

# $V_{cb}$ and $V_{ub}$ CKM Matrix Elements

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See the related review(s):

Semileptonic  $B$  Hadron Decays, Determination of  $V_{cb}$  and  $V_{ub}$

## $V_{cb}$ MEASUREMENTS

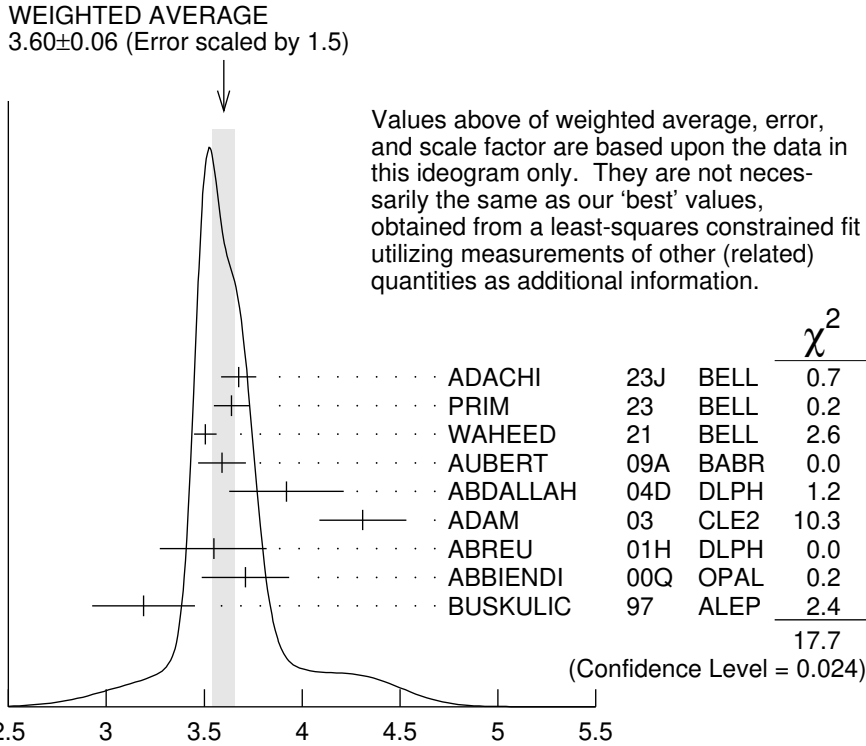
For the discussion of  $V_{cb}$  measurements, which is not repeated here, see the review on “Determination of  $|V_{cb}|$  and  $|V_{ub}|$ .”

The CKM matrix element  $|V_{cb}|$  can be determined by studying the rate of the semileptonic decay  $B \rightarrow D^{(*)} \ell \nu$  as a function of the recoil kinematics of  $D^{(*)}$  mesons. Taking advantage of theoretical constraints on the normalization and a linear  $\omega$  dependence of the form factors ( $F(\omega)$ ,  $G(\omega)$ ) provided by Heavy Quark Effective Theory (HQET), the  $|V_{cb}| \times F(\omega)$  and  $\rho^2$  can be simultaneously extracted from data, where  $\omega$  is the scalar product of the two-meson four velocities,  $F(1)$  is the form factor at zero recoil ( $\omega=1$ ) and  $\rho^2$  is the slope. Using the theoretical input of  $F(1)$ , a value of  $|V_{cb}|$  can be obtained.

