



$$I(J^P) = \frac{1}{2}(0^-)$$

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## CHARGED KAON MASS

Revised 1994 by T.G. Trippe (LBNL).

The average of the six charged kaon mass measurements which we use in the Particle Listings is

$$m_{K^\pm} = 493.677 \pm 0.013 \text{ MeV (S = 2.4) ,} \quad (1)$$

where the error has been increased by the scale factor S. The large scale factor indicates a serious disagreement between different input data. The average before scaling the error is

$$m_{K^\pm} = 493.677 \pm 0.005 \text{ MeV ,}$$

$$\chi^2 = 22.9 \text{ for 5 D.F., Prob. = 0.04% ,} \quad (2)$$

where the high  $\chi^2$  and correspondingly low  $\chi^2$  probability further quantify the disagreement.

The main disagreement is between the two most recent and precise results,

$$m_{K^\pm} = 493.696 \pm 0.007 \text{ MeV} \quad \text{DENISOV 91}$$

$$m_{K^\pm} = 493.636 \pm 0.011 \text{ MeV (S = 1.5) GALL 88}$$

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$$\text{Average} = 493.679 \pm 0.006 \text{ MeV}$$

$$\chi^2 = 21.2 \text{ for 1 D.F., Prob. = 0.0004% ,} \quad (3)$$

both of which are measurements of x-ray energies from kaonic atoms. Comparing the average in Eq. (3) with the overall average in Eq. (2), it is clear that DENISOV 91 and GALL 88 dominate the overall average, and that their disagreement is responsible for most of the high  $\chi^2$ .

The GALL 88 measurement was made using four different kaonic atom transitions,  $K^- \text{Pb}$  ( $9 \rightarrow 8$ ),  $K^- \text{Pb}$  ( $11 \rightarrow 10$ ),  $K^- \text{W}$  ( $9 \rightarrow 8$ ), and  $K^- \text{W}$  ( $11 \rightarrow 10$ ). The  $m_{K^\pm}$  values they obtain from each of these transitions is shown in the Particle Listings and in Fig. 1. Their  $K^- \text{Pb}$  ( $9 \rightarrow 8$ )  $m_{K^\pm}$  is below and somewhat inconsistent with their other three transitions. The average of their four measurements is

