# 13. *CP* Violation in the Quark Sector

Revised August 2021 by T. Gershon (Warwick U.) and Y. Nir (Weizmann Inst.).

The *CP* transformation combines charge conjugation $C$ with parity $P$. Under $C$, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, *e.g.*, $Q \to -Q$ for electromagnetic charge. Under $P$, the handedness of space is reversed, $\vec{x} \to -\vec{x}$. Thus, for example, a left-handed electron $e^-_L$ is transformed under *CP* into a right-handed positron, $e^+_R$.

If *CP* were an exact symmetry, the laws of nature would be the same for matter and for antimatter. We observe that most phenomena are $C$- and $P$-symmetric, and therefore, also *CP*-symmetric. In particular, these symmetries are respected by the electromagnetic and strong interactions. The weak interactions, on the other hand, violate $C$ and $P$ in the strongest possible way. For example, the $W$ bosons couple to left-handed electrons, $e^-_L$, and to their *CP*-conjugate right-handed positrons, $e^+_R$, but to neither their $C$-conjugate left-handed positrons, $e^+_L$, nor their $P$-conjugate right-handed electrons, $e^-_R$. While weak interactions violate $C$ and $P$ separately, *CP* is still preserved in most weak interaction processes. The *CP* symmetry is, however, violated in certain processes involving interference effects, as discovered in neutral $K$ decays in 1964 [1], and established later in $B$ (2001) and $D$ (2019) decays. For example, as discovered in 1967, a $K_L$ meson decays more often to $\pi^- e^+ \nu_e$ than to $\pi^+ e^- \overline{\nu}_e$, thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level. The *CP*-violating effects observed in the $B$ system are larger: the parameter describing the *CP* asymmetry in the decay time distribution of $B^0/\overline{B}^0$ meson transitions to *CP* eigenstates like $J/\psi K_S$ is about 0.7 [2, 3]. These effects are related to $K^0$–$\overline{K}^0$ and $B^0$–$\overline{B}^0$ mixing, but *CP* violation arising solely from decay amplitudes has also been observed, first in $K \to \pi\pi$ decays [4–6], subsequently in $B^0$ [7, 8], $B^+$ [9–11], and $B^0_s$ [12] decays, and most recently in charm decays [13]. All of these observed *CP* asymmetries are consistent with the Standard Model predictions. Similar effects could also occur in decays of baryons, but have not yet been observed. Given that neutrino masses and lepton mixing have been established, it is expected that *CP* is violated also in the lepton sector [14]. Discovering *CP* violation in the lepton sector is one of the main goals of current and near-future experiments. *CP* violation has not yet been observed in processes involving the top quark, nor in flavor-conserving processes such as electric dipole moments; for these, any significant observation would be a clear indication of physics beyond the Standard Model.

In addition to parity and to continuous Lorentz transformations, there is one other spacetime operation that could be a symmetry of the interactions: time reversal $T$, $t \to -t$. Violations of $T$ symmetry have been observed in neutral $K$ decays [15]. More recently, $T$ violation has been observed between states that are not *CP*-conjugate [16], exploiting the fact that for neutral $B$ mesons both flavor tagging and *CP* tagging can be used [17]. Moreover, $T$ violation is expected as a corollary of *CP* violation if the combined *CPT* transformation is a fundamental symmetry of nature [18]. All observations indicate that *CPT* is indeed a symmetry of nature [15]. Furthermore, one cannot build a locally Lorentz-invariant quantum field theory with a Hermitian Hamiltonian that violates *CPT*. (At several points in our discussion, we avoid assumptions about *CPT*, in order to identify cases where evidence for *CP* violation relies on assumptions about *CPT*.)

Within the Standard Model, *CP* symmetry is broken by complex phases in the Yukawa couplings (that is, the couplings of the Higgs scalar to quarks). When all transformations to remove unphysical phases in this model are exhausted, a single *CP*-violating parameter remains [19]. In the basis of mass eigenstates, this single phase appears in the $3 \times 3$ unitary matrix that gives the $W$-boson couplings to an up-type antiquark and a down-type quark. (If the Standard Model is supplemented with Majorana mass terms for the neutrinos, the analogous mixing matrix for leptons has three

*CP*-violating phases.) The beautifully consistent and economical Standard-Model description of *CP* violation in terms of Yukawa couplings, known as the Kobayashi-Maskawa (KM) mechanism [19], agrees with all measurements to date. Furthermore, one can fit the data allowing contributions from beyond the Standard Model (referred to subsequently as new physics) to loop processes to compete with, or even dominate over, the Standard Model amplitudes [20, 21]. Such an analysis provides model-independent proof that the KM phase is different from zero, and that the matrix of three-generation quark mixing is the dominant source of *CP* violation in meson decays.

The current level of experimental accuracy and the theoretical uncertainties involved in the interpretation of the various observations leave room, however, for additional subdominant sources of *CP* violation from new physics. Indeed, almost all extensions of the Standard Model imply that there are such additional sources. Moreover, *CP* violation is a necessary condition for baryogenesis, the process of dynamically generating the matter-antimatter asymmetry of the Universe [22]. Despite the phenomenological success of the KM mechanism, it fails (by several orders of magnitude) to accommodate the observed asymmetry [23]. This discrepancy strongly suggests that nature provides additional sources of *CP* violation beyond the KM mechanism. The evidence for neutrino masses implies that *CP* can be violated also in the lepton sector. This situation makes leptogenesis [24, 25], a scenario where *CP*-violating phases in the Yukawa couplings of the neutrinos play a crucial role in the generation of the baryon asymmetry, a very attractive possibility. The expectation of new sources motivates the large ongoing experimental effort to find deviations from the predictions of the KM mechanism.

*CP* violation can be experimentally searched for in a variety of processes, such as hadron decays, electric dipole moments of neutrons, electrons and nuclei, and neutrino oscillations. Hadron decays via the weak interaction probe flavor-changing *CP* violation. The search for electric dipole moments may find (or constrain) sources of *CP* violation that, unlike the KM phase, are not related to flavor-changing couplings. Following the discovery of the Higgs boson [26, 27], searches for *CP* violation in the Higgs sector are becoming feasible. Future searches for *CP* violation in neutrino oscillations might provide further input on leptogenesis.

The present measurements of *CP* asymmetries provide some of the strongest constraints on the weak couplings of quarks. Future measurements of *CP* violation in $K$, $D$, $B$, and $B_s^0$ meson decays will provide additional constraints on the flavor parameters of the Standard Model, and can probe new physics. In this review, we give the formalism and basic physics motivations that are relevant to present and near future measurements of *CP* violation in the quark sector.

## 13.1 Formalism

The phenomenology of *CP* violation for neutral flavored mesons is particularly interesting, since many of the observables can be cleanly interpreted. Although the phenomenology is superficially different for $K^0$, $D^0$, $B^0$, and $B_s^0$ decays, this is primarily because each of these systems is governed by a different balance between decay rates, oscillations, and lifetime splitting. However, the general considerations presented in this section are identical for all flavored neutral pseudoscalar mesons. The phenomenology of *CP* violation for neutral mesons that do not carry flavor quantum numbers (such as the $\eta^{(\prime)}$ state) is quite different: such states are their own antiparticles and have definite *CP* eigenvalues, so the signature of *CP* violation is simply the decay to a final state with the opposite *CP*. Such decays are mediated by the electromagnetic or (OZI-suppressed [28–30]) strong interaction, where *CP* violation is not expected and has not yet been observed. In the remainder of this review, we restrict ourselves to considerations of weakly decaying hadrons.

In this section, we present a general formalism for, and classification of, *CP* violation in the decay of a weakly decaying hadron, denoted $M$. We pay particular attention to the case that $M$ is a $K^0$, $D^0$, $B^0$, or $B_s^0$ meson. Subsequent sections describe the *CP*-violating phenomenology, approximations, and alternative formalisms that are specific to each system.

### 13.1.1 *Charged- and neutral-hadron decays*

We define decay amplitudes of $M$ (which could be charged or neutral) and its $CP$ conjugate $\overline{M}$ to a multi-particle final state $f$ and its $CP$ conjugate $\overline{f}$ as

$$A_f = \langle f \,|\mathcal{H}|\, M\rangle \ , \quad \overline{A}_f = \langle f \,|\mathcal{H}|\, \overline{M}\rangle \ , \tag{13.1a}$$

$$A_{\overline{f}} = \langle \overline{f} \,|\mathcal{H}|\, M\rangle \ , \quad \overline{A}_{\overline{f}} = \langle \overline{f} \,|\mathcal{H}|\, \overline{M}\rangle \ , \tag{13.1b}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of $CP$ on these states introduces phases $\xi_M$ and $\xi_f$ that depend on their flavor content, according to

$$CP|M\rangle = e^{+i\xi_M}\,|\overline{M}\rangle \ , \quad CP|f\rangle = e^{+i\xi_f}\,|\overline{f}\rangle \ , \tag{13.2a}$$

$$CP|\overline{M}\rangle = e^{-i\xi_M}\,|M\rangle \ , \quad CP|\overline{f}\rangle = e^{-i\xi_f}\,|f\rangle \ , \tag{13.2b}$$

so that $(CP)^2 = 1$. The phases $\xi_M$ and $\xi_f$ are arbitrary and unobservable because of the flavor symmetry of the strong interaction. If $CP$ is conserved by the dynamics, $[CP, \mathcal{H}] = 0$, then $A_f$ and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_M)}\, A_f \ . \tag{13.3}$$

### 13.1.2 *Neutral-meson mixing*

A state that is initially a superposition of $M^0$ and $\overline{M}^0$, say

$$|\psi(0)\rangle = a(0)|M^0\rangle + b(0)|\overline{M}^0\rangle \ , \tag{13.4}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|M^0\rangle + b(t)|\overline{M}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{13.5}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times $t$ under study are much larger than the typical strong interaction scale, then we can use a much simplified formalism [31]. The simplified time evolution is determined by a $2 \times 2$ effective Hamiltonian $\mathbf{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathbf{H}$, can be written in terms of Hermitian matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ as

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\,\mathbf{\Gamma} \ . \tag{13.6}$$

$\mathbf{M}$ and $\mathbf{\Gamma}$ are associated with $(M^0, \overline{M}^0) \leftrightarrow (M^0, \overline{M}^0)$ transitions via off-shell (dispersive), and on-shell (absorptive) intermediate states, respectively. Diagonal elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ are associated with the flavor-conserving transitions $M^0 \to M^0$ and $\overline{M}^0 \to \overline{M}^0$, while off-diagonal elements are associated with flavor-changing transitions $M^0 \leftrightarrow \overline{M}^0$.

The eigenvectors of $\mathbf{H}$ have well-defined masses and decay widths. To specify the components of the strong interaction eigenstates, $M^0$ and $\overline{M}^0$, in the light ($M_L$) and heavy ($M_H$) mass eigenstates, we introduce three complex parameters: $p$, $q$, and, for the case that both $CP$ and $CPT$ are violated in mixing, $z$. Then

$$|M_L\rangle \propto p\sqrt{1-z}\,|M^0\rangle + q\sqrt{1+z}\,|\overline{M}^0\rangle \ , \tag{13.7a}$$

$$|M_H\rangle \propto p\sqrt{1+z}\,|M^0\rangle - q\sqrt{1-z}\,|\overline{M}^0\rangle \ , \tag{13.7b}$$

with the normalization $|q|^2 + |p|^2 = 1$ when $z = 0$. (Another possible choice of labeling, which is in standard usage for $K$ mesons, defines the mass eigenstates according to their lifetimes: $K_S$ for the short-lived and $K_L$ for the long-lived state. The $K_L$ is experimentally found to be the heavier state. Yet another choice is often used for the $D$ mesons [32]: the eigenstates are labeled according to their dominant *CP* content.)

The real and imaginary parts of the eigenvalues $\omega_{L,H}$ corresponding to $|M_{L,H}\rangle$ represent their masses and decay widths, respectively. The mass and width splittings are

$$\Delta m \equiv m_H - m_L = \mathcal{R}e(\omega_H - \omega_L) \,, \tag{13.8a}$$

$$\Delta\Gamma \equiv \Gamma_H - \Gamma_L = -2\,\mathcal{I}m(\omega_H - \omega_L) \,. \tag{13.8b}$$

Note that here $\Delta m$ is positive by definition, while the sign of $\Delta\Gamma$ must be experimentally determined. The sign of $\Delta\Gamma$ has not yet been established for $B^0$ mesons, while $\Delta\Gamma < 0$ is established for $K$ and $B_s^0$ mesons. The Standard Model predicts $\Gamma_L > \Gamma_H$ for $B_{(s)}^0$ mesons; for this reason, $\Delta\Gamma = \Gamma_L - \Gamma_H$, which is still a signed quantity, is often used in the $B_{(s)}^0$ literature and is the convention used in the PDG experimental summaries.

Solving the eigenvalue problem for $\mathbf{H}$ yields

$$\left(\frac{q}{p}\right)^2 = \frac{\mathbf{M}_{12}^* - (i/2)\mathbf{\Gamma}_{12}^*}{\mathbf{M}_{12} - (i/2)\mathbf{\Gamma}_{12}} \tag{13.9}$$

and

$$z \equiv \frac{\delta m - (i/2)\delta\Gamma}{\Delta m - (i/2)\Delta\Gamma} \,, \tag{13.10}$$

where

$$\delta m \equiv \mathbf{M}_{11} - \mathbf{M}_{22} \,, \quad \delta\Gamma \equiv \mathbf{\Gamma}_{11} - \mathbf{\Gamma}_{22} \tag{13.11}$$

are the differences in effective mass and decay-rate expectation values for the strong interaction states $M^0$ and $\overline{M}^0$.

If either *CP* or *CPT* is a symmetry of $\mathbf{H}$ (independently of whether $T$ is conserved or violated), then the values of $\delta m$ and $\delta\Gamma$ are both zero, and hence $z = 0$. We also find that

$$\omega_H - \omega_L = 2\sqrt{\left(\mathbf{M}_{12} - \frac{i}{2}\mathbf{\Gamma}_{12}\right)\left(\mathbf{M}_{12}^* - \frac{i}{2}\mathbf{\Gamma}_{12}^*\right)} \,. \tag{13.12}$$

If either *CP* or $T$ is a symmetry of $\mathbf{H}$ (independently of whether *CPT* is conserved or violated), then $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_M} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \,, \tag{13.13}$$

where $\xi_M$ is the arbitrary unphysical phase introduced in Eq. (13.2). If, and only if, *CP* is a symmetry of $\mathbf{H}$ (independently of *CPT* and $T$), then both of the above conditions hold, with the result that the mass eigenstates are orthogonal

$$\langle M_H | M_L \rangle = |p|^2 - |q|^2 = 0 \,. \tag{13.14}$$

### 13.1.3 *CP-violating observables*

All *CP*-violating observables in $M$ and $\overline{M}$ decays to final states $f$ and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of $A_f$, $\overline{A}_f$, $A_{\overline{f}}$, and $\overline{A}_{\overline{f}}$, together with, for neutral meson decays only, $q/p$. *CP* violation in charged meson and all baryon decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while *CP* violation in flavored neutral meson decays is enriched by $M^0 \leftrightarrow \overline{M}^0$ oscillations, and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

The decay rates of the two neutral kaon mass eigenstates, $K_S$ and $K_L$, are different enough ($\Gamma_S/\Gamma_L \sim 500$) that one can, in most cases, actually study their decays independently. For $D^0$, $B^0$, and $B_s^0$ mesons, however, values of $\Delta\Gamma/\Gamma$ (where $\Gamma \equiv (\Gamma_H + \Gamma_L)/2$) are relatively small, and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|M^0\rangle$ or $|\overline{M}^0\rangle$ after an elapsed proper time $t$ as $|M^0_{\rm phys}(t)\rangle$ or $|\overline{M}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, but not assuming *CPT* to be a good symmetry, we obtain

$$|M^0_{\rm phys}(t)\rangle = (g_+(t) + z\, g_-(t))\,|M^0\rangle - \sqrt{1-z^2}\,\frac{q}{p}\; g_-(t)|\overline{M}^0\rangle \; , \tag{13.15a}$$

$$|\overline{M}^0_{\rm phys}(t)\rangle = (g_+(t) - z\, g_-(t))\,|\overline{M}^0\rangle - \sqrt{1-z^2}\,\frac{p}{q}\; g_-(t)|M^0\rangle \; , \tag{13.15b}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left[\exp\left(-im_Ht - \frac{1}{2}\Gamma_H t\right) \pm \exp\left(-im_Lt - \frac{1}{2}\Gamma_L t\right)\right] \tag{13.16}$$

and $z = 0$ if either *CPT* or *CP* is conserved.