### $|V_{cb}| \times F(1)$ (from $B^0 \rightarrow D^{*-} \ell^+ \nu$ )

VALUE (units $10^{-2}$ )	DOCUMENT ID	TECN	COMMENT
<b>3.534 ± 0.037 OUR EVALUATION</b>	(Produced by HFLAV) with $\rho^2=1.139 \pm 0.020$ and a correlation 0.268. The fitted $\chi^2$ is 63.2 for 27 degrees of freedom.		
<b>3.60 ± 0.06 OUR AVERAGE</b>	Error includes scale factor of 1.5. See the ideogram below.		
3.676 ± 0.028 ± 0.086	<sup>1</sup> ADACHI	23J BELL	$e^+ e^- \rightarrow \Upsilon(4S)$
3.64 ± 0.09	<sup>2</sup> PRIM	23 BELL	$e^+ e^- \rightarrow \Upsilon(4S)$
3.506 ± 0.015 ± 0.056	<sup>3</sup> WAHEED	21 BELL	$e^+ e^- \rightarrow \Upsilon(4S)$
3.59 ± 0.02 ± 0.12	<sup>4</sup> AUBERT	09A BABR	$e^+ e^- \rightarrow \Upsilon(4S)$
3.92 ± 0.18 ± 0.23	<sup>5</sup> ABDALLAH	04D DLPH	$e^+ e^- \rightarrow Z^0$
4.31 ± 0.13 ± 0.18	<sup>6</sup> ADAM	03 CLE2	$e^+ e^- \rightarrow \Upsilon(4S)$
3.55 ± 0.14 <sup>+0.23</sup> / <sub>-0.24</sub>	<sup>7</sup> ABREU	01H DLPH	$e^+ e^- \rightarrow Z$
3.71 ± 0.10 ± 0.20	<sup>8</sup> ABBIENDI	00Q OPAL	$e^+ e^- \rightarrow Z$
3.19 ± 0.18 ± 0.19	<sup>9</sup> BUSKULIC	97 ALEP	$e^+ e^- \rightarrow Z$
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
3.483 ± 0.015 ± 0.056	<sup>3</sup> WAHEED	19 BELL	Repl. by WAHEED 21
3.46 ± 0.02 ± 0.10	<sup>10</sup> DUNGEL	10 BELL	Repl. by WAHEED 19
3.59 ± 0.06 ± 0.14	<sup>11</sup> AUBERT	08AT BABR	Repl. by AUBERT 09A
3.44 ± 0.03 ± 0.11	<sup>12</sup> AUBERT	08R BABR	Repl. by AUBERT 09A
3.55 ± 0.03 ± 0.16	<sup>13</sup> AUBERT	05E BABR	Repl. by AUBERT 08R
3.77 ± 0.11 ± 0.19	<sup>14</sup> ABDALLAH	04D DLPH	$e^+ e^- \rightarrow Z^0$
3.54 ± 0.19 ± 0.18	<sup>15</sup> ABE	02F BELL	Repl. by DUNGEL 10
4.31 ± 0.13 ± 0.18	<sup>16</sup> BRIERE	02 CLE2	$e^+ e^- \rightarrow \Upsilon(4S)$
3.28 ± 0.19 ± 0.22	ACKERSTAFF	97G OPAL	Repl. by ABBIENDI 00Q

3.50 ±0.19 ±0.23	17 ABREU	96P DLPH	Repl. by ABREU 01H
3.51 ±0.19 ±0.20	18 BARISH	95 CLE2	Repl. by ADAM 03
3.14 ±0.23 ±0.25	BUSKULIC	95N ALEP	Repl. by BUSKULIC 97



$$|V_{cb}| \times F(1) \text{ (from } B^0 \rightarrow D^{*-} \ell^+ \nu \text{)}$$