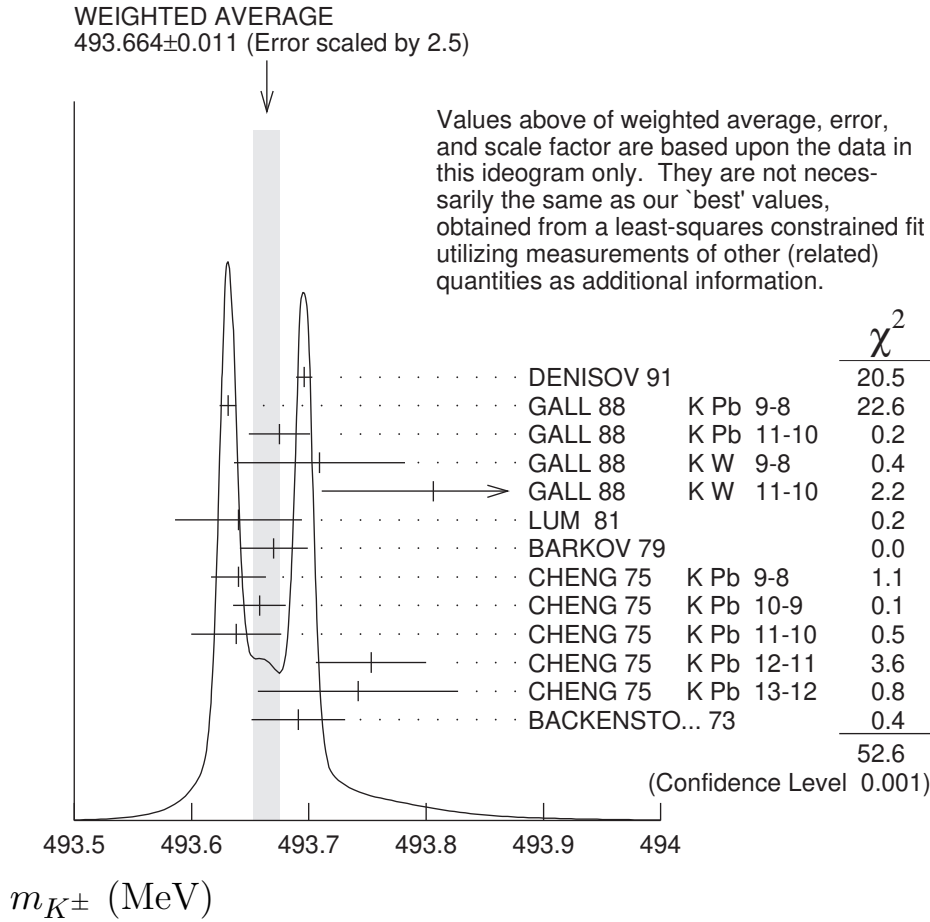
$$m_{K^\pm} = 493.636 \pm 0.007 ,$$

$$\chi^2 = 7.0 \text{ for 3 D.F., Prob. } = 7.2\% . \quad (4)$$

This is a low but acceptable  $\chi^2$  probability so, to be conservative, GALL 88 scaled up the error on their average by  $S=1.5$  to obtain their published error  $\pm 0.011$  shown in Eq. (3) above and used in the Particle Listings average.

The ideogram in Fig. 1 shows that the DENISOV 91 measurement and the GALL 88  $K^- \text{Pb}$  ( $9 \rightarrow 8$ ) measurement yield two well-separated peaks. One might suspect the GALL 88  $K^- \text{Pb}$  ( $9 \rightarrow 8$ ) measurement since it is responsible both for the internal inconsistency in the GALL 88 measurements and the disagreement with DENISOV 91.

To see if the disagreement could result from a systematic problem with the  $K^- \text{Pb}$  ( $9 \rightarrow 8$ ) transition, we have separated the CHENG 75 data, which also used  $K^- \text{Pb}$ , into its separate transitions. Figure 1 shows that the CHENG 75 and GALL 88  $K^- \text{Pb}$  ( $9 \rightarrow 8$ ) values are consistent, suggesting the possibility of a common effect such as contaminant nuclear  $\gamma$  rays near the  $K^- \text{Pb}$  ( $9 \rightarrow 8$ ) transition energy, although the CHENG 75 errors are too large to make a strong conclusion. The average of all 13 measurements has a  $\chi^2$  of 52.6 as shown in Fig. 1 and the first line of Table 1, yielding an unacceptable  $\chi^2$  probability of 0.00005%. The second line of Table 1 excludes



**Figure 1:** Ideogram of  $m_{K^\pm}$  mass measurements. GALL 88 and CHENG 75 measurements are shown separately for each transition they measured.

both the GALL 88 and CHENG 75 measurements of the  $K^- \text{Pb} (9 \rightarrow 8)$  transition and yields a  $\chi^2$  probability of 43%. The third [fourth] line of Table 1 excludes only the GALL 88  $K^- \text{Pb} (9 \rightarrow 8)$  [DENISOV 91] measurement and yields a  $\chi^2$  probability of 20% [8.6%]. Table 1 shows that removing both measurements of the  $K^- \text{Pb} (9 \rightarrow 8)$  transition produces the most consistent set of data, but that excluding only the

GALL 88  $K^-$  Pb (9  $\rightarrow$  8) transition or DENISOV 91 also produces acceptable probabilities.