Defining $x \equiv \Delta m/\Gamma$ and $y \equiv \Delta\Gamma/(2\Gamma)$, and assuming $z = 0$, one obtains the following time-dependent decay rates:

$$\begin{aligned}\frac{d\Gamma\left[M^0_{\rm phys}(t)\to f\right]/dt}{e^{-\Gamma t}\mathcal{N}_f} = {} & \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t) \\ & + 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\;,\end{aligned} \tag{13.17a}$$

$$\begin{aligned}\frac{d\Gamma\left[\overline{M}^0_{\rm phys}(t)\to f\right]/dt}{e^{-\Gamma t}\mathcal{N}_f} = {} & \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t) \\ & + 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\;,\end{aligned} \tag{13.17b}$$

where $\mathcal{N}_f$ is a common, time-independent, normalization factor that can be determined bearing in mind that the range of $t$ is $0 < t < \infty$. Decay rates to the *CP*-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (13.17a) and (13.17b). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $M^0 \leftrightarrow \overline{M}^0$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (13.17a) and (13.17b) are associated with the interference between these two cases. Note that, in multi-body decays such as $D^0 \to K_S\pi^+\pi^-$ or $B^0 \to \pi^+\pi^-\pi^+\pi^-$, amplitudes are functions of variables that describe the phase-space of the final state. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

When neutral pseudoscalar mesons are produced coherently in pairs from the decay of a vector resonance, $V \to M^0\overline{M}^0$ (for example, $\Upsilon(4S) \to B^0\overline{B}^0$, $\psi(3770) \to D^0\overline{D}^0$ or $\phi \to K^0\overline{K}^0$), the time-dependence of their subsequent decays to final states $f_1$ and $f_2$ has a similar form to Eqs. (13.17a) and (13.17b):

$$\frac{d\Gamma\left[V_{\rm phys}(t_1,t_2)\to f_1f_2\right]/d(\Delta t)}{e^{-\Gamma|\Delta t|}\mathcal{N}_{f_1f_2}} = \left(|a_+|^2+|a_-|^2\right)\cosh(y\Gamma\Delta t) + \left(|a_+|^2-|a_-|^2\right)\cos(x\Gamma\Delta t) - 2\,\mathcal{R}e(a_+^*a_-)\sinh(y\Gamma\Delta t) + 2\,\mathcal{I}m(a_+^*a_-)\sin(x\Gamma\Delta t)\ , \tag{13.18}$$

where $\Delta t \equiv t_2 - t_1$ is the difference in the production times, $t_1$ and $t_2$, of $f_1$ and $f_2$, respectively, and the dependence on the average decay time and on decay angles has been integrated out. The normalization factor $\mathcal{N}_{f_1f_2}$ can be evaluated, noting that the range of $\Delta t$ is $-\infty < \Delta t < \infty$. The coefficients in Eq. (13.18) are determined by the amplitudes for no net oscillation from $t_1 \to t_2$, $\overline{A}_{f_1}A_{f_2}$, and $A_{f_1}\overline{A}_{f_2}$, and for a net oscillation, $(q/p)\overline{A}_{f_1}\overline{A}_{f_2}$ and $(p/q)A_{f_1}A_{f_2}$, via

$$a_+ \equiv \overline{A}_{f_1}A_{f_2} - A_{f_1}\overline{A}_{f_2}\ , \tag{13.19a}$$

$$a_- \equiv -\sqrt{1-z^2}\left(\frac{q}{p}\overline{A}_{f_1}\overline{A}_{f_2} - \frac{p}{q}A_{f_1}A_{f_2}\right) + z\left(\overline{A}_{f_1}A_{f_2} + A_{f_1}\overline{A}_{f_2}\right)\ . \tag{13.19b}$$

Assuming *CPT* conservation, $z = 0$, and identifying $\Delta t \to t$ and $f_2 \to f$, we find that Eqs. (13.18) and (13.19) reduce to Eq. (13.17a) with $A_{f_1} = 0$, $\overline{A}_{f_1} = 1$, or to Eq. (13.17b) with $\overline{A}_{f_1} = 0$, $A_{f_1} = 1$. Indeed, this plays an important role in experiments that exploit the coherence of $V \to M^0\overline{M}^0$ production. Final states $f_1$ with $A_{f_1} = 0$ or $\overline{A}_{f_1} = 0$ are called tagging states, because they identify the decaying pseudoscalar meson as, respectively, $\overline{M}^0$ or $M^0$. Before one of $M^0$ or $\overline{M}^0$ decays, they evolve in phase, so that there is always one $M^0$ and one $\overline{M}^0$ present. A tagging decay of one meson sets the clock for the time evolution of the other: it starts at $t_1$ as purely $M^0$ or $\overline{M}^0$, with time evolution that depends only on $t_2 - t_1$.

When $f_1$ is a state that both $M^0$ and $\overline{M}^0$ can decay into, then Eq. (13.18) contains interference terms proportional to $A_{f_1}\overline{A}_{f_1} \neq 0$ that are not present in Eqs. (13.17a) and (13.17b). Even when $f_1$ is dominantly produced by $M^0$ decays rather than $\overline{M}^0$ decays, or vice versa, $A_{f_1}\overline{A}_{f_1}$ can be non-zero owing to doubly-CKM-suppressed decays (with amplitudes suppressed by at least two powers of $\lambda$ relative to the dominant amplitude, in the language of Section 13.3), and these terms should be considered for precision studies of *CP* violation in coherent $V \to M^0\overline{M}^0$ decays [33]. The correlations in $V \to M^0\overline{M}^0$ decays can also be exploited to determine strong phase differences between favored and suppressed decay amplitudes [34].

#### 13.1.4 *Classification of CP-violating effects*

We distinguish three types of *CP*-violating effects that can occur in the quark sector:

I. *CP* violation in decay is defined by

$$|\overline{A}_{\overline{f}}/A_f| \neq 1\ . \tag{13.20}$$

In charged meson (and all baryon) decays, where mixing effects are absent, this is the only possible source of *CP* asymmetries:

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(M^-\to f^-) - \Gamma(M^+\to f^+)}{\Gamma(M^-\to f^-) + \Gamma(M^+\to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1}\ . \tag{13.21}$$

Note that the usual sign convention for *CP* asymmetries of hadrons is for the difference between the rate involving the particle that contains a heavy quark and that which contains an antiquark. Hence, Eq. (13.21) corresponds to the definition for $B^\pm$ mesons, but the opposite sign is used for $D^\pm_{(s)}$ decays.

II. *CP* (and $T$) violation in mixing is defined by

$$|q/p| \neq 1 \; . \tag{13.22}$$

In charged-current semileptonic neutral meson decays $M, \overline{M} \to \ell^\pm X^\mp$ (taking $|A_{\ell^+X^-}| = |\overline{A}_{\ell^-X^+}|$ and $A_{\ell^-X^+} = \overline{A}_{\ell^+X^-} = 0$, as is the case in the Standard Model, to lowest order in $G_F$, and in most of its extensions), this is the only source of *CP* violation, and can be measured via the asymmetry of "wrong-sign" decays induced by oscillations:

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{d\Gamma/dt[\overline{M}^0_{\rm phys}(t) \to \ell^+X^-] - d\Gamma/dt[M^0_{\rm phys}(t) \to \ell^-X^+]}{d\Gamma/dt[\overline{M}^0_{\rm phys}(t) \to \ell^+X^-] + d\Gamma/dt[M^0_{\rm phys}(t) \to \ell^-X^+]} \, , \tag{13.23a}$$

$$= \frac{1-|q/p|^4}{1+|q/p|^4} \, . \tag{13.23b}$$

Note that this asymmetry of time-dependent decay rates is actually time-independent.

III. *CP* violation in interference between a decay without mixing, $M^0 \to f$, and a decay with mixing, $M^0 \to \overline{M}^0 \to f$ (such an effect occurs only in decays to final states that are common to $M^0$ and $\overline{M}^0$, including all *CP* eigenstates), is defined by

$$\arg(\lambda_f) + \arg(\lambda_{\bar f}) \neq 0 \; , \qquad \text{with} \qquad \lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f} \; . \tag{13.24}$$

For final *CP* eigenstates, $f_{CP}$, the condition Eq. (13.24) simplifies to

$$\mathcal{I}m(\lambda_{f_{CP}}) \neq 0 \; , \tag{13.25}$$

This form of *CP* violation can be observed, for example, using the asymmetry of neutral meson decay rates into *CP* eigenstates

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{M}^0_{\rm phys}(t) \to f_{CP}] - d\Gamma/dt[M^0_{\rm phys}(t) \to f_{CP}]}{d\Gamma/dt[\overline{M}^0_{\rm phys}(t) \to f_{CP}] + d\Gamma/dt[M^0_{\rm phys}(t) \to f_{CP}]} \; . \tag{13.26}$$

If $\Delta\Gamma = 0$, as expected to a good approximation for $B^0$ mesons but not for $K^0$ and $B^0_s$ mesons, and $|q/p| = 1$, then $\mathcal{A}_{f_{CP}}$ has a particularly simple form (see Eq. (13.75), below). If, in addition, the decay amplitudes fulfill $|\overline{A}_{f_{CP}}| = |A_{f_{CP}}|$, the interference between decays with and without mixing is the only source of asymmetry and $\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_{f_{CP}})\sin(x\Gamma t)$.

Examples of these three types of *CP* violation will be given in Sections 13.4, 13.5, and 13.6.

## 13.2 Theoretical Interpretation: General Considerations

Consider the $M \to f$ decay amplitude $A_f$, and the *CP* conjugate process, $\overline{M} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the *CP*-conjugate amplitude. Thus, their phases appear in $A_f$ and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons, and hence, are often called "weak phases." The weak phase of any single term is convention-dependent. However, the difference between the weak phases in two different terms in $A_f$ is convention-independent. A second type of phase can appear in scattering or decay amplitudes, even when the Lagrangian is real. This phase originates from the possible contribution from intermediate on-shell states in the decay process. Since such phases are generated by *CP*-invariant interactions, they are the same in $A_f$ and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions; hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The "weak" and "strong" phases discussed here appear in addition to the spurious *CP*-transformation phases of Eq. (13.3). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any *CP* violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution $a_i$ to $A_f$ in three parts: its magnitude $|a_i|$, its weak phase $\phi_i$, and its strong phase $\delta_i$. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$A_f = |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}\,, \tag{13.27a}$$

$$\overline{A}_{\overline{f}} = |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}\,. \tag{13.27b}$$

Similarly, for neutral mesons, it is useful to write

$$\mathbf{M}_{12} = |\mathbf{M}_{12}| e^{i\phi_M}\,, \quad \mathbf{\Gamma}_{12} = |\mathbf{\Gamma}_{12}| e^{i\phi_\Gamma}\,. \tag{13.28}$$

Each of the phases appearing in Eqs. (13.27) and (13.28) is convention-dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$, and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

It is now straightforward to evaluate the various asymmetries in terms of the theoretical parameters introduced here. We will do so with approximations that are often relevant to the most interesting measured asymmetries.

1. The *CP* asymmetry in charged meson and all baryon decays [Eq. (13.21)] is given by

$$\mathcal{A}_f = -\frac{2|a_1 a_2| \sin(\delta_2-\delta_1)\sin(\phi_2-\phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1 a_2|\cos(\delta_2-\delta_1)\cos(\phi_2-\phi_1)}\,. \tag{13.29}$$

The quantity of most interest to theory is the weak phase difference $\phi_2 - \phi_1$. Its extraction from the asymmetry requires, however, that the amplitude ratio $|a_2/a_1|$ and the strong phase difference $\delta_2 - \delta_1$ are known. Both quantities depend on non-perturbative hadronic parameters that are difficult to calculate, but in some cases can be obtained from experiment.

2. In the approximation that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| \ll 1$ (valid for $B^0$ and $B_s^0$ mesons), the *CP* asymmetry in semileptonic neutral-meson decays [Eq. (13.23)] is given by

$$\mathcal{A}_{\rm SL} = -\left|\frac{\mathbf{\Gamma}_{12}}{\mathbf{M}_{12}}\right| \sin(\phi_M - \phi_\Gamma)\,. \tag{13.30}$$

The quantity of most interest to theory is the weak phase $\phi_M - \phi_\Gamma$. Its extraction from the asymmetry requires, however, that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}|$ is known. State of the art calculations of this quantity for the $B^0$ and $B_s^0$ mesons have uncertainties of around 15–20% [35].

3. In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f+\phi_f)}$, and that $|\mathbf{\Gamma}_{12}/\mathbf{M}_{12}| = 0$, we obtain $|\lambda_f| = 1$, and the *CP* asymmetries in decays to a final *CP* eigenstate $f$ [Eq. (13.26)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f)\,. \tag{13.31}$$

Note that the phase measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$ .

The discussion above allows us to introduce another classification of *CP*-violating effects:

1. *Indirect CP violation* is consistent with taking $\phi_M \neq 0$ and setting all other *CP* violating phases to zero. *CP* violation in mixing (type II) belongs to this class.
2. *Direct CP violation* cannot be accounted for by just $\phi_M \neq 0$. *CP* violation in decay (type I) belongs to this class.

The historical significance of this classification is related to theory. In superweak models [36], *CP* violation appears only in diagrams that contribute to $\mathbf{M}_{12}$, hence predicting no direct *CP* violation. In most models and, in particular, in the Standard Model, *CP* violation is both direct and indirect. As concerns type III *CP* violation, a single observation of such an effect would be consistent with indirect *CP* violation, but observing $\eta_{f_1}\mathcal{I}m(\lambda_{f_1}) \neq \eta_{f_2}\mathcal{I}m(\lambda_{f_2})$ (for the same decaying meson and two different final *CP* eigenstates $f_1$ and $f_2$) would establish direct *CP* violation. The experimental observation of $\epsilon' \neq 0$, which was achieved by establishing that $\mathcal{I}m(\lambda_{\pi^+\pi^-}) \neq \mathcal{I}m(\lambda_{\pi^0\pi^0})$ (see Section 13.4), excluded the superweak scenario.