- <sup>1</sup> Measured from differential shapes of exclusive  $B \rightarrow D^* \ell^- \nu_\ell$  ( $\ell = e$  or  $\mu$ ) decays. Using CNL form factor parametrization and the zero-recoil lattice QCD point  $F(1) = 0.906 \pm 0.013$  ADACHI 23J finds  $|V_{cb}|_{CNL} = (40.57 \pm 0.31 \pm 0.95 \pm 0.58) \times 10^{-3}$  where the last uncertainty is due to the prediction of  $F(1)$ . Also reports a measurement of  $|V_{cb}|_{BGL} = (40.13 \pm 0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$  using BGL form factors parametrization.
- <sup>2</sup> Measured from differential shapes of exclusive  $B \rightarrow D^* \ell^- \nu_\ell$  decays with hadronic tag-side reconstruction and extracting the CNL and BGL form factor parameters. PRIM 23 finds  $|V_{cb}|_{CNL} = (40.2 \pm 0.9) \times 10^{-3}$  with the zero-recoil lattice QCD point  $F(1) = 0.906 \pm 0.013$ . PRIM 23 provides also a measurement of  $|V_{cb}|_{BGL} = (40.7 \pm 1.0) \times 10^{-3}$ .
- <sup>3</sup> WAHEED 21 uses fully reconstructed  $D^{*-} \ell^+ \nu$  events ( $\ell = e$  or  $\mu$ ) and  $\eta_{EW} = 1.0066$ .
- <sup>4</sup> Obtained from a global fit to  $B \rightarrow D^{(*)} \ell \nu_\ell$  events, with reconstructed  $D^0 \ell$  and  $D^+ \ell$  final states and  $\rho^2 = 1.22 \pm 0.02 \pm 0.07$ .
- <sup>5</sup> Measurement using fully reconstructed  $D^*$  sample with a  $\rho^2 = 1.32 \pm 0.15 \pm 0.33$ .
- <sup>6</sup> Average of the  $B^0 \rightarrow D^*(2010)^- \ell^+ \nu$  and  $B^+ \rightarrow \bar{D}^*(2007) \ell^+ \nu$  modes with  $\rho^2 = 1.61 \pm 0.09 \pm 0.21$  and  $f_{+-} = 0.521 \pm 0.012$ .
- <sup>7</sup> ABREU 01H measured using about 5000 partial reconstructed  $D^*$  sample with a  $\rho^2 = 1.34 \pm 0.14^{+0.24}_{-0.22}$ .
- <sup>8</sup> ABBIENDI 00Q: measured using both inclusively and exclusively reconstructed  $D^{*\pm}$  samples with a  $\rho^2 = 1.21 \pm 0.12 \pm 0.20$ . The statistical and systematic correlations between  $|V_{cb}| \times F(1)$  and  $\rho^2$  are 0.90 and 0.54 respectively.

- <sup>9</sup> BUSKULIC 97: measured using exclusively reconstructed  $D^{*\pm}$  with a  $a^2=0.31 \pm 0.17 \pm 0.08$ . The statistical correlation is 0.92.
- <sup>10</sup> Uses fully reconstructed  $D^{*-} \ell^+ \nu$  events ( $\ell = e$  or  $\mu$ ).
- <sup>11</sup> Measured using the dependence of  $B^- \rightarrow D^{*0} e^- \bar{\nu}_e$  decay differential rate and the form factor description by CAPRINI 98 with  $\rho^2 = 1.16 \pm 0.06 \pm 0.08$ .
- <sup>12</sup> Measured using fully reconstructed  $D^*$  sample and a simultaneous fit to the Caprini-Lellouch-Neubert form factor parameters:  $\rho^2 = 1.191 \pm 0.048 \pm 0.028$ ,  $R_1(1) = 1.429 \pm 0.061 \pm 0.044$ , and  $R_2(1) = 0.827 \pm 0.038 \pm 0.022$ .
- <sup>13</sup> Measurement using fully reconstructed  $D^*$  sample with a  $\rho^2 = 1.29 \pm 0.03 \pm 0.27$ .
- <sup>14</sup> Combines with previous partial reconstructed  $D^*$  measurement with a  $\rho^2 = 1.39 \pm 0.10 \pm 0.33$ .
- <sup>15</sup> Measured using exclusive  $B^0 \rightarrow D^*(892)^- e^+ \nu$  decays with  $\rho^2 = 1.35 \pm 0.17 \pm 0.19$  and a correlation of 0.91.
- <sup>16</sup> BRIERE 02 result is based on the same analysis and data sample reported in ADAM 03.
- <sup>17</sup> ABREU 96P: measured using both inclusively and exclusively reconstructed  $D^{*\pm}$  samples.
- <sup>18</sup> BARISH 95: measured using both exclusive reconstructed  $B^0 \rightarrow D^{*-} \ell^+ \nu$  and  $B^+ \rightarrow D^{*0} \ell^+ \nu$  samples. They report their experiment's uncertainties  $\pm 0.0019 \pm 0.0018 \pm 0.0008$ , where the first error is statistical, the second is systematic, and the third is the uncertainty in the lifetimes. We combine the last two in quadrature.