**Table 1:**  $m_{K^\pm}$  averages for some combinations of Fig. 1 data.

| $m_{K^\pm}$ (MeV)   | $\chi^2$ | D.F. | Prob. (%) | Measurements used                      |
|---------------------|----------|------|-----------|--|
| $493.664 \pm 0.004$ | 52.6     | 12   | 0.00005   | all 13 measurements                    |
| $493.690 \pm 0.006$ | 10.1     | 10   | 43        | no $K^-$ Pb(9 $\rightarrow$ 8)         |
| $493.687 \pm 0.006$ | 14.6     | 11   | 20        | no GALL 88 $K^-$ Pb(9 $\rightarrow$ 8) |
| $493.642 \pm 0.006$ | 17.8     | 11   | 8.6       | no DENISOV 91                          |

Yu.M. Ivanov, representing DENISOV 91, has estimated corrections needed for the older experiments because of improved  $^{192}\text{Ir}$  and  $^{198}\text{Au}$  calibration  $\gamma$ -ray energies. He estimates that CHENG 75 and BACKENSTOSS 73  $m_{K^\pm}$  values could be raised by about 15 keV and 22 keV, respectively. With these estimated corrections, Table 1 becomes Table 2. The last line of Table 2 shows that if such corrections are assumed, then GALL 88  $K^-$  Pb (9  $\rightarrow$  8) is inconsistent with the rest of the data even when DENISOV 91 is excluded. Yu.M. Ivanov warns that these are rough estimates. Accordingly, we do not use Table 2 to reject the GALL 88  $K^-$  Pb (9  $\rightarrow$  8) transition, but we note that a future reanalysis of the CHENG 75 data could be useful because it might provide supporting evidence for such a rejection.

The GALL 88 measurement uses a Ge semiconductor spectrometer which has a resolution of about 1 keV, so they run the risk of some contaminant nuclear  $\gamma$  rays. Studies of  $\gamma$  rays following stopped  $\pi^-$  and  $\Sigma^-$  absorption in nuclei (unpublished) do not show any evidence for contaminants according to GALL 88 spokesperson, B.L. Roberts. The DENISOV 91

measurement uses a crystal diffraction spectrometer with a resolution of 6.3 eV for radiation at 22.1 keV to measure the 4f-3d transition in  $K^-$   $^{12}\text{C}$ . The high resolution and the light nucleus reduce the probability for overlap by contaminant  $\gamma$  rays, compared with the measurement of GALL 88. The DENISOV 91 measurement is supported by their high-precision measurement of the 4d-2p transition energy in  $\pi^-$   $^{12}\text{C}$ , which is in good agreement with the calculated energy.

**Table 2:**  $m_{K^\pm}$  averages for some combinations of Fig. 1 data after raising CHENG 75 and BACKENSTOSS 73 values by 0.015 and 0.022 MeV respectively.

| $m_{K^\pm}$ (MeV)   | $\chi^2$ | D.F. | Prob. (%) | Measurements used                      |
|---------------------|----------|------|-----------|--|
| $493.666 \pm 0.004$ | 53.9     | 12   | 0.00003   | all 13 measurements                    |
| $493.693 \pm 0.006$ | 9.0      | 10   | 53        | no $K^-$ Pb(9 $\rightarrow$ 8)         |
| $493.690 \pm 0.006$ | 11.5     | 11   | 40        | no GALL 88 $K^-$ Pb(9 $\rightarrow$ 8) |
| $493.645 \pm 0.006$ | 23.0     | 11   | 1.8       | no DENISOV 91                          |

While we suspect that the GALL 88  $K^-$  Pb (9  $\rightarrow$  8) measurements could be the problem, we are unable to find clear grounds for rejecting it. Therefore, we retain their measurement in the average and accept the large scale factor until further information can be obtained from new measurements and/or from reanalysis of GALL 88 and CHENG 75 data.

We thank B.L. Roberts (Boston Univ.) and Yu.M. Ivanov (Petersburg Nuclear Physics Inst.) for their extensive help in understanding this problem.

