

## 66. *CPT Invariance Tests in Neutral Kaon Decay*

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*CPT* theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in  $K^0 - \bar{K}^0$  system, described by the equation

$$i \frac{d}{dt} \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = [M - i\Gamma/2] \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix},$$

where  $M$  and  $\Gamma$  are hermitian matrices (see PDG review [1], references [2, 3], and KLOE paper [4] for notations and previous literature), allows a very accurate test of *CPT* symmetry; indeed since *CPT* requires  $M_{11} = M_{22}$  and  $\Gamma_{11} = \Gamma_{22}$ , the mass and width eigenstates,  $K_{S,L}$ , have a *CPT*-violating piece,  $\delta$ , in addition to the usual *CPT*-conserving parameter  $\epsilon$ :

$$\begin{aligned} K_{S,L} &= \frac{1}{\sqrt{2(1 + |\epsilon_{S,L}|^2)}} \left[ (1 + \epsilon_{S,L}) K^0 \pm (1 - \epsilon_{S,L}) \bar{K}^0 \right] \\ \epsilon_{S,L} &= \frac{-i\Im(M_{12}) - \frac{1}{2}\Im(\Gamma_{12}) \mp \frac{1}{2} \left[ M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22}) \right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2} \\ &\equiv \epsilon \pm \delta. \end{aligned} \tag{66.1}$$

Using the phase convention  $\Im(\Gamma_{12}) = 0$ , we determine the phase of  $\epsilon$  to be  $\varphi_{SW} \equiv \arctan \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$ . Imposing unitarity to an arbitrary combination of  $K^0$  and  $\bar{K}^0$  wave functions, we obtain the Bell-Steinberger relation [5] connecting *CP* and *CPT* violation in the mass matrix to *CP* and *CPT* violation in the decay; in fact, neglecting  $\mathcal{O}(\epsilon)$  corrections to the coefficient of the *CPT*-violating parameter,  $\delta$ , we can write [4]

$$\begin{aligned} \left[ \frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \right] \left[ \frac{\Re(\epsilon)}{1 + |\epsilon|^2} - i\Im(\delta) \right] &= \\ \frac{1}{\Gamma_S - \Gamma_L} \sum_f A_L(f) A_S^*(f), & \end{aligned} \tag{66.2}$$

where  $A_{L,S}(f) \equiv A(K_{L,S} \rightarrow f)$ . We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (66.2); in fact, defining for the hadronic modes

$$\begin{aligned} \alpha_i &\equiv \frac{1}{\Gamma_S} \langle \mathcal{A}_L(i) \mathcal{A}_S^*(i) \rangle = \eta_i \mathcal{B}(K_S \rightarrow i), \\ i &= \pi^0 \pi^0, \pi^+ \pi^- (\gamma), 3\pi^0, \pi^0 \pi^+ \pi^- (\gamma), \end{aligned} \tag{66.3}$$

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. [4] has been updated by using the recent measurements of  $K_L$

branching ratios from KTeV [6, 7], NA48 [8, 9], the results described in the *CP* violation in  $K_L$  decays minireview, and the KLOE result [10])

$$\begin{aligned}
\alpha_{\pi^+\pi^-} &= ((1.121 \pm 0.010) + i(1.061 \pm 0.010)) \times 10^{-3}, \\
\alpha_{\pi^0\pi^0} &= ((0.493 \pm 0.005) + i(0.471 \pm 0.005)) \times 10^{-3}, \\
\alpha_{\pi^+\pi^-\pi^0} &= ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6}, \\
|\alpha_{\pi^0\pi^0\pi^0}| &< 1.5 \times 10^{-6} \text{ at 95\% CL}.
\end{aligned} \tag{66.4}$$

The semileptonic contribution to the right-handed side of Eq. (66.2) requires the determination of several observables: we define [2, 3]

$$\begin{aligned}
\mathcal{A}(K^0 \rightarrow \pi^- l^+ \nu) &= \mathcal{A}_0(1 - y), \\
\mathcal{A}(K^0 \rightarrow \pi^+ l^- \nu) &= \mathcal{A}_0^*(1 + y^*)(x_+ - x_-)^*, \\
\mathcal{A}(\bar{K}^0 \rightarrow \pi^+ l^- \nu) &= \mathcal{A}_0^*(1 + y^*), \\
\mathcal{A}(\bar{K}^0 \rightarrow \pi^- l^+ \nu) &= \mathcal{A}_0(1 - y)(x_+ + x_-),
\end{aligned} \tag{66.5}$$