### $|V_{cb}| \times G(1)$ (from $B \rightarrow D^- \ell^+ \nu$ )

VALUE (units  $10^{-2}$ )      DOCUMENT ID      TECN      COMMENT

**4.121 ± 0.100 OUR EVALUATION** (Produced by HFLAV) with  $\rho^2 = 1.128 \pm 0.033$  and a correlation 0.747. The fitted  $\chi^2$  is 4.8 for 8 degrees of freedom.

#### **4.22 ± 0.10 OUR AVERAGE**

4.229 ± 0.137	<sup>1</sup> GLATTAUER	16	BELL	$e^+ e^- \rightarrow \Upsilon(4S)$
4.23 ± 0.19 ± 0.14	<sup>2</sup> AUBERT	10	BABR	$e^+ e^- \rightarrow \Upsilon(4S)$
4.31 ± 0.08 ± 0.23	<sup>3</sup> AUBERT	09A	BABR	$e^+ e^- \rightarrow \Upsilon(4S)$
4.16 ± 0.47 ± 0.37	<sup>4</sup> BARTELT	99	CLE2	$e^+ e^- \rightarrow \Upsilon(4S)$
2.78 ± 0.68 ± 0.65	<sup>5</sup> BUSKULIC	97	ALEP	$e^+ e^- \rightarrow Z$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
4.11 ± 0.44 ± 0.52	<sup>6</sup> ABE	02E	BELL	Repl. by GLATTAUER 16
3.37 ± 0.44 $\begin{smallmatrix} +0.72 \\ -0.49 \end{smallmatrix}$	<sup>7</sup> ATHANAS	97	CLE2	Repl. by BARTELT 99

- <sup>1</sup> Obtained from a fit to the combined partially reconstructed  $B \rightarrow \bar{D} \ell \nu_\ell$  sample while tagged by the other fully reconstructed  $B$  meson in the event. Also reports fitted  $\rho^2 = 1.09 \pm 0.05$ .
- <sup>2</sup> Obtained from a fit to the combined  $B \rightarrow \bar{D} \ell^+ \nu_\ell$  sample in which a hadronic decay of the second  $B$  meson is fully reconstructed and  $\rho^2 = 1.20 \pm 0.09 \pm 0.04$ .
- <sup>3</sup> Obtained from a global fit to  $B \rightarrow D^{(*)} \ell \nu_\ell$  events, with reconstructed  $D^0 \ell$  and  $D^+ \ell$  final states and  $\rho^2 = 1.20 \pm 0.04 \pm 0.07$ .
- <sup>4</sup> BARTELT 99: measured using both exclusive reconstructed  $B^0 \rightarrow D^- \ell^+ \nu$  and  $B^+ \rightarrow D^0 \ell^+ \nu$  samples.
- <sup>5</sup> BUSKULIC 97: measured using exclusively reconstructed  $D^\pm$  with a  $a^2 = -0.05 \pm 0.53 \pm 0.38$ . The statistical correlation is 0.99.
- <sup>6</sup> Using the missing energy and momentum to extract kinematic information about the undetected neutrino in the  $B^0 \rightarrow D^- \ell^+ \nu$  decay.
- <sup>7</sup> ATHANAS 97: measured using both exclusive reconstructed  $B^0 \rightarrow D^- \ell^+ \nu$  and  $B^+ \rightarrow D^0 \ell^+ \nu$  samples with a  $\rho^2 = 0.59 \pm 0.22 \pm 0.12 \begin{smallmatrix} +0.59 \\ -0 \end{smallmatrix}$ . They report their experiment's uncertainties  $\pm 0.0044 \pm 0.0048 \begin{smallmatrix} +0.0053 \\ -0.0012 \end{smallmatrix}$ , where the first error is statistical, the second is systematic, and the third is the uncertainty due to the form factor model variations. We combine the last two in quadrature.