## $K^\pm$ MASS

VALUE (MeV)                      DOCUMENT ID    TECN    CHG    COMMENT

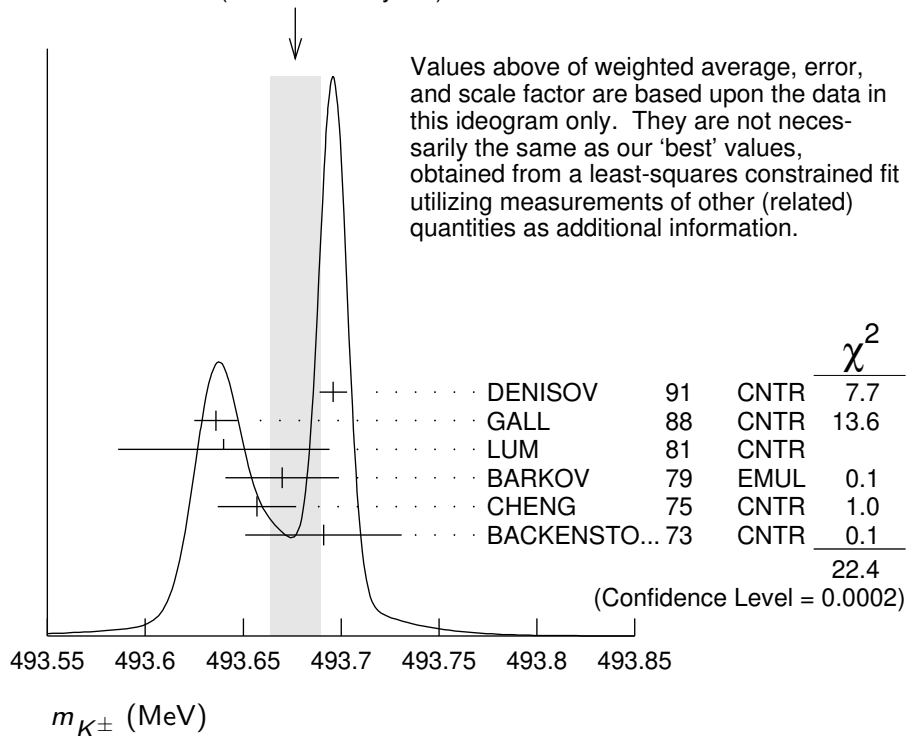
**493.677 ± 0.016 OUR FIT**    Error includes scale factor of 2.8.  
**493.677 ± 0.013 OUR AVERAGE**    Error includes scale factor of 2.4. See the ideogram below.

|                 |   |                |    |      |   |                               |
|-----------------|---|----------------|----|------|---|-------------------------------|
| 493.696 ± 0.007 | 1 | DENISOV        | 91 | CNTR | — | Kaonic atoms                  |
| 493.636 ± 0.011 | 2 | GALL           | 88 | CNTR | — | Kaonic atoms                  |
| 493.640 ± 0.054 |   | LUM            | 81 | CNTR | — | Kaonic atoms                  |
| 493.670 ± 0.029 |   | BARKOV         | 79 | EMUL | ± | $e^+ e^- \rightarrow K^+ K^-$ |
| 493.657 ± 0.020 | 2 | CHENG          | 75 | CNTR | — | Kaonic atoms                  |
| 493.691 ± 0.040 |   | BACKENSTO...73 |    | CNTR | — | Kaonic atoms                  |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                         |   |       |    |      |   |                    |
|-------------------------|---|-------|----|------|---|--------------------|
| 493.631 ± 0.007         |   | GALL  | 88 | CNTR | — | $K^-$ Pb (9 → 8)   |
| 493.675 ± 0.026         |   | GALL  | 88 | CNTR | — | $K^-$ Pb (11 → 10) |
| 493.709 ± 0.073         |   | GALL  | 88 | CNTR | — | $K^-$ W (9 → 8)    |
| 493.806 ± 0.095         |   | GALL  | 88 | CNTR | — | $K^-$ W (11 → 10)  |
| 493.640 ± 0.022 ± 0.008 | 3 | CHENG | 75 | CNTR | — | $K^-$ Pb (9 → 8)   |
| 493.658 ± 0.019 ± 0.012 | 3 | CHENG | 75 | CNTR | — | $K^-$ Pb (10 → 9)  |
| 493.638 ± 0.035 ± 0.016 | 3 | CHENG | 75 | CNTR | — | $K^-$ Pb (11 → 10) |
| 493.753 ± 0.042 ± 0.021 | 3 | CHENG | 75 | CNTR | — | $K^-$ Pb (12 → 11) |
| 493.742 ± 0.081 ± 0.027 | 3 | CHENG | 75 | CNTR | — | $K^-$ Pb (13 → 12) |

WEIGHTED AVERAGE  
 493.677 ± 0.013 (Error scaled by 2.4)



<sup>1</sup> Error increased from 0.0059 based on the error analysis in IVANOV 92.