where  $x_+$  ( $x_-$ ) describes the violation of the  $\Delta S = \Delta Q$  rule in *CPT*-conserving (violating) decay amplitudes, and  $y$  parameterizes *CPT* violation for  $\Delta S = \Delta Q$  transitions. Taking advantage of their tagged  $K^0(\bar{K}^0)$  beams, CPLEAR has measured  $\Im(x_+)$ ,  $\Re(x_-)$ ,  $\Im(\delta)$ , and  $\Re(\delta)$  [11]. These determinations have been improved in Ref. [4] by including the information  $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$  (valid at first order in the small parameters), where  $A_{L,S}$  are the  $K_L$  and  $K_S$  semileptonic charge asymmetries, respectively, from the PDG [12] and the new KLOE semileptonic measurement [13]. Here we are also including the *T*-violating asymmetry measurement from CPLEAR [14] with a finer binning than appearing in the published article.

**Table 66.1:** Values, errors, and correlation coefficients for  $\Re(\delta)$ ,  $\Im(\delta)$ ,  $\Re(x_-)$ ,  $\Im(x_+)$ , and  $A_S + A_L$  obtained from a combined fit, including KLOE [4, 13] and CPLEAR [14].

	value	Correlations coefficients			
$\Re(\delta)$	$(4.3 \pm 2.7) \times 10^{-4}$	1			
$\Im(\delta)$	$(-0.9 \pm 0.6) \times 10^{-2}$	-0.40	1		
$\Re(x_-)$	$(-0.22 \pm 0.10) \times 10^{-2}$	-0.14	-0.30	1	
$\Im(x_+)$	$(0.06 \pm 0.19) \times 10^{-2}$	-0.12	-0.02	0.34	1
$A_S + A_L$	$(-0.23 \pm 0.38) \times 10^{-2}$	-0.12	-0.29	0.94	0.18

The value  $A_S + A_L$  in Table 66.1 can be directly included in the semileptonic contributions to the Bell Steinberger relations in Eq. (66.2)

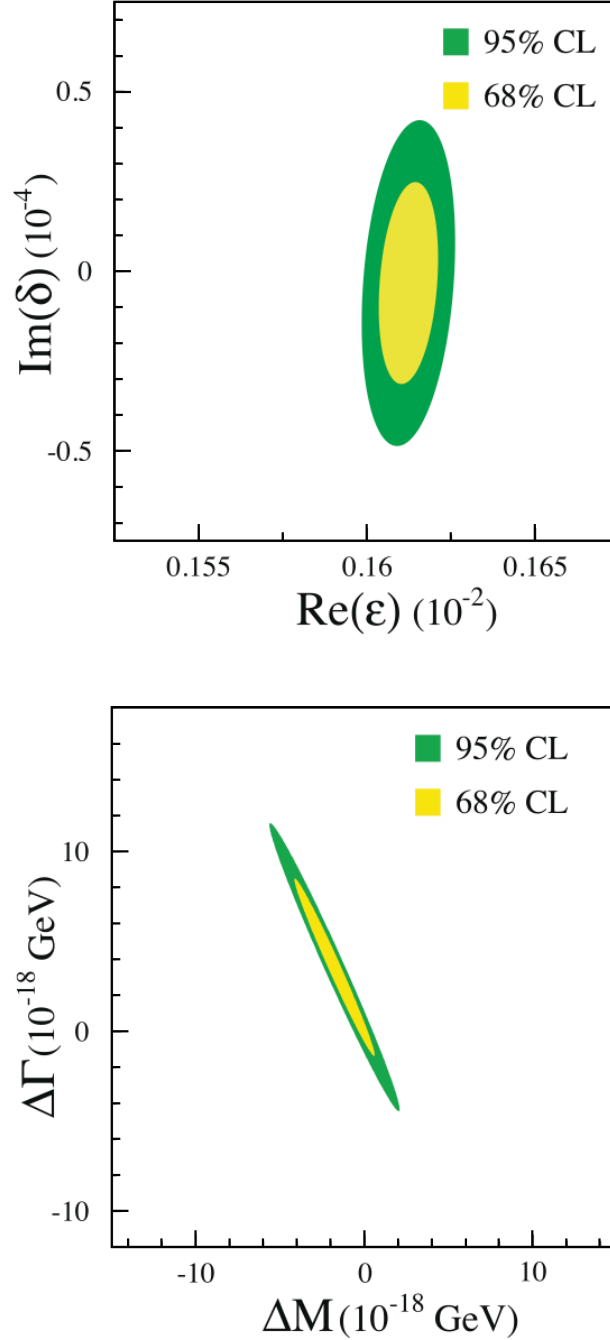
$$\begin{aligned}
&\sum_{\pi l \nu} \langle \mathcal{A}_L(\pi l \nu) \mathcal{A}_S^*(\pi l \nu) \rangle \\
&= 2\Gamma(K_L \rightarrow \pi l \nu)(\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))) \\
&= 2\Gamma(K_L \rightarrow \pi l \nu)((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))).
\end{aligned} \tag{66.6}$$