**$|V_{cb}|$  (from  $D_s^{*-} \mu^+ \nu_\mu$ )**

VALUE (units $10^{-3}$ )	DOCUMENT ID	TECN	COMMENT
<b><math>41.4 \pm 0.6 \pm 0.9 \pm 1.2</math></b>	<sup>1</sup> AAIJ	20E	LHCB $pp$ at 7, 8 TeV

<sup>1</sup> Measured from an inclusive sample of  $D_s^- \mu^+$  candidates using CNL parameterization of the form factor. AAIJ 20E provides also measurement of  $|V_{cb}| = (42.3 \pm 0.8 \pm 0.9 \pm 1.2) \times 10^{-3}$  using BGL parameterization of the form factor. The third uncertainty is due to the external inputs used in the measurement.

 **$V_{ub}$  MEASUREMENTS**

For the discussion of  $V_{ub}$  measurements, which is not repeated here, see the review on "Determination of  $|V_{cb}|$  and  $|V_{ub}|$ ."

The CKM matrix element  $|V_{ub}|$  can be determined by studying the rate of the charmless semileptonic decay  $b \rightarrow u \ell \nu$ . The relevant branching ratio measurements based on exclusive and inclusive decays can be found in the  $B$  Listings, and are not repeated here.

 **$V_{cb}$  and  $V_{ub}$  CKM Matrix Elements REFERENCES**

ADACHI	23J	PR D108 092013	I. Adachi <i>et al.</i>	(BELLE II Collab.)
PRIM	23	PR D108 012002	M.T. Prim <i>et al.</i>	(BELLE Collab.)
WAHEED	21	PR D103 079901	E. Waheed <i>et al.</i>	(BELLE Collab.)
AAIJ	20E	PR D101 072004	R. Aaij <i>et al.</i>	(LHCb Collab.)
WAHEED	19	PR D100 052007	E. Waheed <i>et al.</i>	(BELLE Collab.)
GLATTAUER	16	PR D93 032006	R. Glattauer <i>et al.</i>	(BELLE Collab.)
AUBERT	10	PRL 104 011802	B. Aubert <i>et al.</i>	(BABAR Collab.)
DUNGEL	10	PR D82 112007	W. Dungen <i>et al.</i>	(BELLE Collab.)
AUBERT	09A	PR D79 012002	B. Aubert <i>et al.</i>	(BABAR Collab.)
AUBERT	08AT	PRL 100 231803	B. Aubert <i>et al.</i>	(BABAR Collab.)
AUBERT	08R	PR D77 032002	B. Aubert <i>et al.</i>	(BABAR Collab.)
AUBERT	05E	PR D71 051502	B. Aubert <i>et al.</i>	(BABAR Collab.)
ABDALLAH	04D	EPJ C33 213	J. Abdallah <i>et al.</i>	(DELPHI Collab.)
ADAM	03	PR D67 032001	N.E. Adam <i>et al.</i>	(CLEO Collab.)
ABE	02E	PL B526 258	K. Abe <i>et al.</i>	(BELLE Collab.)
ABE	02F	PL B526 247	K. Abe <i>et al.</i>	(BELLE Collab.)
BRIERE	02	PRL 89 081803	R. Briere <i>et al.</i>	(CLEO Collab.)
ABREU	01H	PL B510 55	P. Abreu <i>et al.</i>	(DELPHI Collab.)
ABBIENDI	00Q	PL B482 15	G. Abbiendi <i>et al.</i>	(OPAL Collab.)
BARTELT	99	PRL 82 3746	J. Bartelt <i>et al.</i>	(CLEO Collab.)
CAPRINI	98	NP B530 153	I. Caprini, L. Lellouch, M. Neubert	(BCIP, CERN)
ACKERSTAFF	97G	PL B395 128	K. Ackerstaff <i>et al.</i>	(OPAL Collab.)
ATHANAS	97	PRL 79 2208	M. Athanas <i>et al.</i>	(CLEO Collab.)
BUSKULIC	97	PL B395 373	D. Buskulic <i>et al.</i>	(ALEPH Collab.)
ABREU	96P	ZPHY C71 539	P. Abreu <i>et al.</i>	(DELPHI Collab.)
BARISH	95	PR D51 1014	B.C. Barish <i>et al.</i>	(CLEO Collab.)
BUSKULIC	95N	PL B359 236	D. Buskulic <i>et al.</i>	(ALEPH Collab.)