<sup>2</sup> This value is the authors' combination of all of the separate transitions listed for this paper.

<sup>3</sup>The CHENG 75 values for separate transitions were calculated from their Table 7 transition energies. The first error includes a 20% systematic error in the noncircular contaminant shift. The second error is due to a  $\pm 5$  eV uncertainty in the theoretical transition energies.

### $m_{K^+} - m_{K^-}$

Test of *CPT*.

| VALUE (MeV)           | EVTS | DOCUMENT ID       | TECN | CHG    |
|-----------------------|------|-------------------|------|--------|
| <b>-0.032 ± 0.090</b> | 1.5M | <sup>1</sup> FORD | 72   | ASPK ± |

<sup>1</sup>FORD 72 uses  $m_{\pi^+} - m_{\pi^-} = +28 \pm 70$  keV.

### $K^\pm$ MEAN LIFE

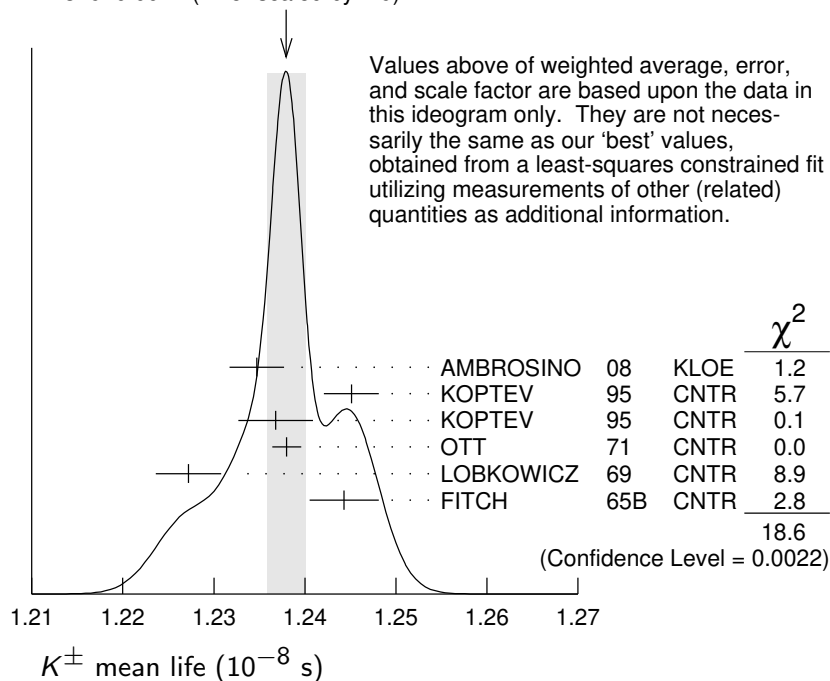
| VALUE ( $10^{-8}$ s) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|----------------------|------|-------------|------|-----|---------|
|----------------------|------|-------------|------|-----|---------|