Defining

$$\alpha_{\pi l \nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi l \nu} \langle \mathcal{A}_L(\pi l \nu) \mathcal{A}_S^*(\pi l \nu) \rangle + 2i \frac{\tau_{K_S}}{\tau_{K_L}} \mathcal{B}(K_L \rightarrow \pi l \nu) \Im(\delta), \tag{66.7}$$

we find:

$$\alpha_{\pi l\nu} = ((-0.1 \pm 0.2) + i(-0.1 \pm 0.5)) \times 10^{-5}. \quad (66.8)$$



**Figure 66.1:** Top: allowed region at 68% and 95% C.L. in the  $\Re(\epsilon)$ ,  $\Im(\delta)$  plane. Bottom: allowed region at 68% and 95% C.L. in the  $\Delta M$ ,  $\Delta\Gamma$  plane.

The analysis of semileptonic decay asymmetries implicitly assumes Lepton Flavour Universality (LFU) of any effect violating CPT or  $\Delta S = \Delta Q$ , through the use of a single set of  $x$ ,  $y$  parameters,

**Table 66.2:** Summary of results: values, errors, and correlation coefficients for  $\Re(\epsilon)$ ,  $\Im(\delta)$ ,  $\Re(\delta)$ , and  $\Re(x_-)$ .

	value	Correlations coefficients			
$\Re(\epsilon)$	$(161.2 \pm 0.5) \times 10^{-5}$	+1			
$\Im(\delta)$	$(-0.3 \pm 1.4) \times 10^{-5}$	+0.08	1		
$\Re(\delta)$	$(2.6 \pm 2.5) \times 10^{-4}$	+0.00	-0.05	1	
$\Re(x_-)$	$(-2.7 \pm 1.0) \times 10^{-3}$	+0.05	0.13	-0.30	1

consistently with the availability of experimental information on  $A_S$  for the electron mode only. The explicit LFU assumption in the input BR measurements has been lifted exploiting the measurement of  $BR(K_S \rightarrow \pi\mu\nu)$  Ref. [15], with no effect on the numerical results.

Inserting the values of the  $\alpha$  parameters into Eq. (66.2), we find

$$\begin{aligned}\Re(\epsilon) &= (161.2 \pm 0.5) \times 10^{-5}, \\ \Im(\delta) &= (-0.3 \pm 1.4) \times 10^{-5}.\end{aligned}\tag{66.9}$$

The complete information on Eq. (66.9) is given in Table 66.2.

Now the agreement with *CPT* conservation,  $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$ , is at 18% C.L.

The allowed region in the  $\Re(\epsilon) - \Im(\delta)$  plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 66.1.

The process giving the largest contribution to the size of the allowed region is  $K_L \rightarrow \pi^+\pi^-$ , through the uncertainty on  $\phi_{+-}$ .

The limits on  $\Im(\delta)$  and  $\Re(\delta)$  can be used to constrain the  $K^0 - \bar{K}^0$  mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\bar{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{\Gamma_S - \Gamma_L} \cos \phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)].$$

The allowed region in the  $\Delta M = (m_{K^0} - m_{\bar{K}^0})$ ,  $\Delta\Gamma = (\Gamma_{K^0} - \Gamma_{\bar{K}^0})$  plane is shown in the bottom panel of Fig. 66.1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. [12]) and in the limit  $\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0$  we obtain

$$-4.0 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\bar{K}^0} < 4.0 \times 10^{-19} \text{ GeV} \quad \text{at 95 \% C.L.}$$

## References

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