## 13.3 Theoretical Interpretation: The KM Mechanism

Of all the Standard Model quark parameters, only the Kobayashi-Maskawa (KM) phase is *CP*-violating. Having a single source of *CP* violation, the Standard Model is very predictive for *CP* asymmetries: some vanish, and those that do not are correlated.

To be precise, *CP* could be violated also by strong interactions. The experimental upper bound on the electric-dipole moment of the neutron [37] implies, however, that $\theta_{\rm QCD}$, the non-perturbative parameter that determines the strength of this type of *CP* violation, is tiny, if not zero [38]. The smallness of $\theta_{\rm QCD}$ constitutes a theoretical puzzle, known as "the strong *CP* problem." This, however, is irrelevant to our discussion of hadron decays.

The charged current interactions (that is, the $W^\pm$ interactions) for quarks are given by

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}}\ \overline{u_{Li}}\ \gamma^\mu\ (V_{\rm CKM})_{ij}\ d_{Lj}\ W_\mu^+ + \text{h.c.} \tag{13.32}$$

Here $i, j = 1, 2, 3$ are generation numbers. The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks is a $3 \times 3$ unitary matrix [39]. Ordering the quarks by their masses, *i.e.*, $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$, the elements of $V_{\rm CKM}$ are written as follows:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{13.33}$$

While a general $3 \times 3$ unitary matrix depends on three real angles and six phases, the freedom to redefine the phases of the quark mass eigenstates can be used to remove five of the phases, leaving a single physical phase, the Kobayashi-Maskawa phase, that is responsible for all *CP* violation in the Standard Model.

The fact that one can parameterize $V_{\rm CKM}$ by three real and only one imaginary physical parameters can be made manifest by choosing an explicit parametrization. The Wolfenstein parametrization [40, 41] is particularly useful:

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3[1 - (1 - \frac{1}{2}\lambda^2)(\rho + i\eta)] & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} . \tag{13.34}$$

Here $\lambda \approx 0.23$ (not to be confused with $\lambda_f$), the sine of the Cabibbo angle, plays the role of an expansion parameter, and $\eta$ represents the *CP*-violating phase. Terms of $\mathcal{O}(\lambda^6)$ have been neglected.

The unitarity of the CKM matrix, $(VV^\dagger)_{ij} = (V^\dagger V)_{ij} = \delta_{ij}$, leads to twelve distinct complex relations among the matrix elements. The six relations with $i \neq j$ can be represented geometrically as triangles in the complex plane. Two of these,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \; , \tag{13.35a}$$
$$V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* = 0 \; , \tag{13.35b}$$

have terms of equal order, $\mathcal{O}(A\lambda^3)$, and so have corresponding triangles whose interior angles are all $\mathcal{O}(1)$ physical quantities that can be independently measured. The angles of the first triangle (see Fig. 13.1) are given by

$$\alpha \equiv \varphi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \simeq \arg\left(-\frac{1-\rho-i\eta}{\rho+i\eta}\right) \; , \tag{13.36a}$$
$$\beta \equiv \varphi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \simeq \arg\left(\frac{1}{1-\rho-i\eta}\right) \; , \tag{13.36b}$$
$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \simeq \arg\left(\rho+i\eta\right) \; . \tag{13.36c}$$

The angles of the second triangle are equal to $(\alpha, \beta, \gamma)$ up to corrections of $\mathcal{O}(\lambda^2)$. The notations $(\alpha, \beta, \gamma)$ and $(\varphi_1, \varphi_2, \varphi_3)$ are both in common usage but, for convenience, we only use the first convention in the following.

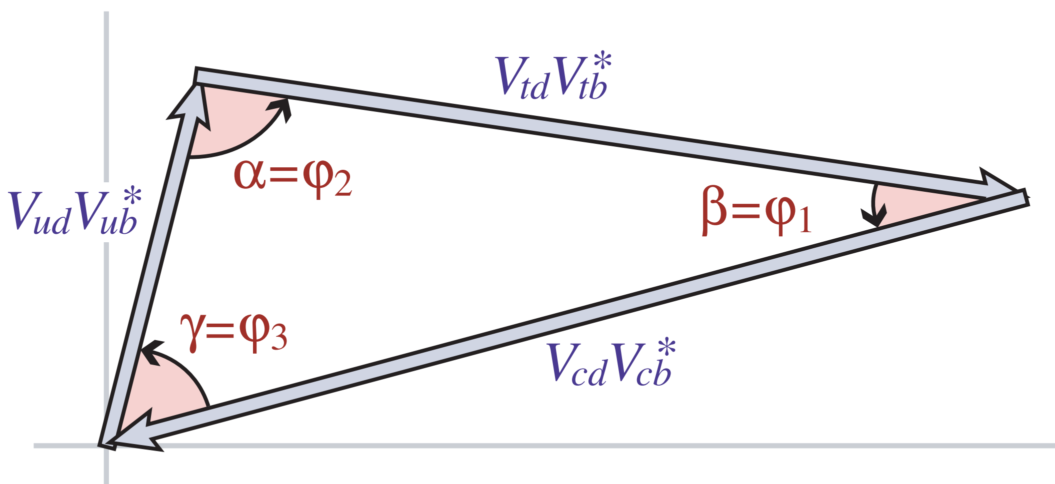



**Figure 13.1:** Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

Another relation that can be represented as a triangle,

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \; , \tag{13.37}$$

and, in particular, its small angle, of $\mathcal{O}(\lambda^2)$,

$$\beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right) , \tag{13.38}$$

is convenient for analyzing $CP$ violation in the $B_s^0$ sector.

All unitarity triangles have the same area, commonly denoted by $J/2$ [42]. If *CP* is violated, $J$ is different from zero and can be taken as the single *CP*-violating parameter. In the Wolfenstein parametrization of Eq. (13.34), $J \simeq \lambda^6 A^2 \eta$.

## 13.4 Kaons

*CP* violation was discovered in $K \to \pi\pi$ decays in 1964 [1]. The same mode provided the first observation of direct *CP* violation [4–6].

The decay amplitudes actually measured in neutral $K$ decays refer to the mass eigenstates $K_L$ and $K_S$, rather than to the $K$ and $\overline{K}$ states referred to in Eq. (13.1). The final $\pi^+\pi^-$ and $\pi^0\pi^0$ states are *CP*-even. In the *CP* conservation limit, $K_S$ ($K_L$) would be *CP*-even (odd), and therefore would (would not) decay to two pions. We define *CP*-violating amplitude ratios for two-pion final states,

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle}\,, \quad \eta_{+-} \equiv \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle}\,. \tag{13.39}$$

Another important observable is the asymmetry of time-integrated semileptonic decay rates:

$$\delta_L \equiv \frac{\Gamma(K_L \to \ell^+\nu_\ell\pi^-) - \Gamma(K_L \to \ell^-\overline{\nu}_\ell\pi^+)}{\Gamma(K_L \to \ell^+\nu_\ell\pi^-) + \Gamma(K_L \to \ell^-\overline{\nu}_\ell\pi^+)}\,. \tag{13.40}$$

*CP* violation has been observed as an appearance of $K_L$ decays to two-pion final states [43],

$$|\eta_{00}| = (2.220 \pm 0.011) \times 10^{-3}\,, \tag{13.41a}$$

$$|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3}\,, \tag{13.41b}$$

$$|\eta_{00}/\eta_{+-}| = 0.9950 \pm 0.0007\,, \tag{13.41c}$$

where the *CP*-conserving phases $\phi_{ij}$ of the amplitude ratios $\eta_{ij}$ have been determined both assuming *CPT* invariance:

$$\phi_{00} = (43.52 \pm 0.05)^\circ\,, \qquad \phi_{+-} = (43.51 \pm 0.05)^\circ\,, \tag{13.42}$$

and without assuming *CPT* invariance:

$$\phi_{00} = (43.7 \pm 0.6)^\circ\,, \qquad \phi_{+-} = (43.4 \pm 0.5)^\circ\,. \tag{13.43}$$

*CP* violation has also been observed in semileptonic $K_L$ decays [43]

$$\delta_L = (3.32 \pm 0.06) \times 10^{-3}\,, \tag{13.44}$$

where $\delta_L$ is a weighted average of muon and electron measurements, as well as in $K_L$ decays to $\pi^+\pi^-\gamma$ and $\pi^+\pi^-e^+e^-$ [43]. *CP* violation in $K \to 3\pi$ decays has not yet been observed [43, 44].

Historically, *CP* violation in neutral $K$ decays has been described in terms of the complex parameters $\epsilon$ and $\epsilon'$. The observables $\eta_{00}$, $\eta_{+-}$, and $\delta_L$ are related to these parameters, and to those of Section 13.1, by

$$\eta_{00} = \frac{1-\lambda_{\pi^0\pi^0}}{1+\lambda_{\pi^0\pi^0}} = \epsilon - 2\epsilon'\,, \tag{13.45a}$$

$$\eta_{+-} = \frac{1-\lambda_{\pi^+\pi^-}}{1+\lambda_{\pi^+\pi^-}} = \epsilon + \epsilon'\,, \tag{13.45b}$$

$$\delta_L = \frac{1-|q/p|^2}{1+|q/p|^2} = \frac{2\mathcal{R}e(\epsilon)}{1+|\epsilon|^2}\,, \tag{13.45c}$$

where, in the last line, we have assumed that $|A_{\ell^+\nu_\ell\pi^-}| = \left|\overline{A}_{\ell^-\overline{\nu}_\ell\pi^+}\right|$ and $|A_{\ell^-\overline{\nu}_\ell\pi^+}| = \left|\overline{A}_{\ell^+\nu_\ell\pi^-}\right| = 0$. (The convention-dependent parameter $\tilde{\epsilon} \equiv (1-q/p)/(1+q/p)$, sometimes used in the literature, is, in general, different from $\epsilon$ but yields a similar expression, $\delta_L = 2\mathcal{R}e(\tilde{\epsilon})/(1+|\tilde{\epsilon}|^2)$.) A fit to the $K \to \pi\pi$ data yields [43]

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}\,, \tag{13.46a}$$

$$\mathcal{R}e(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3}\,. \tag{13.46b}$$

In discussing two-pion final states, it is useful to express the amplitudes $A_{\pi^0\pi^0}$ and $A_{\pi^+\pi^-}$ in terms of their isospin components via

$$A_{\pi^0\pi^0} = \sqrt{\frac{1}{3}}\,|A_0|\,e^{i(\delta_0+\phi_0)} - \sqrt{\frac{2}{3}}\,|A_2|\,e^{i(\delta_2+\phi_2)}\,, \tag{13.47a}$$

$$A_{\pi^+\pi^-} = \sqrt{\frac{2}{3}}\,|A_0|\,e^{i(\delta_0+\phi_0)} + \sqrt{\frac{1}{3}}\,|A_2|\,e^{i(\delta_2+\phi_2)}\,, \tag{13.47b}$$

where we parameterize the amplitude $A_I(\overline{A}_I)$ for $K^0(\overline{K}^0)$ decay into two pions with total isospin $I = 0$ or 2 as

$$A_I \equiv \langle(\pi\pi)_I\,|\mathcal{H}|\,K^0\rangle = |A_I|\,e^{i(\delta_I+\phi_I)}\,, \tag{13.48a}$$

$$\overline{A}_I \equiv \langle(\pi\pi)_I\,|\mathcal{H}|\,\overline{K}^0\rangle = |A_I|\,e^{i(\delta_I-\phi_I)}\,. \tag{13.48b}$$

The smallness of $|\eta_{00}|$ and $|\eta_{+-}|$ allows us to approximate

$$\epsilon \simeq \frac{1}{2}(1-\lambda_{(\pi\pi)_{I=0}})\,, \qquad \epsilon' \simeq \frac{1}{6}\left(\lambda_{\pi^0\pi^0} - \lambda_{\pi^+\pi^-}\right)\,. \tag{13.49}$$

The parameter $\epsilon$ represents indirect *CP* violation, while $\epsilon'$ parameterizes direct *CP* violation: $\mathcal{R}e(\epsilon')$ measures *CP* violation in decay (type I), $\mathcal{R}e(\epsilon)$ measures *CP* violation in mixing (type II), and $\mathcal{I}m(\epsilon)$ and $\mathcal{I}m(\epsilon')$ measure the interference between decays with and without mixing (type III).

The following expressions for $\epsilon$ and $\epsilon'$ are useful for theoretical evaluations:

$$\epsilon \simeq \frac{e^{i\pi/4}}{\sqrt{2}}\frac{\mathcal{I}m(\mathbf{M}_{12})}{\Delta m}\,, \qquad \epsilon' = \frac{i}{\sqrt{2}}\left|\frac{A_2}{A_0}\right| e^{i(\delta_2-\delta_0)}\sin(\phi_2-\phi_0)\,. \tag{13.50}$$

The expression for $\epsilon$ is only valid in a phase convention where $\phi_2 = 0$, corresponding to a real $V_{ud}V_{us}^*$, and in the approximation that also $\phi_0 = 0$. The phase of $\epsilon$, $\arg(\epsilon) \approx \arctan(-2\Delta m/\Delta\Gamma)$, is determined by non-perturbative QCD dynamics and is experimentally determined to be about $\pi/4$. The calculation of $\epsilon$ benefits from the fact that $\mathcal{I}m(\mathbf{M}_{12})$ is dominated by short distance physics. Consequently, the main sources of uncertainty in theoretical interpretations of $\epsilon$ are the values of matrix elements, such as $\langle K^0\,|(\overline{s}d)_{V-A}(\overline{s}d)_{V-A}|\,\overline{K}^0\rangle$. The expression for $\epsilon'$ is valid to first order in $|A_2/A_0| \sim 1/20$. The phase of $\epsilon'$ is experimentally determined, $\pi/2 + \delta_2 - \delta_0 \approx \pi/4$, and is independent of the model of electroweak interactions. Note that, accidentally, $\epsilon'/\epsilon$ is real to a good approximation. Determination of weak phase information from the measurement of $\mathcal{R}e(\epsilon'/\epsilon)$ given in Eq. (13.46) has until now been precluded by uncertainties in the hadronic parameters, but recent advances in lattice QCD calculations [45, 46] suggest that it may become possible [47].

A future measurement of much interest is that of *CP* violation in the rare $K \to \pi\nu\overline{\nu}$ decays. The signal for *CP* violation is simply observing the $K_L \to \pi^0\nu\overline{\nu}$ decay. The effect here is that of interference between decays with and without mixing (type III) [48]:

$$\frac{\Gamma(K_L \to \pi^0\nu\overline{\nu})}{\Gamma(K^+ \to \pi^+\nu\overline{\nu})} = \frac{1}{2}\left[1 + |\lambda_{\pi\nu\overline{\nu}}|^2 - 2\,\mathcal{R}e(\lambda_{\pi\nu\overline{\nu}})\right] \simeq 1 - \mathcal{R}e(\lambda_{\pi\nu\overline{\nu}})\,, \tag{13.51}$$

where in the last equation we neglect $CP$ violation in decay and in mixing (expected, model-independently, to be of order $10^{-5}$ and $10^{-3}$, respectively). Such a measurement is experimentally very challenging but would be theoretically very rewarding [49]. Similar to the $CP$ asymmetry in $B^0 \to J/\psi K_S$, the $CP$ violation in $K \to \pi\nu\overline{\nu}$ decay is predicted to be large (that is, the ratio in Eq. (13.51) is neither CKM- nor loop-suppressed) and can be very cleanly interpreted. In particular, the independent determinations of the CKM parameters via $B$-meson and $K$-meson decays and mixing will over-constrain the unitarity triangle and provide a stringent text of the KM mechanism.