**1.2380 ± 0.0020 OUR FIT** Error includes scale factor of 1.8.

**1.2379 ± 0.0021 OUR AVERAGE** Error includes scale factor of 1.9. See the ideogram below.

|   |      |                        |     |        |                            |
|---|------|------------------------|-----|--------|----------------------------|
| 1.2347 ± 0.0030   | 15M  | <sup>1</sup> AMBROSINO | 08  | KLOE ± | $\phi \rightarrow K^+ K^-$ |
| 1.2451 ± 0.0030   | 250k | KOPTEV                 | 95  | CNTR   | $K$ at rest, U target      |
| 1.2368 ± 0.0041   | 150k | KOPTEV                 | 95  | CNTR   | $K$ at rest, Cu target     |
| 1.2380 ± 0.0016   | 3M   | OTT                    | 71  | CNTR + | $K$ at rest                |
| 1.2272 ± 0.0036   |      | LOBKOWICZ              | 69  | CNTR + | $K$ in flight              |
| 1.2443 ± 0.0038   |      | FITCH                  | 65B | CNTR + | $K$ at rest                |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |      |                        |     |        |                            |
| 1.2415 ± 0.0024   | 400k | <sup>2</sup> KOPTEV    | 95  | CNTR   | $K$ at rest                |
| 1.221 ± 0.011   |      | FORD                   | 67  | CNTR ± |                            |
| 1.231 ± 0.011   |      | BOYARSKI               | 62  | CNTR + |                            |

WEIGHTED AVERAGE  
1.2379 ± 0.0021 (Error scaled by 1.9)



<sup>1</sup> Result obtained by averaging the decay length and decay time analyses taking correlations into account.

<sup>2</sup> KOPTEV 95 report this weighted average of their U-target and Cu-target results, where they have weighted by  $1/\sigma$  rather than  $1/\sigma^2$ .

$$(\tau_{K^+} - \tau_{K^-}) / \tau_{\text{average}}$$

This quantity is a measure of *CPT* invariance in weak interactions.

| VALUE (%)                      | DOCUMENT ID                         | TECN |
|--------------------------------|-------------------------------------|------|
| <b>0.10 ± 0.09 OUR AVERAGE</b> | Error includes scale factor of 1.2. |      |
| -0.4 ± 0.4                     | AMBROSINO 08                        | KLOE |
| 0.090 ± 0.078                  | LOBKOWICZ 69                        | CNTR |
| 0.47 ± 0.30                    | FORD 67                             | CNTR |

See the related review(s):

[Rare Kaon Decays](#)

### $K^+$ DECAY MODES

$K^-$  modes are charge conjugates of the modes below.

| Mode  | Fraction ( $\Gamma_i/\Gamma$ )           | Scale factor/<br>Confidence level |
|---|--|-----------------------------------|
| <b>Leptonic and semileptonic modes</b>                        |  |                                   |
| $\Gamma_1$ $e^+ \nu_e$  | $( 1.582 \pm 0.007 ) \times 10^{-5}$     |                                   |
| $\Gamma_2$ $\mu^+ \nu_\mu$                                    | $( 63.56 \pm 0.11 ) \%$                  | S=1.2                             |
| $\Gamma_3$ $\pi^0 e^+ \nu_e$<br>Called $K_{e3}^+$             | $( 5.07 \pm 0.04 ) \%$                   | S=2.1                             |
| $\Gamma_4$ $\pi^0 \mu^+ \nu_\mu$<br>Called $K_{\mu 3}^+$      | $( 3.352 \pm 0.033 ) \%$                 | S=1.9                             |
| $\Gamma_5$ $\pi^0 \pi^0 e^+ \nu_e$                            | $( 2.55 \pm 0.04 ) \times 10^{-5}$       | S=1.1                             |
| $\Gamma_6$ $\pi^+ \pi^- e^+ \nu_e$                            | $( 4.247 \pm 0.024 ) \times 10^{-5}$     |                                   |
| $\Gamma_7$ $\pi^+ \pi^- \mu^+ \nu_\mu$                        | $( 1.4 \pm 0.9 ) \times 10^{-5}$         |                                   |
| $\Gamma_8$ $\pi^0 \pi^0 \pi^0 e^+ \nu_e$                      | $< 3.5 \times 10^{-6}$                   | CL=90%                            |
| <b>Hadronic modes</b>   |  |                                   |
| $\Gamma_9$ $\pi^+ \pi^0$                                      | $( 20.67 \pm 0.08 ) \%$                  | S=1.2                             |
| $\Gamma_{10}$ $\pi^+ \pi^0 \pi^0$                             | $( 1.760 \pm 0.023 ) \%$                 | S=1.1                             |
| $\Gamma_{11}$ $\pi^+ \pi^+ \pi^-$                             | $( 5.583 \pm 0.024 ) \%$                 |                                   |
| <b>Leptonic and semileptonic modes with photons</b>           |  |                                   |
| $\Gamma_{12}$ $\mu^+ \nu_\mu \gamma$                          | [a,b] $( 6.2 \pm 0.8 ) \times 10^{-3}$   |                                   |
| $\Gamma_{13}$ $\mu^+ \nu_\mu \gamma (SD^+)$                   | [c,d] $( 1.33 \pm 0.22 ) \times 10^{-5}$ |                                   |
| $\Gamma_{14}$ $\mu^+ \nu_\mu \gamma (SD^+ \text{INT})$        | [c,d] $< 2.7 \times 10^{-5}$             | CL=90%                            |
| $\Gamma_{15}$ $\mu^+ \nu_\mu \gamma (SD^- + SD^- \text{INT})$ | [c,d] $< 2.6 \times 10^{-4}$             | CL=90%                            |



