eq. (21.50) from general second-order tensor-scalar theories, Eq. (21.49), one is led to conclude that the coupling function $G_5(\varphi, X)$ has to be ignored and that the coupling function $G_4(\varphi, X)$ has to be restricted to depend only on φ. This drastically reduces the viable tensor-scalar modified-gravity models [173–176].

Let us finally mention that four pulsar timing arrays have recently given tantalizing evidence for the existence of a stochastic background of gravitational waves with frequency $\sim 10^{-9}$ Hz [177–180].

21.8 Conclusions

All present experimental tests are compatible with the predictions of the current "standard" theory of gravitation, Einstein's General Relativity. Let us recap the main tests. The universality of the coupling between matter and gravity (Equivalence Principle) has been verified at around the 10^{-15} level. Solar system experiments have tested the weak-field predictions of Einstein's theory at the few times 10^{-5} level. The propagation properties (in the near zone) of relativistic gravity, as well as several of its static strong-field aspects, have been verified at the 10^{-4} level (or better) in several binary pulsar experiments. Interferometric detectors of gravitational radiation have given direct observational proofs of the existence, and properties, of gravitational waves (in the wave zone), and of the existence of coalescing black holes, and they have already set strong limits on possible deviations; in particular: an upper bound $|\delta\hat{p}_{-2}| < 7.3 \times 10^{-4}$ on a possible dipolar contribution to the GW flux; the $O(10^{-15})$ bound of Eq. (21.70) on the speed of gravity; and confirmation of the tensor polarization structure of gravitational waves. In addition, laboratory experiments have set strong constraints on sub-millimeter modifications of Newtonian gravity, while many different cosmological data sets have been used to set limits on possible GR deviations on cosmological scales [29]. In spite of the uneasiness of having to assume the existence of dark matter, and the presence of an unnaturally small cosmological constant (as dark energy), General Relativity stands

out as a uniquely successful description of gravity on all the scales that have been explored so far. There are no modified-gravity models which naturally pass all existing experimental tests, while either explaining away the need for dark matter or for dark energy.

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