Within the Standard Model, the $K_L \to \pi^0\nu\overline{\nu}$ decay is dominated by an intermediate top quark contribution and, consequently, can be interpreted in terms of CKM parameters [50]. (For the charged mode, $K^+ \to \pi^+\nu\overline{\nu}$, the contribution from an intermediate charm quark is not negligible, and constitutes a source of hadronic uncertainty.) In particular, $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ provides a theoretically clean way to determine the Wolfenstein parameter $\eta$ [51]:

$$\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) = \kappa_L[X(m_t^2/m_W^2)]^2 A^4\eta^2\,, \tag{13.52}$$

where the hadronic parameter $\kappa_L \sim 2 \times 10^{-10}$ incorporates the value of the four-fermion matrix element which is deduced, using isospin relations, from $\mathcal{B}(K^+ \to \pi^0 e^+\nu_e)$, and $X(m_t^2/m_W^2)$ is a known function of the top mass. An explicit calculation gives $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) = (3.00 \pm 0.30) \times 10^{-11}$ [52].

Currently the most stringent experimental limit is $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) < 3.0 \times 10^{-9}$ [53, 54], which does not yet reach the bound that can be derived from Eq. (13.51), $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu}) < 4.4 \times \mathcal{B}(K^+ \to \pi^+\nu\overline{\nu})$ [48], with the most precise result for the charged kaon decay being $\mathcal{B}(K^+ \to \pi^+\nu\overline{\nu}) = \left(10.6\,^{+4.0}_{-3.4} \pm 0.9\right) \times 10^{-11}$ [55]. Significant further progress is anticipated from experiments searching for $K \to \pi\nu\overline{\nu}$ decays in the next few years.

## 13.5 Charm

The existence of $D^0$–$\overline{D}^0$ mixing is well established [56–60], with the latest experimental constraints giving [61, 62] $x \equiv \Delta m/\Gamma = (0.41 \pm 0.05) \times 10^{-2}$ and $y \equiv \Delta\Gamma/(2\Gamma) = (0.62 \pm 0.06) \times 10^{-2}$. Long-distance contributions make it difficult to calculate Standard Model predictions for the $D^0$–$\overline{D}^0$ mixing parameters. Therefore, the goal of the search for $D^0$–$\overline{D}^0$ mixing is not to constrain the CKM parameters, but rather to probe new physics. Here $CP$ violation plays an important role. Within the Standard Model, the $CP$-violating effects are predicted to be small, since the mixing and the relevant decays are described, to an excellent approximation, by the physics of the first two generations only. The expectation is that the Standard Model size of $CP$ violation in $D$ decays is $\mathcal{O}(10^{-3})$ or less. At present, the most sensitive searches involve the $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$ and $D^0 \to K^\pm\pi^\mp$ modes.

The neutral $D$ mesons decay via a singly-Cabibbo-suppressed transition to the $CP$ eigenstates $K^+K^-$ and $\pi^+\pi^-$. These decays are dominated by Standard-Model tree diagrams. Thus, we can write, for $f = K^+K^-$ or $\pi^+\pi^-$,

$$A_f = A_f^T e^{+i\phi_f^T}\left[1 + r_f e^{i(\delta_f + \phi_f)}\right]\,, \tag{13.53a}$$

$$\bar{A}_f = A_f^T e^{-i\phi_f^T}\left[1 + r_f e^{i(\delta_f - \phi_f)}\right]\,, \tag{13.53b}$$

where $A_f^T e^{\pm i\phi_f^T}$ is the Standard Model tree-level contribution, $\phi_f^T$ and $\phi_f$ are weak, $CP$ violating phases, $\delta_f$ is a strong phase difference, and $r_f$ is the ratio between a subleading ($r_f \ll 1$) contribution with a weak phase different from $\phi_f^T$ and the Standard Model tree-level contribution. Neglecting $r_f$, $\lambda_f$ is universal, and we can define an observable phase $\phi_D$ via

$$\lambda_f \equiv -|q/p| e^{i\phi_D}\,. \tag{13.54}$$

(In the limit of *CP* conservation, choosing $\phi_D = 0$ is equivalent to defining the mass eigenstates by their *CP* eigenvalue: $|D_\mp\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$, with $D_-$ ($D_+$) being the *CP*-odd (*CP*-even) state; that is, the state that does not (does) decay into $K^+K^-$.)

We define the time integrated *CP* asymmetry for a final *CP* eigenstate $f$ as follows:

$$a_f \equiv \frac{\int_0^\infty \Gamma(D^0_{\rm phys}(t) \to f)dt - \int_0^\infty \Gamma(\overline{D}^0_{\rm phys}(t) \to f)dt}{\int_0^\infty \Gamma(D^0_{\rm phys}(t) \to f)dt + \int_0^\infty \Gamma(\overline{D}^0_{\rm phys}(t) \to f)dt}\,. \tag{13.55}$$

(This expression corresponds to the $D$ meson being tagged at production, hence the integration goes from 0 to $+\infty$; measurements are also possible with $\psi(3770) \to D^0\overline{D}^0$, in which case the integration goes from $-\infty$ to $+\infty$ giving slightly different results; see the discussion in Section 13.1.3.) We take $x, y, r_f \ll 1$ and expand to leading order in these parameters. We can then separate the contribution to $a_f$ into three parts [63],

$$a_f = a_f^d + a_f^m + a_f^i\,, \tag{13.56}$$

with the following underlying mechanisms:

1. $a_f^d$ signals *CP* violation in decay (similar to Eq. (13.21)):

$$a_f^d = 2r_f \sin\phi_f \sin\delta_f\,. \tag{13.57}$$

2. $a_f^m$ signals *CP* violation in mixing (similar to Eq. (13.30)). With our approximations, it is universal:

$$a^m = -\frac{y}{2}\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\cos\phi_D\,. \tag{13.58}$$

3. $a_f^i$ signals *CP* violation in the interference of mixing and decay (similar to Eq. (13.31)). With our approximations, it is universal:

$$a^i = \frac{x}{2}\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\sin\phi_D\,. \tag{13.59}$$

In the SM, both $a^m$ and $a^i$ are $\mathcal{O}(10^{-5})$ or less, while $a^d$ could be up to two orders of magnitude larger.

One can isolate the effects of direct *CP* violation by taking the difference between the *CP* asymmetries in the $K^+K^-$ and $\pi^+\pi^-$ modes:

$$\Delta a_{CP} \equiv a_{K^+K^-} - a_{\pi^+\pi^-} = a^d_{K^+K^-} - a^d_{\pi^+\pi^-}\,, \tag{13.60}$$

where we neglected a residual, experiment-dependent, contribution from indirect *CP* violation due to the fact that there may be a decay time-dependent acceptance function that can be different for the $K^+K^-$ and $\pi^+\pi^-$ channels. The current average gives [13, 61]:

$$a^d_{K^+K^-} - a^d_{\pi^+\pi^-} = (-0.164 \pm 0.028) \times 10^{-2}\,, \tag{13.61}$$

demonstrating *CP* violation in charm decay. While the asymmetry is somewhat larger than the theoretical predictions that preceded the measurement, it can in principle be explained by non-perturbative QCD effects.

One can also isolate the effects of indirect *CP* violation in the following way. Consider the time-dependent decay rates in Eq. (13.17a) and Eq. (13.17b). The mixing processes modify the time dependence from a pure exponential. However, given the small values of $x$ and $y$, the time dependences can be recast, to a good approximation, into purely exponential form, but with modified decay-rate parameters [64,65] (given here for the $K^+K^-$ final state):

$$\Gamma_{D^0\to K^+K^-} = \Gamma \times [1 + |q/p| (y\cos\phi_D - x\sin\phi_D)] \,, \tag{13.62a}$$
$$\Gamma_{\overline{D}^0\to K^+K^-} = \Gamma \times [1 + |p/q| (y\cos\phi_D + x\sin\phi_D)] \,. \tag{13.62b}$$

One can define *CP*-conserving and *CP*-violating combinations of these two observables (normalized to the true width $\Gamma$):

$$\begin{aligned} y_{CP} &\equiv \frac{\Gamma_{\overline{D}^0\to K^+K^-} + \Gamma_{D^0\to K^+K^-}}{2\Gamma} - 1 \\ &= (y/2)\,(|q/p| + |p/q|)\cos\phi_D - (x/2)\,(|q/p| - |p/q|)\sin\phi_D \,, \end{aligned} \tag{13.63a}$$
$$\begin{aligned} A_\Gamma &\equiv \frac{\Gamma_{D^0\to K^+K^-} - \Gamma_{\overline{D}^0\to K^+K^-}}{2\Gamma} \\ &= -(a^m + a^i) \,. \end{aligned} \tag{13.63b}$$

In the limit of *CP* conservation (and, in particular, within the Standard Model), $y_{CP} = (\Gamma_+ - \Gamma_-)/2\Gamma = y$ (where $\Gamma_+(\Gamma_-)$ is the decay width of the *CP*-even (-odd) mass eigenstate) and $A_\Gamma = 0$. Indeed, present measurements imply that *CP* violation is small [61],

$$y_{CP} = (+0.72 \pm 0.11) \times 10^{-2} \,, \tag{13.64a}$$
$$A_\Gamma = (0.009 \pm 0.011) \times 10^{-2} \,. \tag{13.64b}$$

The $K^\pm\pi^\mp$ states are not *CP* eigenstates, but they are still common final states for $D^0$ and $\overline{D}^0$ decays. Since $D^0(\overline{D}^0) \to K^-\pi^+$ is a Cabibbo-favored (doubly-Cabibbo-suppressed) process, these processes are particularly sensitive to $x$ and/or $y = \mathcal{O}(\lambda^2)$. Taking into account that $|\lambda_{K^-\pi^+}|, \left|\lambda^{-1}_{K^+\pi^-}\right| \ll 1$ and $x, y \ll 1$, assuming that there is no direct *CP* violation (these are Standard Model tree-level decays dominated by a single weak phase, and there is no contribution from penguin-like and chromomagnetic operators), and expanding the time-dependent rates for $xt, yt \lesssim \Gamma^{-1}$, one obtains

$$\begin{aligned} \Gamma[D^0_{\rm phys}(t) \to K^+\pi^-] =& e^{-\Gamma t}|\overline{A}_{K^-\pi^+}|^2 \\ &\times \left[ r_d^2 + r_d \left|\frac{q}{p}\right| (y'\cos\phi_D - x'\sin\phi_D)\Gamma t + \left|\frac{q}{p}\right|^2 \frac{y^2+x^2}{4}(\Gamma t)^2 \right] \,, \end{aligned} \tag{13.65a}$$
$$\begin{aligned} \Gamma[\overline{D}^0_{\rm phys}(t) \to K^-\pi^+] =& e^{-\Gamma t}|\overline{A}_{K^-\pi^+}|^2 \\ &\times \left[ r_d^2 + r_d \left|\frac{p}{q}\right| (y'\cos\phi_D + x'\sin\phi_D)\Gamma t + \left|\frac{p}{q}\right|^2 \frac{y^2+x^2}{4}(\Gamma t)^2 \right] \,, \end{aligned} \tag{13.65b}$$

where

$$y' \equiv y\cos\delta - x\sin\delta \quad \text{and} \quad x' \equiv x\cos\delta + y\sin\delta \,. \tag{13.66}$$

The weak phase $\phi_D$ is the same as that of Eq. (13.54) (a consequence of neglecting direct *CP* violation) and $r_d = \mathcal{O}(\tan^2\theta_c)$ is the amplitude ratio, $r_d = \left|\overline{A}_{K^-\pi^+}/A_{K^-\pi^+}\right| = \left|A_{K^+\pi^-}/\overline{A}_{K^+\pi^-}\right|$, that is, $\lambda_{K^-\pi^+} = r_d|q/p|e^{-i(\delta-\phi_D)}$ and $\lambda^{-1}_{K^+\pi^-} = r_d|p/q|e^{-i(\delta+\phi_D)}$. The parameter $\delta$ is a strong-phase difference for these processes, that can be obtained from measurements of quantum correlated $\psi(3770) \to D^0\overline{D}^0$ decays [66,67]. By fitting to the six coefficients of the various time-dependences, one can determine $r_d$, $|q/p|$, $(x^2+y^2)$, $y'\cos\phi_D$, and $x'\sin\phi_D$. In particular, finding *CP* violation ($|q/p|\neq 1$ and/or $\sin\phi_D\neq 0$) at a level much higher than $10^{-3}$ would constitute evidence for new physics. The most stringent constraints to date on *CP* violation in charm mixing have been obtained with this method [68] and from the $A_\Gamma$ measurement [69].

A fit to all data [61], including also results from time-dependent analyses of $D^0\to K_S\pi^+\pi^-$ decays, from which $x$, $y$, $|q/p|$ and $\phi_D$ can be determined directly, yields no evidence for indirect *CP* violation:

$$1-|q/p| = +0.005\pm 0.016\,, \tag{13.67a}$$
$$\phi_D = (-2.5\pm 1.2)^\circ\ . \tag{13.67b}$$

With the additional assumption of no direct *CP* violation in doubly-Cabibbo-suppressed $D$ decays [70–72], more stringent constraints are obtained:

$$1-|q/p| = -0.005\pm 0.007\,, \tag{13.68a}$$
$$\phi_D = (-0.2\pm 0.3)^\circ\ . \tag{13.68b}$$

More details on various theoretical and experimental aspects of $D^0$–$\overline{D}^0$ mixing can be found in Ref. [32].

Searches for *CP* violation in charged $D_{(s)}$ decays have been performed in many modes. Searches in decays mediated by Cabibbo-suppressed amplitudes are particularly interesting, since in other channels effects are likely to be too small to be observable in current experiments. Examples of relevant two-body modes are $D^+\to\pi^+\pi^0$, $K_SK^+$, $\phi\pi^+$ and $D_s^+\to K^+\pi^0$, $K_S\pi^+$, $\phi K^+$. The most precise results are $\mathcal{A}_{D^+\to K_SK^+} = +0.0011\pm 0.0017$ and $\mathcal{A}_{D_s^+\to K_S\pi^+} = +0.0038\pm 0.0048$ [61]. The precision of experiments is now sufficient that the effect from *CP* violation in the neutral kaon system can be seen in $D^+\to K_S\pi^+$ decays [73, 74].

Three- and four-body final states provide additional possibilities to search for *CP* violation, since effects may vary over the phase-space [75]. A number of methods have been proposed to exploit this feature and search for *CP* violation in ways that do not require modelling of the decay distribution [76–79]. Such methods are useful for analysis of charm decays since they are less sensitive to biases from production asymmetries, and are well suited to address the issue of whether or not *CP* violation effects are present. They can also be applied to tagged neutral $D$ meson as well as to charged $D_{(s)}$ decays (flavor tagging is typically achieved from the charge of the pion produced in $D^{*+}\to D^0\pi^+$ decays). The results of all searches to date are consistent with the absence of *CP* violation, with the most significant hint at the level of $2.7\sigma$ [80].