|               |                                |       |                     |                  |        |
|---------------|--------------------------------|-------|---------------------|------------------|--------|
| $\Gamma_{16}$ | $e^+ \nu_e \gamma$             |       | ( 9.4 $\pm$ 0.4 )   | $\times 10^{-6}$ |        |
| $\Gamma_{17}$ | $\pi^0 e^+ \nu_e \gamma$       | [a,b] | ( 2.56 $\pm$ 0.16 ) | $\times 10^{-4}$ |        |
| $\Gamma_{18}$ | $\pi^0 e^+ \nu_e \gamma$ (SD)  | [c,d] | < 5.3               | $\times 10^{-5}$ | CL=90% |
| $\Gamma_{19}$ | $\pi^0 \mu^+ \nu_\mu \gamma$   | [a,b] | ( 1.25 $\pm$ 0.25 ) | $\times 10^{-5}$ |        |
| $\Gamma_{20}$ | $\pi^0 \pi^0 e^+ \nu_e \gamma$ |       | < 5                 | $\times 10^{-6}$ | CL=90% |

### Hadronic modes with photons or $\ell\bar{\ell}$ pairs

|               |                            |       |  |                  |        |
|---------------|----------------------------|-------|--|------------------|--------|
| $\Gamma_{21}$ | $\pi^+ \pi^0 \gamma$ (INT) |       | ( - 4.2 $\pm$ 0.9 )  | $\times 10^{-6}$ |        |
| $\Gamma_{22}$ | $\pi^+ \pi^0 \gamma$ (DE)  | [a,e] | ( 6.0 $\pm$ 0.4 )  | $\times 10^{-6}$ |        |
| $\Gamma_{23}$ | $\pi^+ \pi^0 e^+ e^-$      |       | ( 4.24 $\pm$ 0.14 )  | $\times 10^{-6}$ |        |
| $\Gamma_{24}$ | $\pi^+ \pi^0 \pi^0 \gamma$ | [a,b] | ( 7.6 $\begin{smallmatrix} +6.0 \\ -3.0 \end{smallmatrix}$ ) | $\times 10^{-6}$ |        |
| $\Gamma_{25}$ | $\pi^+ \pi^+ \pi^- \gamma$ | [a,b] | ( 7.1 $\pm$ 0.5 )  | $\times 10^{-6}$ |        |
| $\Gamma_{26}$ | $\pi^+ \gamma \gamma$      | [a]   | ( 1.01 $\pm$ 0.06 )  | $\times 10^{-6}$ |        |
| $\Gamma_{27}$ | $\pi^+ 3\gamma$            | [a]   | < 1.0  | $\times 10^{-4}$ | CL=90% |
| $\Gamma_{28}$ | $\pi^+ e^+ e^- \gamma$     |       | ( 1.19 $\pm$ 0.13 )  | $\times 10^{-8}$ |        |