## 13.6 Beauty

### 13.6.1 *CP violation in mixing of* $B^0$ *and* $B_s^0$ *mesons*

The upper bound on the *CP* asymmetry in semileptonic $B$ decays [81] implies that *CP* violation in $B^0$–$\overline{B}^0$ mixing is a small effect (we use $\mathcal{A}_{\rm SL}/2\approx 1-|q/p|$, see Eq. (13.23)):

$$\mathcal{A}^d_{\rm SL} = (-2.1\pm 1.7)\times 10^{-3} \quad\Longrightarrow\quad |q/p| = 1.0010\pm 0.0008\ . \tag{13.69}$$

The Standard Model prediction is

$$\mathcal{A}^d_{\rm SL} = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta\right] \lesssim 0.001\ . \tag{13.70}$$

An explicit calculation gives $(-4.7\pm 0.6)\times 10^{-4}$ [35].

The experimental constraint on *CP* violation in $B_s^0$–$\overline{B}_s^0$ mixing is somewhat weaker than that in the $B^0$–$\overline{B}^0$ system [81]

$$\mathcal{A}_{\rm SL}^s = (-0.6 \pm 2.8) \times 10^{-3} \quad \Longrightarrow \quad |q/p| = 1.0003 \pm 0.0014 \; . \tag{13.71}$$

The Standard Model prediction is $\mathcal{A}_{\rm SL}^s = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta_s\right] \lesssim 10^{-4}$, with an explicit calculation giving $(2.22 \pm 0.27) \times 10^{-5}$ [35].

The fit to experimental data that results in the averages quoted above has a $\chi^2$ probability of 4.5% indicating some tension between the different measurements [61]. This originates in part from a result from the D0 collaboration for the inclusive same-sign dimuon asymmetry that deviates from the Standard Model prediction by $3.6\sigma$ [82]. As yet, this has not been confirmed by independent studies.

In models where $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is approximately real, such as the Standard Model, an upper bound on $\Delta\Gamma/\Delta m \approx \mathcal{R}e(\mathbf{\Gamma}_{12}/\mathbf{M}_{12})$ provides yet another upper bound on the deviation of $|q/p|$ from one. This constraint does not hold if $\mathbf{\Gamma}_{12}/\mathbf{M}_{12}$ is approximately imaginary. (An alternative parameterization uses $q/p = (1-\tilde{\epsilon}_B)/(1+\tilde{\epsilon}_B)$, leading to $\mathcal{A}_{\rm SL} \simeq 4\mathcal{R}e(\tilde{\epsilon}_B)$.)

### 13.6.2 *CP violation in interference of $B^0$ decays with and without mixing*

The small deviation (less than one percent) of $|q/p|$ from 1 implies that, at the present level of experimental precision, *CP* violation in $B^0$ mixing is a negligible effect. Thus, for the purpose of analyzing *CP* asymmetries in hadronic $B^0$ decays, we can use

$$\lambda_f = e^{-i\phi_{M(B^0)}}(\overline{A}_f/A_f) \; , \tag{13.72}$$

where $\phi_{M(B^0)}$ refers to the phase of $\mathbf{M}_{12}$ appearing in Eq. (13.28) that is appropriate for $B^0$–$\overline{B}^0$ oscillations. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_{M(B^0)}} = (V_{tb}^* V_{td})/(V_{tb} V_{td}^*) \; . \tag{13.73}$$

The class of *CP* violation effects in interference between mixing and decay is studied with final states that are common to $B^0$ and $\overline{B}^0$ decays [83–85]. It is convenient to rewrite Eq. (13.26) for $B^0$ decays as [86–88]

$$\mathcal{A}_f(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t) \; , \tag{13.74}$$

$$S_f \equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2} \; , \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} \; , \tag{13.75}$$

where we assume that $\Delta\Gamma = 0$ and $|q/p| = 1$. An alternative notation in use is $A_f \equiv -C_f$ – this $A_f$ should not be confused with the $A_f$ of Eq. (13.1), but in the limit that $|q/p| = 1$ is equivalent with the $\mathcal{A}_f$ of Eq. (13.21).

A large class of interesting processes proceed via quark transitions of the form $\overline{b} \to \overline{q}q\overline{q}'$ with $q' = s$ or $d$. For $q = c$ or $u$, there are contributions from both tree ($t$) and penguin ($p^{q_u}$, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 13.2) which carry different weak phases:

$$A_f = \left(V_{qb}^* V_{qq'}\right) t_f + \sum_{q_u = u,c,t} \left(V_{q_u b}^* V_{q_u q'}\right) p_f^{q_u} \; . \tag{13.76}$$

(The distinction between tree and penguin contributions is a heuristic one; the separation by the operator that enters is more precise. A more detailed discussion of the operator product expansion approach, which also includes higher order QCD corrections, can be found in Ref. [89, 90] for example.) Using CKM unitarity, the various decay amplitudes can always be written in terms of just two CKM combinations. For example, for $f = \pi\pi$, which proceeds via a $\overline{b} \to \overline{u}u\overline{d}$ transition, we can write

$$A_{\pi\pi} = (V_{ub}^* V_{ud})\, T_{\pi\pi} + (V_{tb}^* V_{td})\, P_{\pi\pi}^t \ , \tag{13.77}$$

where $T_{\pi\pi} = t_{\pi\pi} + p_{\pi\pi}^u - p_{\pi\pi}^c$ and $P_{\pi\pi}^t = p_{\pi\pi}^t - p_{\pi\pi}^c$. *CP*-violating phases in Eq. (13.77) appear only in the CKM elements, so that

$$\frac{\overline{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{(V_{ub} V_{ud}^*)\, T_{\pi\pi} + (V_{tb} V_{td}^*)\, P_{\pi\pi}^t}{(V_{ub}^* V_{ud})\, T_{\pi\pi} + (V_{tb}^* V_{td})\, P_{\pi\pi}^t} \ . \tag{13.78}$$

For $f = J/\psi K$, which proceeds via a $\overline{b} \to \overline{c}c\overline{s}$ transition, we can write

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u \ , \tag{13.79}$$

where $T_{\psi K} = t_{\psi K} + p_{\psi K}^c - p_{\psi K}^t$ and $P_{\psi K}^u = p_{\psi K}^u - p_{\psi K}^t$. A subtlety arises in this decay that is related to the fact that $B^0$ decays into a final $J/\psi K^0$ state while $\overline{B}^0$ decays into a final $J/\psi \overline{K}^0$ state. A common final state, *e.g.*, $J/\psi K_S$, is reached only via $K^0$–$\overline{K}^0$ mixing. Consequently, the phase factor (defined in Eq. (13.28)) corresponding to neutral $K$ mixing, $e^{-i\phi_{M(K)}} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V_{cs}^*)\, T_{\psi K} + (V_{ub} V_{us}^*)\, P_{\psi K}^u}{(V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \ . \tag{13.80}$$

For $q = s$ or $d$, there are only penguin contributions to $A_f$, that is, $t_f = 0$ in Eq. (13.76). (The tree $\overline{b} \to \overline{u}u\overline{q}'$ transition followed by $\overline{u}u \to \overline{q}q$ rescattering is included below in the $P^u$ terms.) Again, CKM unitarity allows us to write $A_f$ in terms of two CKM combinations. For example, for $f = \phi K_S$, which proceeds via a $\overline{b} \to \overline{s}s\overline{s}$ transition, we can write

$$\frac{\overline{A}_{\phi K_S}}{A_{\phi K_S}} = -\frac{(V_{cb} V_{cs}^*)\, P_{\phi K}^c + (V_{ub} V_{us}^*)\, P_{\phi K}^u}{(V_{cb}^* V_{cs})\, P_{\phi K}^c + (V_{ub}^* V_{us})\, P_{\phi K}^u} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \ , \tag{13.81}$$

where $P_{\phi K}^c = p_{\phi K}^c - p_{\phi K}^t$ and $P_{\phi K}^u = p_{\phi K}^u - p_{\phi K}^t$.

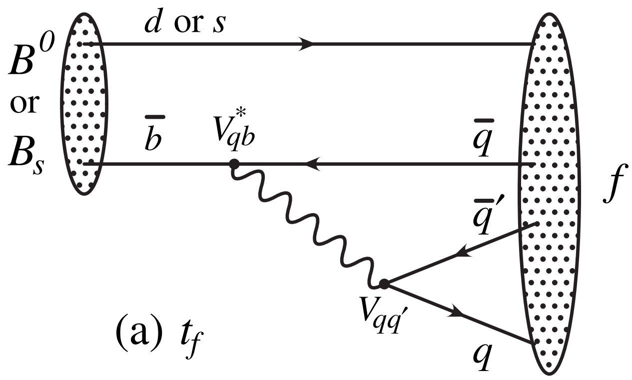
Since in general the amplitude $A_f$ involves two different weak phases, the corresponding decays can exhibit both *CP* violation in the interference of decays with and without mixing, $S_f \neq 0$, and *CP* violation in decay, $C_f \neq 0$. (At the present level of experimental precision, the contribution to $C_f$ from *CP* violation in mixing is negligible, see Eq. (13.69).) If the contribution from a second weak phase is suppressed, then the interpretation of $S_f$ in terms of Lagrangian *CP*-violating parameters is clean, while $C_f$ is small. If such a second contribution is not suppressed, $S_f$ depends on hadronic parameters and, if the relevant strong phase difference is large, $C_f$ is large.

A summary of $\bar{b} \to \bar{q}q\bar{q}'$ modes with $q' = s$ or $d$ is given in Table 13.1. The $\overline{b} \to \overline{d}d\overline{q}$ transitions lead to final states that are similar to those from $\overline{b} \to \overline{u}u\overline{q}$ transitions and have similar phase dependence. Final states that consist of two vector mesons ($\psi\phi$ and $\phi\phi$) are not *CP* eigenstates, and angular analysis is needed to separate the *CP*-even from the *CP*-odd contributions.

The cleanliness of the theoretical interpretation of $S_f$ can be assessed from the information in the last column of Table 13.1. In case of small uncertainties, the expression for $S_f$ in terms of CKM phases can be deduced from the fourth column of Table 13.1 in combination with Eq. (13.73) (and, for $b \to q\bar{q}s$ decays, the example in Eq. (13.80)). Here we consider several interesting examples.

For $B^0 \to J/\psi K_S$ and other $\overline{b} \to \overline{c}c\overline{s}$ processes, we can neglect the $P^u$ contribution to $A_f$, in the Standard Model, to an approximation that is better than one percent, giving

$$\lambda_{\psi K_S} = -e^{-2i\beta} \ \Rightarrow\ S_{\psi K_S} = \sin(2\beta)\ , \quad C_{\psi K_S} = 0\ . \tag{13.82}$$



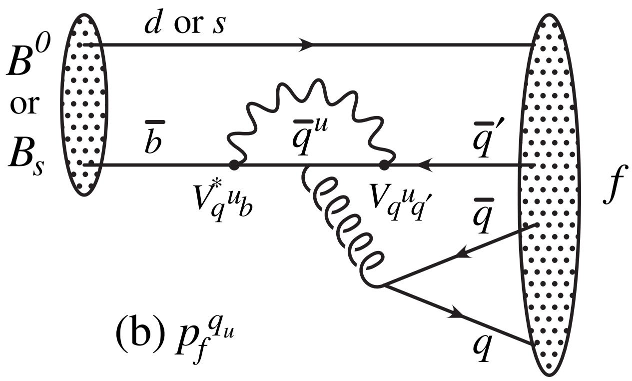




**Figure 13.2:** Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s^0 \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.

**Table 13.1:** Summary of $\bar{b} \to \bar{q}q\bar{q}'$ modes with $q' = s$ or $d$. The second and third columns give examples of hadronic final states (usually those which are experimentally most convenient to study). The fourth column gives the CKM dependence of the amplitude $A_f$, using the notation of Eqs. ((13.77), (13.79), (13.81)), with the dominant term first and the subdominant second. The suppression factor of the second term compared to the first is given in the last column. "Loop" refers to a penguin versus tree-suppression factor (it is mode-dependent and roughly $\mathcal{O}(0.2-0.3)$) and $\lambda \simeq 0.23$ is the expansion parameter of Eq. (13.34).

| $\bar{b} \to \bar{q}q\bar{q}'$ | $B^0 \to f$ | $B_s^0 \to f$ | CKM dependence of $A_f$ | Suppression |
|---|---|---|---|---|
| $\bar{b} \to \bar{c}c\bar{s}$ | $\psi K_S$ | $\psi\phi$ | $(V_{cb}^*V_{cs})T + (V_{ub}^*V_{us})P^u$ | loop $\times\ \lambda^2$ |
| $\bar{b} \to \bar{s}s\bar{s}$ | $\phi K_S$ | $\phi\phi$ | $(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})P^u$ | $\lambda^2$ |
| $\bar{b} \to \bar{u}u\bar{s}$ | $\pi^0 K_S$ | $K^+K^-$ | $(V_{cb}^*V_{cs})P^c + (V_{ub}^*V_{us})T$ | $\lambda^2$/loop |
| $\bar{b} \to \bar{c}c\bar{d}$ | $D^+D^-$ | $\psi K_S$ | $(V_{cb}^*V_{cd})T + (V_{tb}^*V_{td})P^t$ | loop |
| $\bar{b} \to \bar{s}s\bar{d}$ | $K_S K_S$ | $\phi K_S$ | $(V_{tb}^*V_{td})P^t + (V_{cb}^*V_{cd})P^c$ | $\lesssim 1$ |
| $\bar{b} \to \bar{u}u\bar{d}$ | $\pi^+\pi^-$ | $\rho^0 K_S$ | $(V_{ub}^*V_{ud})T + (V_{tb}^*V_{td})P^t$ | loop |
| $\bar{b} \to \bar{c}u\bar{d}$ | $D_{CP}\pi^0$ | $D_{CP}K_S$ | $(V_{cb}^*V_{ud})T + (V_{ub}^*V_{cd})T'$ | $\lambda^2$ |
| $\bar{b} \to \bar{c}u\bar{s}$ | $D_{CP}K_S$ | $D_{CP}\phi$ | $(V_{cb}^*V_{us})T + (V_{ub}^*V_{cs})T'$ | $\lesssim 1$ |

It is important to verify experimentally the level of suppression of the penguin contribution. Methods based on flavor symmetries [91–94] allow limits to be obtained. All are currently consistent with the $P^u$ term being negligible. Explicit calculations [94–97] also support this conclusion.

In the presence of new physics, $A_f$ is still likely to be dominated by the $T$ term, but the mixing amplitude might be modified. Thus, model-independently, $C_f \approx 0$ while $S_f$ cleanly determines the mixing phase $(\phi_M - 2\arg(V_{cb}V_{cd}^*))$. The experimental measurement [61], $S_{\psi K} = +0.699 \pm 0.017$, gave the first precision test of the Kobayashi-Maskawa mechanism, and its consistency with the predictions for $\sin 2\beta$ makes it very likely that this mechanism is indeed the dominant source of *CP* violation in the quark sector.

For $B^0 \to \phi K_S$ and other $\bar{b} \to \bar{s}s\bar{s}$ processes (as well as some $\bar{b} \to \bar{u}u\bar{s}$ processes), we can neglect the subdominant contributions, in the Standard Model, to an approximation that is good to the order of a few percent:

$$\lambda_{\phi K_S} = -e^{-2i\beta} \;\Rightarrow\; S_{\phi K_S} = \sin 2\beta\,, \quad C_{\phi K_S} = 0\,. \tag{13.83}$$

A review of explicit calculations of the effects of subleading amplitudes can be found in Ref. [98]. In the presence of new physics, both $A_f$ and $\mathbf{M}_{12}$ can have contributions that are comparable in size to those of the Standard Model and carry new weak phases. Such a situation gives several interesting consequences for penguin-dominated $\bar{b} \to \bar{q}q\bar{s}$ decays $(q = u, d, s)$ to a final state $f$:

1. The value of $-\eta_f S_f$ may be different from $S_{\psi K_S}$ by more than a few percent, where $\eta_f$ is the *CP* eigenvalue of the final state.
2. The values of $\eta_f S_f$ for different final states $f$ may be different from each other by more than a few percent (for example, $S_{\phi K_S} \neq S_{\eta' K_S}$).
3. The value of $C_f$ may be different from zero by more than a few percent.

While a clear interpretation of such signals in terms of Lagrangian parameters will be difficult because, under these circumstances, hadronic parameters play a role, any of the above three options will clearly signal new physics. In addition, flavor symmetry relations, such as those that relate observables in $B \to K\pi$ decays [99,100] can be used to provide further tests of the Standard Model. Fig. 13.3 summarizes the present experimental results: none of the possible signatures listed above is unambiguously established, but there is definitely still room for new physics.

For the $\bar{b} \to \bar{u}u\bar{d}$ process $B \to \pi\pi$ and other related channels, the penguin-to-tree ratio can be estimated using SU(3) relations and experimental data on related $B \to K\pi$ decays. The result (for $\pi\pi$) is that the suppression is at the level of $0.2 - 0.3$ and so cannot be neglected. The expressions for $S_{\pi\pi}$ and $C_{\pi\pi}$ to leading order in $R_{PT} \equiv (|V_{tb}V_{td}|\, P^t_{\pi\pi})/(|V_{ub}V_{ud}|\, T_{\pi\pi})$ are:

$$\lambda_{\pi\pi} = e^{2i\alpha}\left[(1 - R_{PT}e^{-i\alpha})/(1 - R_{PT}e^{+i\alpha})\right] \;\Rightarrow \tag{13.84}$$

$$S_{\pi\pi} \approx \sin 2\alpha + 2\,\mathcal{R}e(R_{PT})\cos 2\alpha \sin\alpha\,, \quad C_{\pi\pi} \approx 2\,\mathcal{I}m(R_{PT})\sin\alpha\,. \tag{13.85}$$

Note that $R_{PT}$ is mode-dependent and, in particular, could be different for $\pi^+\pi^-$ and $\pi^0\pi^0$. If strong phases can be neglected, then $R_{PT}$ is real, resulting in $C_{\pi\pi} = 0$. The size of $C_{\pi\pi}$ is an indicator of how large the strong phase is. The present experimental average is $C_{\pi^+\pi^-} = -0.31 \pm 0.03$ [61]. As concerns $S_{\pi\pi}$, it is clear from Eq. (13.85) that the relative size or strong phase of the penguin contribution must be known to extract $\alpha$. The theoretical uncertainty stemming from $|R_{PT}| \not\ll 1$ is referred to in the literature as penguin pollution.

The cleanest solution involves isospin relations among the $B \to \pi\pi$ amplitudes [101]:

$$\frac{1}{\sqrt{2}}A_{\pi^+\pi^-} + A_{\pi^0\pi^0} = A_{\pi^+\pi^0}\,. \tag{13.86}$$

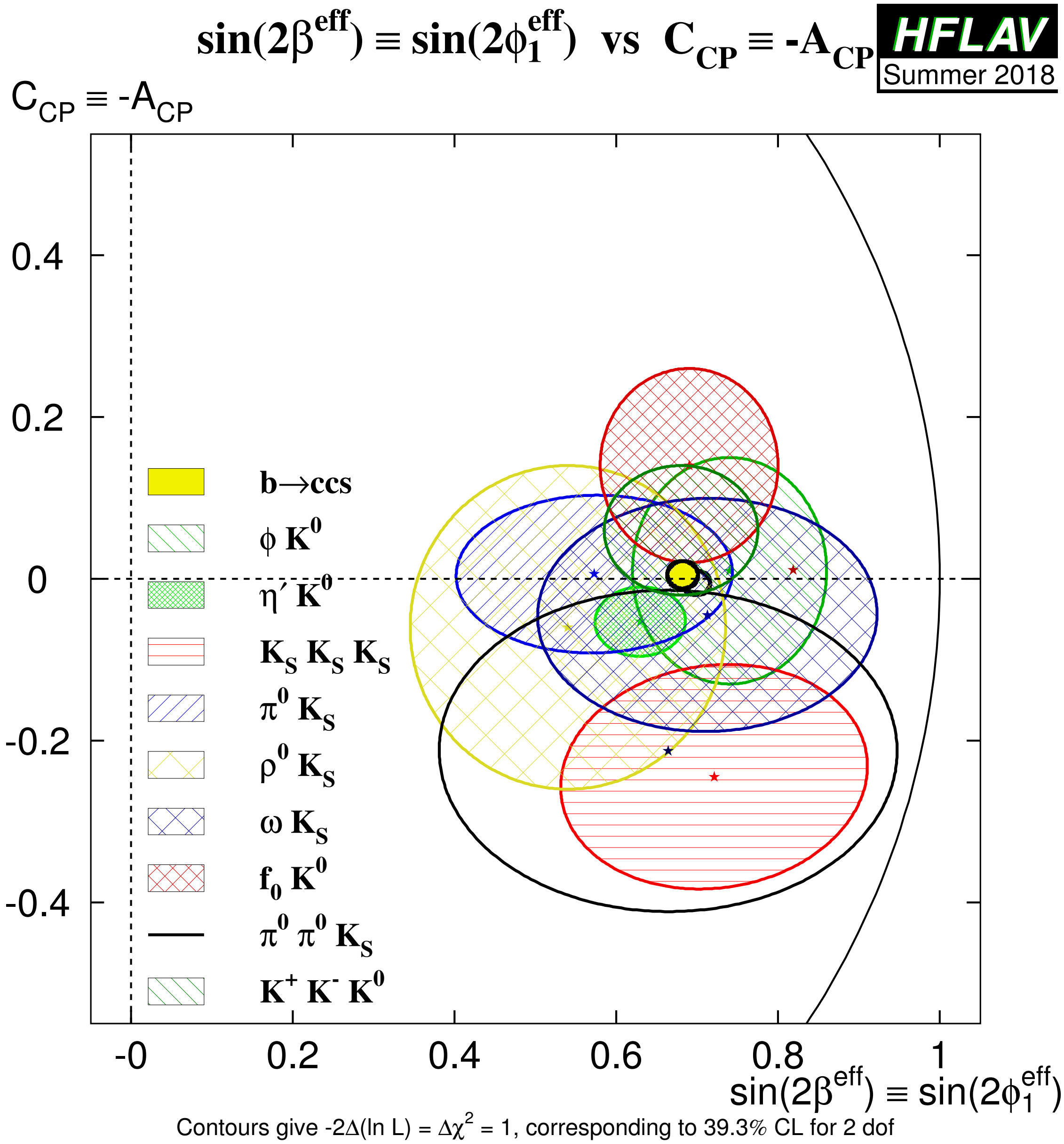



**Figure 13.3:** Summary of the results [61] of time-dependent analyses of $b \to q\bar{q}s$ decays, which are potentially sensitive to new physics.

The method exploits the fact that the penguin contribution to $P^t_{\pi\pi}$ is pure $\Delta I = 1/2$ (this is not true for the electroweak penguins which, however, are expected to be small), while the tree contribution to $T_{\pi\pi}$ contains amplitudes that are both $\Delta I = 1/2$ and $\Delta I = 3/2$. A simple geometric construction then allows one to find $R_{PT}$ and extract $\alpha$ cleanly from $S_{\pi^+\pi^-}$. The key experimental difficulty is that one must measure accurately the separate rates for $B^0$ and $\overline{B}^0 \to \pi^0\pi^0$.

*CP* asymmetries in $B \to \rho\pi$ and $B \to \rho\rho$ can also be used to determine $\alpha$. In particular, the $B \to \rho\rho$ measurements are presently very significant in constraining $\alpha$. The extraction proceeds via isospin analysis similar to that of $B \to \pi\pi$. There are, however, several important differences. First, due to the finite width of the $\rho$ mesons, a final $(\rho\rho)_{I=1}$ state is possible [102]. The effect is, however, of the order of $(\Gamma_\rho/m_\rho)^2 \sim 0.04$. Second, due to the presence of three helicity states for the two vector mesons, angular analysis is needed to separate the *CP*-even and *CP*-odd components. The theoretical expectation is that the *CP*-odd component is small. This is supported by experiments which find that the $\rho^+\rho^-$ and $\rho^\pm\rho^0$ modes are dominantly longitudinally polarized. Third, an important advantage of the $\rho\rho$ modes is that the penguin contribution is expected to be small due to different hadronic dynamics. This expectation is confirmed by the smallness of $\mathcal{B}(B^0 \to \rho^0\rho^0) = (0.95 \pm 0.16) \times 10^{-6}$ compared to $\mathcal{B}(B^0 \to \rho^+\rho^-) = (24.2 \pm 3.1) \times 10^{-6}$ [61]. Thus, $S_{\rho^+\rho^-}$ is not far from $\sin 2\alpha$. Finally, both $S_{\rho^0\rho^0}$ and $C_{\rho^0\rho^0}$ are experimentally accessible, which may allow a precise determination of $\alpha$. However, a full isospin analysis should allow that the fractions of longitudinal polarization in $B$ and $\overline{B}$ decays may differ, which has not yet been done by the experiments.

Detailed discussion of the determination of $\alpha$ with these methods, and the latest world average, can be found in Refs. [103,104]. The consistency between the range of $\alpha$ determined by the $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism.

All modes discussed in this Section so far have possible contributions from penguin amplitudes. As shown in Table 13.1, *CP* violation can also be studied with final states, typically containing charmed mesons, where no such contribution is possible. The neutral charmed meson must be reconstructed in a final state, such as a *CP* eigenstate, common to $D^0$ and $\overline{D}^0$ so that the amplitudes for the $B$ and $\overline{B}$ meson decays interfere. Although there is a second tree amplitude with a different weak phase, the contributions of the different diagrams can in many cases be separated experimentally (for example by exploiting different decays of the neutral $D$ mesons) making these channels very clean theoretically. The first determination of $\sin(2\beta)$, with significance of *CP* violation over $5\sigma$, with this method has recently been reported [105]. Moreover, the interference between the two tree diagrams gives sensitivity to $\gamma$, as will be discussed in Section 13.6.4.

### 13.6.3 *CP violation in interference of $B_s^0$ decays with and without mixing*

As discussed in Section 13.6.1, the world average for $|q/p|$ in the $B_s^0$ system currently deviates from the Standard Model expectation due to an anomalous value of the dimuon asymmetry. Attributing the dimuon asymmetry result to a fluctuation, we again neglect the deviation of $|q/p|$ from 1, and use

$$\lambda_f = e^{-i\phi_M(B_s^0)}(\overline{A}_f/A_f) \; . \tag{13.87}$$

Within the Standard Model,

$$e^{-i\phi_{M(B_s^0)}} = (V_{tb}^* V_{ts})/(V_{tb} V_{ts}^*) \; . \tag{13.88}$$

Note that $\mathit{\Delta\Gamma}/\mathit{\Gamma} = 0.132 \pm 0.008$ [61] and therefore $y$ should not be put to zero in Eqs. (13.17a) and (13.17b). However, $|q/p| = 1$ is expected to hold to an even better approximation than for $B^0$ mesons. One therefore obtains

$$\mathcal{A}_f(t) = \frac{S_f \sin(\mathit{\Delta} mt) - C_f \cos(\mathit{\Delta} mt)}{\cosh(\mathit{\Delta\Gamma} t/2) - A_f^{\mathit{\Delta\Gamma}} \sinh(\mathit{\Delta\Gamma} t/2)} \; , \tag{13.89}$$

$$A_f^{\mathit{\Delta\Gamma}} \equiv \frac{-2\,\mathcal{R}e(\lambda_f)}{1 + |\lambda_f|^2} \; . \tag{13.90}$$

The presence of the $A_f^{\mathit{\Delta\Gamma}}$ term implies that information on $\lambda_f$ can be obtained from analyses that do not use tagging of the initial flavor, through so-called effective lifetime measurements [106].

The $B_s^0 \to J/\psi\phi$ decay proceeds via the $\overline{b} \to \overline{c}c\overline{s}$ transition. The *CP* asymmetry in this mode thus determines (with angular analysis to disentangle the *CP*-even and *CP*-odd components of the final state) $\sin 2\beta_s$, where $\beta_s$ is defined in Eq. (13.38) [107]. The $B_s^0 \to J/\psi\pi^+\pi^-$ decay, which has a large contribution from $J/\psi f_0(980)$ and is assumed to also proceed dominantly via the $\overline{b} \to \overline{c}c\overline{s}$

transition, has also been used to determine $\beta_s$. In this case no angular analysis is necessary, since the final state has been shown to be dominated by the *CP*-odd component [108]. The combination of measurements yields [61]

$$2\beta_s = 0.050 \pm 0.019\,, \tag{13.91}$$

consistent with the Standard Model prediction, assuming negligible penguin contributions, $\beta_s = 0.0184 \pm 0.0004$ [20].

A time-dependent *CP* asymmetry was established in $B_s^0 \to K^+K^-$ decay, which proceeds via the $\overline{b} \to \overline{u}u\overline{s}$ transition [109]:

$$C_{KK} = +0.172 \pm 0.039,\ , \quad S_{KK} = +0.139 \pm 0.032\ . \tag{13.92}$$

For both $C_{KK}$ and $S_{KK}$, the hadronic ratio $(T/P^c)$ plays an important role (see Table 13.1), making a clean theoretical interpretation challenging. First results on the $\overline{b} \to \overline{q}q\overline{s}$ decays $B_s^0 \to \phi\phi$ and $K^{*0}\overline{K}^{*0}$ have also been reported. Parameters of *CP* violation have also been determined from the decay-time distributions of $B_s^0 \to D_s^{\mp}K^{\pm}$ and $D_s^{\mp}K^{\pm}\pi^+\pi^-$ decays, involving interference between $\overline{b} \to \overline{c}u\overline{s}$ and $\overline{b} \to \overline{u}c\overline{s}$ amplitudes.

### 13.6.4 *Direct CP violation in the B system*

An interesting class of decay modes is that of the tree-level decays $B^{\pm} \to D^{(*)}K^{\pm}$, which allow a theoretically pristine determination of the angle $\gamma$ [110–115]. The method uses the decays $B^+ \to D^0K^+$, which proceeds via the quark transition $\overline{b} \to \overline{u}c\overline{s}$, and $B^+ \to \overline{D}^0K^+$, which proceeds via the quark transition $\overline{b} \to \overline{c}u\overline{s}$, with the $D^0$ and $\overline{D}^0$ decaying into a common final state. The decays into common final states, such $(\pi^0K_S)_DK^+$, involve interference effects between the two amplitudes, with sensitivity to the relative phase, $\delta + \gamma$ ($\delta$ is the relevant strong phase difference). The *CP*-conjugate processes are sensitive to $\delta - \gamma$. Measurements of branching ratios and *CP* asymmetries allow the determination of $\gamma$ and $\delta$ from amplitude triangle relations. The method suffers from discrete ambiguities but, since all hadronic parameters can be determined from the data, has negligible theoretical uncertainty [116].

Unfortunately, the smallness of the CKM-suppressed $b \to u$ transitions makes it difficult to use the simplest methods alone [110–112] to determine $\gamma$. These difficulties are overcome (and the discrete ambiguities are removed) by performing a Dalitz plot analysis for multi-body $D$ decays [113–115]. Detailed discussion of the determination of $\gamma$ with these methods can be found in Ref. [104].

Constraints on $\gamma$ from combinations of results on various $B \to D^{(*)}K^{(*)}$ processes have been obtained by experiments [117, 118]. The latest world average is [61, 104]

$$\gamma = \left(66.2\,^{+3.4}_{-3.6}\right)^{\circ}\ . \tag{13.93}$$

The consistency between the range of $\gamma$ determined by the $B \to DK$ measurements and the range allowed by CKM fits (excluding these direct determinations) provides further support to the Kobayashi-Maskawa mechanism. As more data become available, determinations of $\gamma$ from $B_s^0 \to D_s^{\mp}K^{\pm}$ [119, 120] and $B^0 \to DK^{*0}$ [121–124] are expected to also give competitive measurements.

Decays to the final state $K^{\mp}\pi^{\pm}$ provided the first observations of direct *CP* violation in both $B^0$ and $B_s^0$ systems. The asymmetry arises due to interference between tree and penguin diagrams [125], similar to the effect discussed in Section 13.6.2. In principle, measurements of $\mathcal{A}_{B^0\to K^+\pi^-}$ and $\mathcal{A}_{B_s^0\to K^-\pi^+}$ could be used to determine the weak phase difference $\gamma$, but lack of knowledge of the relative magnitude and strong phase of the contributing amplitudes limits the achievable precision. The uncertainties on these hadronic parameters can be reduced by exploiting flavor symmetries, which predict a number of relations between asymmetries in different modes. One such relation is that the partial rate differences for $B^0$ and $B_s^0$ decays to $K^{\mp}\pi^{\pm}$ are expected to be approximately equal and opposite [126], which is consistent with current data. It is also expected that the partial rate asymmetries for $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$ should be approximately equal; however, the experimental results currently show a significant discrepancy [61]:

$$\mathcal{A}_{B^0\to K^+\pi^-} = -0.083 \pm 0.004\,, \quad \mathcal{A}_{B^+\to K^+\pi^0} = +0.029 \pm 0.013\,. \tag{13.94}$$

It is therefore of great interest to understand whether this originates from Standard Model QCD corrections, or whether it is a signature of new dynamics. Improved tests of a more precise relation between the partial rate differences of all four $K\pi$ final states [127–130], currently limited by knowledge of the *CP* asymmetry in $\overline{B}^0 \to K_S\pi^0$ decays, may help to resolve the situation.

It is also of interest to investigate whether similar patterns appear among the *CP* violating asymmetries in $B$ meson decays to final states containing one pseudoscalar and one vector meson. Since the vector resonance decays to two particles, such channels can be studied through Dalitz plot analysis of the three-body final state. Model-independent analyses of $B^+ \to K^+K^-K^+$, $\pi^+\pi^-K^+$, $\pi^+\pi^-\pi^+$ and $K^+K^-\pi^+$ decays have revealed large *CP* violation effects in certain regions of phase space [131]. For the $B^+ \to K^+K^-\pi^+$ decay, an amplitude analysis has established a large *CP* violation effect associated with $\pi\pi \leftrightarrow KK$ S-wave rescattering [132]. In $B^+ \to \pi^+\pi^-\pi^+$ decays, amplitude analysis has established *CP* violation effects in the decay amplitude involving the $f_2(1270)$ resonance, in the $\pi^+\pi^-$ S-wave at low invariant mass, and in the interference between the $\pi^+\pi^-$ S-wave and the P-wave $B^+ \to \rho(770)^0\pi^+$ amplitude [133, 134]. For the other channels it remains to be seen whether the *CP* violation effects are associated to particular resonances or to interference effects, which will be necessary to understand the underlying dynamics.

## 13.7 Summary and Outlook

*CP* violation has been experimentally established in $K$, $D$ and $B$ meson decays. A full list of *CP* asymmetries that have been measured at a level higher than $5\sigma$ is given in the introduction to this review. In Section 13.1.4 we introduced three types of *CP*-violating effects. Examples of these three types include the following:

1. All three types of *CP* violation have been observed in $K \to \pi\pi$ decays:

$$\mathcal{R}e(\epsilon') = \frac{1}{6}\left(\left|\frac{\overline{A}_{\pi^0\pi^0}}{A_{\pi^0\pi^0}}\right| - \left|\frac{\overline{A}_{\pi^+\pi^-}}{A_{\pi^+\pi^-}}\right|\right) = (2.5 \pm 0.4) \times 10^{-6}\,, \quad \text{(I)} \tag{13.95a}$$

$$\mathcal{R}e(\epsilon) = \frac{1}{2}\left(1 - \left|\frac{q}{p}\right|\right) = (1.66 \pm 0.02) \times 10^{-3}\,, \quad \text{(II)} \tag{13.95b}$$

$$\mathcal{I}m(\epsilon) = -\frac{1}{2}\mathcal{I}m(\lambda_{(\pi\pi)_{I=0}}) = (1.57 \pm 0.02) \times 10^{-3}\,. \quad \text{(III)} \tag{13.95c}$$

2. For $D$ mesons, *CP* violation in decay has been established in the difference of asymmetries for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays.

$$\Delta a_{CP} = \frac{|\overline{A}_{K^+K^-}/A_{K^+K^-}|^2 - 1}{|\overline{A}_{K^+K^-}/A_{K^+K^-}|^2 + 1} - \frac{|\overline{A}_{\pi^+\pi^-}/A_{\pi^+\pi^-}|^2 - 1}{|\overline{A}_{\pi^+\pi^-}/A_{\pi^+\pi^-}|^2 + 1} = (-0.164 \pm 0.028) \times 10^{-3}\,, \quad \text{(I)} \tag{13.96}$$

3. In the $B$ meson system, *CP* violation in decay has been observed in, for example, $B^0 \to K^+\pi^-$ transitions, while *CP* violation in interference of decays with and without mixing has been

observed in, for example, the $B^0 \to J/\psi K_S$ channel:

$$\mathcal{A}_{K^+\pi^-} = \frac{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 - 1}{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 + 1} = -0.083 \pm 0.004\,, \qquad \text{(I)} \qquad (13.97a)$$

$$S_{\psi K} = \mathcal{I}m(\lambda_{\psi K}) = +0.699 \pm 0.017\,. \qquad \text{(III)} \qquad (13.97b)$$

Based on Standard Model predictions, further observations of *CP* violation in $B^0$, $B^+$ and $B_s^0$ decays seem likely in the near future, at both LHCb and its upgrades [135–137] as well as the Belle II experiment [138]. The first observation of *CP* violation in $b$ baryons is also likely to be within reach of LHCb. Further improvements in the sensitivity to *CP* violation effects in the charm sector can also be anticipated, though uncertainty in the Standard Model predictions makes it difficult to forecast whether or not additional discoveries will be forthcoming. A number of upcoming experiments have potential to make significant progress on rare kaon decays. Observables that are subject to clean theoretical interpretation, such as $\beta$ from $S_{\psi K_S}$, $\beta_s$ from $B_s^0 \to J/\psi\phi$, $\mathcal{B}(K_L \to \pi^0\nu\overline{\nu})$ and $\gamma$ from *CP* violation in $B \to DK$ decays, are of particular value for constraining the values of the CKM parameters and probing the flavor sector of extensions to the Standard Model. Progress in lattice QCD calculations is also needed to complement the anticipated experimental results. Other probes of *CP* violation now being pursued experimentally include the electric dipole moments of the neutron and electron, and the decays of tau leptons. Additional processes that are likely to play an important role in future *CP* studies include top-quark production and decay, Higgs boson decays and neutrino oscillations.

All measurements of *CP* violation to date are consistent with the predictions of the Kobayashi-Maskawa mechanism of the Standard Model. In fact, it is now established that the KM mechanism plays a dominant role in the *CP* violation measured in the quark sector. However, a dynamically-generated matter-antimatter asymmetry of the universe requires additional sources of *CP* violation, and such sources are naturally generated by extensions to the Standard Model. New sources might eventually reveal themselves as small deviations from the predictions of the KM mechanism, or else might not be observable in the quark sector at all, but observable with future probes such as neutrino oscillations or electric dipole moments. The fundamental nature of *CP* violation demands a vigorous search.

A number of excellent reviews of *CP* violation are available [139–146], where the interested reader may find a detailed discussion of the various topics that are briefly reviewed here.

We thank David Kirkby for significant contributions to earlier versions of this review.

## 13.8 Observed *CP* violation effects

We conclude by listing the observables where *CP* violation has been observed at a level above $5\sigma$ [43, 61, 81]:

- Indirect *CP* violation in $K \to \pi\pi$ and $K \to \pi\ell\nu$ decays, and in the $K_L \to \pi^+\pi^-e^+e^-$ decay, is given by

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}\,. \qquad (13.98)$$

- Direct *CP* violation in $K \to \pi\pi$ decays is given by

$$\mathcal{R}e(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}\,. \qquad (13.99)$$

- *CP* violation in the interference of mixing and decay in the tree-dominated $b \to c\bar{c}s$ transitions, such as $B^0 \to \psi K^0$, is given by (we use $K^0$ throughout to denote results that combine $K_S$ and $K_L$ modes, but use the sign appropriate to $K_S$):

$$S_{\psi K^0} = +0.699 \pm 0.017\,. \qquad (13.100)$$

- $CP$ violation in the interference of mixing and decay in modes governed by the tree-dominated $b \to c\bar{u}d$ transitions is given by

$$S_{D^{(*)}h^0} = +0.71 \pm 0.09\,, \tag{13.101}$$

- $CP$ violation in the interference of mixing and decay in various modes related to $b \to c\bar{c}d$ transitions is given by

$$\begin{aligned} S_{\psi\pi^0} &= -0.86 \pm 0.14\,, \\ S_{D^+D^-} &= -0.84 \pm 0.12\,, \\ S_{D^{*\pm}D^{\mp}} &= -0.81 \pm 0.06\,, \\ S_{D^{*+}D^{*-}} &= -0.71 \pm 0.09\,. \end{aligned} \tag{13.102}$$

- $CP$ violation in the interference of mixing and decay in various modes related to $b \to q\bar{q}s$ (penguin) transitions is given by

$$\begin{aligned} S_{\phi K^0} &= +0.74\,^{+0.11}_{-0.13}\,, \\ S_{\eta' K^0} &= +0.63 \pm 0.06\,, \\ S_{f_0 K^0} &= +0.69\,^{+0.10}_{-0.12}\,, \\ S_{K^+K^-K_S} &= +0.68\,^{+0.09}_{-0.10}\,. \end{aligned} \tag{13.103}$$

- $CP$ violation in the interference of mixing and decay in the $B^0 \to \pi^+\pi^-$ mode is given by

$$S_{\pi^+\pi^-} = -0.67 \pm 0.03\,. \tag{13.104}$$

- Direct $CP$ violation in the $B^0 \to \pi^+\pi^-$ mode is given by

$$C_{\pi^+\pi^-} = -0.31 \pm 0.03\,. \tag{13.105}$$

- Direct $CP$ violation in the $B^0_s \to K^+K^-$ mode is given by

$$C_{K^+K^-} = 0.17 \pm 0.03\,. \tag{13.106}$$

- Direct $CP$ violation in $B^+ \to D^{(*)}_+ K^+$ decays ($D^{(*)}_+$ is the $CP$-even neutral $D^{(*)}$ state) are given by

$$\mathcal{A}_{B^+ \to D_+ K^+} = +0.139 \pm 0.009 \quad \text{and} \quad \mathcal{A}_{B^+ \to D^*_+ K^+} = -0.109 \pm 0.019\,, \tag{13.107}$$

while the corresponding quantity in the case that the neutral $D$ meson is reconstructed in the suppressed $K^-\pi^+$ final state is

$$\mathcal{A}_{B^+ \to D_{K^-\pi^+} K^+} = -0.453 \pm 0.026\,, \tag{13.108}$$

- Direct $CP$ violation has also been observed in $B^+ \to DK^+$ decays through differences between the Dalitz plot distributions of subsequent $D \to K_S\pi^+\pi^-$ decays.
- Direct $CP$ violation in the $B^0 \to K^+\pi^-$ mode is given by

$$\mathcal{A}_{B^0 \to K^+\pi^-} = -0.083 \pm 0.004\,. \tag{13.109}$$

- Direct $CP$ violation in the $B^0_s \to K^-\pi^+$ mode is given by

$$\mathcal{A}_{B^0_s \to K^-\pi^+} = +0.225 \pm 0.012\,. \tag{13.110}$$

- Direct $CP$ violation in $B^+ \to K^+K^-\pi^+$ decays is given by

$$\mathcal{A}_{B^+\to K^+K^-\pi^+} = -0.118 \pm 0.022\,. \tag{13.111}$$

- Large $CP$ violation effects have been observed model-independently in certain regions of the phase space of $B^+ \to K^+K^-K^+$, $K^+K^-\pi^+$, $\pi^+\pi^-K^+$ and $\pi^+\pi^-\pi^+$ decays. An amplitude analysis has established a large $CP$ violation effect associated with $\pi\pi \leftrightarrow KK$ S-wave rescattering in $B^+ \to K^+K^-\pi^+$ decays. In $B^+ \to \pi^+\pi^-\pi^+$ decays, amplitude analysis has established $CP$ violation effects in the decay amplitude involving the $f_2(1270)$ resonance, in the $\pi^+\pi^-$ S-wave at low invariant mass, and in the interference between the $\pi^+\pi^-$ S-wave and the P-wave $B^+ \to \rho(770)^0\pi^+$ amplitude.
- Direct $CP$ violation has been established in the difference of asymmetries for $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays

$$\Delta a_{CP} = (-0.164 \pm 0.028) \times 10^{-3}\,. \tag{13.112}$$

***References***

[1] J. H. Christenson *et al.*, Phys. Rev. Lett. **13**, 138 (1964).

[2] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **87**, 091801 (2001), [hep-ex/0107013].

[3] K. Abe *et al.* (Belle), Phys. Rev. Lett. **87**, 091802 (2001), [hep-ex/0107061].

[4] H. Burkhardt *et al.* (NA31), Phys. Lett. **B206**, 169 (1988).

[5] V. Fanti *et al.* (NA48), Phys. Lett. **B465**, 335 (1999), [hep-ex/9909022].

[6] A. Alavi-Harati *et al.* (KTeV), Phys. Rev. Lett. **83**, 22 (1999), [hep-ex/9905060].

[7] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **93**, 131801 (2004), [hep-ex/0407057].

[8] Y. Chao *et al.* (Belle), Phys. Rev. Lett. **93**, 191802 (2004), [hep-ex/0408100].

[9] A. Poluektov *et al.* (Belle), Phys. Rev. **D81**, 112002 (2010), [arXiv:1003.3360].

[10] P. del Amo Sanchez *et al.* (BaBar), Phys. Rev. **D82**, 072004 (2010), [arXiv:1007.0504].

[11] R. Aaij *et al.* (LHCb), Phys. Lett. **B712**, 203 (2012), [Erratum-ibid. **B713**, 351 (2012)], [arXiv:1203.3662].

[12] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **110**, 221601 (2013), [arXiv:1304.6173].

[13] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **122**, 211803 (2019), [arXiv:1903.08726].

[14] See the review on "Neutrino Masses, Mixing, and Oscillations," in this *Review*.

[15] See the review on "Tests of Conservation Laws," in this *Review*.

[16] J. P. Lees *et al.* (BaBar), Phys. Rev. Lett. **109**, 211801 (2012), [arXiv:1207.5832].

[17] J. Bernabeu, F. Martinez-Vidal and P. Villanueva-Perez, JHEP **08**, 064 (2012), [arXiv:1203.0171].

[18] See, for example, R. F. Streater and A. S. Wightman, *CPT, Spin and Statistics, and All That,* reprinted by Addison-Wesley, New York (1989).

[19] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[20] J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. **C41**, 1 (2005), updated results and plots available at: `http://ckmfitter.in2p3.fr`, [hep-ph/0406184].

[21] M. Bona *et al.* (UTfit), JHEP **10**, 081 (2006), updated results and plots available at: `http://www.utfit.org/UTfit`, [hep-ph/0606167].

[22] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967), [Usp. Fiz. Nauk161,no.5,61(1991)].

[23] A. Riotto, in "Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June 29-July 17, 1998," 326–436 (1998), [hep-ph/9807454].

[24] M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).

[25] S. Davidson, E. Nardi and Y. Nir, Phys. Rept. **466**, 105 (2008), [arXiv:0802.2962].

[26] G. Aad *et al.* (ATLAS), Phys. Lett. **B716**, 1 (2012), [arXiv:1207.7214].

[27] S. Chatrchyan *et al.* (CMS), Phys. Lett. **B716**, 30 (2012), [arXiv:1207.7235].

[28] S. Okubo, Phys. Lett. **5**, 165 (1963).

[29] G. Zweig (1964), *An $SU_3$ model for strong interaction symmetry and its breaking; Version 2*, CERN-TH-412.

[30] J. Iizuka, Prog. Theor. Phys. Suppl. **37**, 21 (1966).

[31] V. Weisskopf and E. P. Wigner, Z. Phys. **63**, 54 (1930).

[32] See the review on "$D^0$–$\overline{D}^0$ Mixing" in this *Review*.

[33] O. Long *et al.*, Phys. Rev. **D68**, 034010 (2003), [hep-ex/0303030].

[34] M. Gronau, Y. Grossman and J. L. Rosner, Phys. Lett. **B508**, 37 (2001), [hep-ph/0103110].

[35] M. Artuso, G. Borissov and A. Lenz, Rev. Mod. Phys. **88**, 045002 (2016), [arXiv:1511.09466].

[36] L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).

[37] C. Abel *et al.* (nEDM), Phys. Rev. Lett. **124**, 081803 (2020), [arXiv:2001.11966].

[38] R. J. Crewther *et al.*, Phys. Lett. **B88**, 123 (1979), [Erratum-ibid. **B91**, 487 (1980)].

[39] See the review on "Cabibbo-Kobayashi-Maskawa Mixing Matrix," in this *Review*.

[40] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[41] A. J. Buras, M. E. Lautenbacher and G. Ostermaier, Phys. Rev. **D50**, 3433 (1994), [hep-ph/9403384].

[42] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).

[43] See the $K$-meson Listings in this *Review*.

[44] See the review on "$CP$ violation in $K_S \to 3\pi$," in this *Review*.

[45] T. Blum *et al.*, Phys. Rev. **D91**, 074502 (2015), [arXiv:1502.00263].

[46] Z. Bai *et al.* (RBC, UKQCD), Phys. Rev. Lett. **115**, 212001 (2015), [arXiv:1505.07863].

[47] A. J. Buras *et al.*, JHEP **11**, 202 (2015), [arXiv:1507.06345].

[48] Y. Grossman and Y. Nir, Phys. Lett. **B398**, 163 (1997), [hep-ph/9701313].

[49] L. S. Littenberg, Phys. Rev. **D39**, 3322 (1989).

[50] A. J. Buras, Phys. Lett. **B333**, 476 (1994), [hep-ph/9405368].

[51] G. Buchalla and A. J. Buras, Nucl. Phys. **B400**, 225 (1993).

[52] A. J. Buras *et al.*, JHEP **11**, 033 (2015), [arXiv:1503.02693].

[53] J. K. Ahn *et al.* (KOTO), Phys. Rev. Lett. **122**, 021802 (2019), [arXiv:1810.09655].

[54] J. K. Ahn *et al.* (KOTO), Phys. Rev. Lett. **126**, 121801 (2021), [arXiv:2012.07571].

[55] E. Cortina Gil *et al.* (NA62), JHEP **06**, 093 (2021), [arXiv:2103.15389].

[56] B. Aubert *et al.* (BaBar), Phys. Rev. Lett. **98**, 211802 (2007), [hep-ex/0703020].

[57] M. Staric *et al.* (Belle), Phys. Rev. Lett. **98**, 211803 (2007), [hep-ex/0703036].

[58] T. Aaltonen *et al.* (CDF), Phys. Rev. Lett. **100**, 121802 (2008), [arXiv:0712.1567].

[59] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **110**, 101802 (2013), [arXiv:1211.1230].

[60] R. Aaij *et al.* (LHCb) (2021), to appear in PRL, [arXiv:2106.03744].

[61] Y. S. Amhis *et al.* (HFLAV), Eur. Phys. J. **C81**, 226 (2021), updated results and plots available at `https://hflav.web.cern.ch/`, [arXiv:1909.12524].

[62] See the $D$-meson Listings in this *Review*.

[63] Y. Grossman, A. L. Kagan and Y. Nir, Phys. Rev. **D75**, 036008 (2007), [hep-ph/0609178].

[64] S. Bergmann *et al.*, Phys. Lett. **B486**, 418 (2000), [hep-ph/0005181].

[65] M. Gersabeck *et al.*, J. Phys. **G39**, 045005 (2012), [arXiv:1111.6515].

[66] D. M. Asner *et al.* (CLEO), Phys. Rev. **D78**, 012001 (2008), [arXiv:0802.2268].

[67] M. Ablikim *et al.* (BESIII), Phys. Lett. **B734**, 227 (2014), [arXiv:1404.4691].

[68] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **111**, 251801 (2013), [arXiv:1309.6534].

[69] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **118**, 261803 (2017), [arXiv:1702.06490].

[70] M. Ciuchini *et al.*, Phys. Lett. **B655**, 162 (2007), [hep-ph/0703204].

[71] Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **103**, 071602 (2009), [arXiv:0904.0305].

[72] A. L. Kagan and M. D. Sokoloff, Phys. Rev. **D80**, 076008 (2009), [arXiv:0907.3917].

[73] Y. Grossman and Y. Nir, JHEP **04**, 002 (2012), [arXiv:1110.3790].

[74] B. R. Ko *et al.* (Belle), Phys. Rev. Lett. **109**, 021601 (2012), [Erratum-ibid. **109**, 119903 (2012)], [arXiv:1203.6409].

[75] See the "Review of Multibody Charm Analyses" in this *Review*.

[76] B. Aubert *et al.* (BaBar), Phys. Rev. **D78**, 051102 (2008), [arXiv:0802.4035].

[77] I. Bediaga *et al.*, Phys. Rev. **D80**, 096006 (2009), [arXiv:0905.4233].

[78] I. Bediaga *et al.*, Phys. Rev. **D86**, 036005 (2012), [arXiv:1205.3036].

[79] M. Williams, Phys. Rev. **D84**, 054015 (2011), [arXiv:1105.5338].

[80] R. Aaij *et al.* (LHCb), Phys. Lett. **B769**, 345 (2017), [arXiv:1612.03207].

[81] See the $B$-meson Listings in this *Review*.

[82] V. M. Abazov *et al.* (D0), Phys. Rev. **D82**, 032001 (2010), [arXiv:1005.2757].

[83] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980).

[84] A. B. Carter and A. I. Sanda, Phys. Rev. **D23**, 1567 (1981).

[85] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. **B193**, 85 (1981).

[86] I. Dunietz and J. L. Rosner, Phys. Rev. **D34**, 1404 (1986).

[87] Y. I. Azimov, N. G. Uraltsev and V. A. Khoze, Sov. J. Nucl. Phys. **45**, 878 (1987), [Yad. Fiz. **45**, 1412 (1987)].

[88] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. **B281**, 41 (1987).

[89] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996), [hep-ph/9512380].

[90] A. J. Buras and L. Silvestrini, Nucl. Phys. **B569**, 3 (2000), [hep-ph/9812392].

[91] R. Fleischer, Eur. Phys. J. **C10**, 299 (1999), [hep-ph/9903455].

[92] M. Ciuchini, M. Pierini and L. Silvestrini, Phys. Rev. Lett. **95**, 221804 (2005), [hep-ph/0507290].

[93] S. Faller *et al.*, Phys. Rev. **D79**, 014030 (2009), [arXiv:0809.0842].

[94] M. Jung, Phys. Rev. **D86**, 053008 (2012), [arXiv:1206.2050].

[95] H.-n. Li and S. Mishima, JHEP **03**, 009 (2007), [hep-ph/0610120].
[96] K. De Bruyn and R. Fleischer, JHEP **03**, 145 (2015), [arXiv:1412.6834].
[97] P. Frings, U. Nierste and M. Wiebusch, Phys. Rev. Lett. **115**, 061802 (2015), [arXiv:1503.00859].
[98] L. Silvestrini, Ann. Rev. Nucl. Part. Sci. **57**, 405 (2007), [arXiv:0705.1624].
[99] R. Fleischer *et al.*, Phys. Rev. **D78**, 111501 (2008), [arXiv:0806.2900].
[100] R. Fleischer *et al.*, Eur. Phys. J. **C78**, 943 (2018), [arXiv:1806.08783].
[101] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
[102] A. F. Falk *et al.*, Phys. Rev. **D69**, 011502 (2004), [hep-ph/0310242].
[103] J. Charles *et al.*, Eur. Phys. J. **C77**, 574 (2017), [arXiv:1705.02981].
[104] See the review on "Determination of CKM Angles from $B$ hadrons," in this *Review*.
[105] A. Abdesselam *et al.* (BaBar, Belle), Phys. Rev. Lett. **115**, 121604 (2015), [arXiv:1505.04147].
[106] R. Fleischer and R. Knegjens, Eur. Phys. J. **C71**, 1789 (2011), [arXiv:1109.5115].
[107] A. S. Dighe, I. Dunietz and R. Fleischer, Eur. Phys. J. **C6**, 647 (1999), [hep-ph/9804253].
[108] R. Aaij *et al.* (LHCb), Phys. Rev. **D89**, 092006 (2014), [arXiv:1402.6248].
[109] R. Aaij *et al.* (LHCb), JHEP **03**, 075 (2021), [arXiv:2012.05319].
[110] M. Gronau and D. London, Phys. Lett. **B253**, 483 (1991).
[111] M. Gronau and D. Wyler, Phys. Lett. **B265**, 172 (1991).
[112] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78**, 3257 (1997), [hep-ph/9612433].
[113] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. **D63**, 036005 (2001), [hep-ph/0008090].
[114] A. Giri *et al.*, Phys. Rev. **D68**, 054018 (2003), [hep-ph/0303187].
[115] A. Bondar, *Proceedings of BINP special analysis meeting on Dalitz analysis*, 24-26 Sep. 2002, unpublished.
[116] J. Brod and J. Zupan, JHEP **01**, 051 (2014), [arXiv:1308.5663].
[117] J. P. Lees *et al.* (BaBar), Phys. Rev. **D87**, 052015 (2013), [arXiv:1301.1029].
[118] R. Aaij *et al.* (LHCb), JHEP **12**, 087 (2016), [arXiv:1611.03076].
[119] R. Aleksan, I. Dunietz and B. Kayser, Z. Phys. **C54**, 653 (1992).
[120] R. Fleischer, Nucl. Phys. **B671**, 459 (2003), [hep-ph/0304027].
[121] I. Dunietz, Phys. Lett. **B270**, 75 (1991).
[122] M. Gronau, Phys. Lett. **B557**, 198 (2003), [hep-ph/0211282].
[123] T. Gershon, Phys. Rev. **D79**, 051301 (2009), [arXiv:0810.2706].
[124] T. Gershon and M. Williams, Phys. Rev. **D80**, 092002 (2009), [arXiv:0909.1495].
[125] M. Bander, D. Silverman and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
[126] X.-G. He, Eur. Phys. J. **C9**, 443 (1999), [hep-ph/9810397].
[127] D. Atwood and A. Soni, Phys. Rev. **D58**, 036005 (1998), [hep-ph/9712287].
[128] M. Gronau and J. L. Rosner, Phys. Rev. **D59**, 113002 (1999), [hep-ph/9809384].
[129] H. J. Lipkin, Phys. Lett. **B445**, 403 (1999), [hep-ph/9810351].
[130] M. Gronau, Phys. Lett. **B627**, 82 (2005), [hep-ph/0508047].
[131] R. Aaij *et al.* (LHCb), Phys. Rev. **D90**, 112004 (2014), [arXiv:1408.5373].
[132] R. Aaij *et al.* (LHCb) (2019), [arXiv:1905.09244].

[133] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **124**, 031801 (2020), [arXiv:1909.05211].

[134] R. Aaij *et al.* (LHCb), Phys. Rev. **D101**, 012006 (2020), [arXiv:1909.05212].

[135] A. A. Alves, Jr. *et al.* (LHCb), JINST **3**, S08005 (2008).

[136] I. Bediaga *et al.* (LHCb) (2012), CERN-LHCC-2012-007.

[137] R. Aaij *et al.* (LHCb) (2017), CERN-LHCC-2017-003.

[138] W. Altmannshofer *et al.* (Belle-II), PTEP **2019**, 123C01 (2019), [Erratum-ibid. **2020**, 029201 (2020)], [arXiv:1808.10567].

[139] G. C. Branco, L. Lavoura and J. P. Silva, Int. Ser. Monogr. Phys. **103**, 1 (1999).

[140] I. I. Bigi and A. I. Sanda (2000), [Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.9,1(2009)].

[141] A. J. Bevan *et al.* (BaBar, Belle), Eur. Phys. J. **C74**, 3026 (2014), [arXiv:1406.6311].

[142] H.R. Quinn and Y. Nir, *"The Mystery of the Missing Antimatter,"* Princeton University Press, Princeton (2008).

[143] T. E. Browder *et al.*, Rev. Mod. Phys. **81**, 1887 (2009), [arXiv:0802.3201].

[144] M. Ciuchini and A. Stocchi, Ann. Rev. Nucl. Part. Sci. **61**, 491 (2011), [arXiv:1110.3920].

[145] R. Aaij *et al.* (LHCb), Eur. Phys. J. **C73**, 2373 (2013), [arXiv:1208.3355].

[146] T. Gershon and V. V. Gligorov, Rept. Prog. Phys. **80**, 046201 (2017), [arXiv:1607.06746].