### Leptonic modes with $\ell\bar{\ell}$ pairs

|               |                               |  |                     |                  |        |
|---------------|-------------------------------|--|---------------------|------------------|--------|
| $\Gamma_{29}$ | $e^+ \nu_e \nu \bar{\nu}$     |  | < 6                 | $\times 10^{-5}$ | CL=90% |
| $\Gamma_{30}$ | $\mu^+ \nu_\mu \nu \bar{\nu}$ |  | < 2.4               | $\times 10^{-6}$ | CL=90% |
| $\Gamma_{31}$ | $e^+ \nu_e e^+ e^-$           |  | ( 2.48 $\pm$ 0.20 ) | $\times 10^{-8}$ |        |
| $\Gamma_{32}$ | $\mu^+ \nu_\mu e^+ e^-$       |  | ( 7.06 $\pm$ 0.31 ) | $\times 10^{-8}$ |        |
| $\Gamma_{33}$ | $e^+ \nu_e \mu^+ \mu^-$       |  | ( 1.7 $\pm$ 0.5 )   | $\times 10^{-8}$ |        |
| $\Gamma_{34}$ | $\mu^+ \nu_\mu \mu^+ \mu^-$   |  | < 4.1               | $\times 10^{-7}$ | CL=90% |

### Lepton family number (LF), Lepton number (L), $\Delta S = \Delta Q$ (SQ) violating modes, or $\Delta S = 1$ weak neutral current (S1) modes

|               |                                   |     |  |                   |        |
|---------------|-----------------------------------|-----|--|-------------------|--------|
| $\Gamma_{35}$ | $\pi^+ \pi^+ e^- \bar{\nu}_e$     | SQ  | < 1.3  | $\times 10^{-8}$  | CL=90% |
| $\Gamma_{36}$ | $\pi^+ \pi^+ \mu^- \bar{\nu}_\mu$ | SQ  | < 3.0  | $\times 10^{-6}$  | CL=95% |
| $\Gamma_{37}$ | $\pi^+ e^+ e^-$                   | S1  | ( 3.00 $\pm$ 0.09 )                                    | $\times 10^{-7}$  |        |
| $\Gamma_{38}$ | $\pi^+ \mu^+ \mu^-$               | S1  | ( 9.4 $\pm$ 0.6 )                                      | $\times 10^{-8}$  | S=2.6  |
| $\Gamma_{39}$ | $\pi^+ \nu \bar{\nu}$             | S1  | ( 8 $\begin{smallmatrix} +6 \\ -4 \end{smallmatrix}$ ) | $\times 10^{-11}$ |        |
| $\Gamma_{40}$ | $\pi^+ \pi^0 \nu \bar{\nu}$       | S1  | < 4.3  | $\times 10^{-5}$  | CL=90% |
| $\Gamma_{41}$ | $\mu^- \nu e^+ e^+$               | LF  | < 2.1  | $\times 10^{-8}$  | CL=90% |
| $\Gamma_{42}$ | $\mu^+ \nu_e$                     | LF  | [f] < 4  | $\times 10^{-3}$  | CL=90% |
| $\Gamma_{43}$ | $\pi^+ \mu^+ e^-$                 | LF  | < 1.3  | $\times 10^{-11}$ | CL=90% |
| $\Gamma_{44}$ | $\pi^+ \mu^- e^+$                 | LF  | < 5.2  | $\times 10^{-10}$ | CL=90% |
| $\Gamma_{45}$ | $\pi^- \mu^+ e^+$                 | L   | < 5.0  | $\times 10^{-10}$ | CL=90% |
| $\Gamma_{46}$ | $\pi^- e^+ e^+$                   | L   | < 2.2  | $\times 10^{-10}$ | CL=90% |
| $\Gamma_{47}$ | $\pi^- \mu^+ \mu^+$               | L   | < 4.2  | $\times 10^{-11}$ | CL=90% |
| $\Gamma_{48}$ | $\mu^+ \bar{\nu}_e$               | L   | [f] < 3.3  | $\times 10^{-3}$  | CL=90% |
| $\Gamma_{49}$ | $\pi^0 e^+ \bar{\nu}_e$           | L   | < 3  | $\times 10^{-3}$  | CL=90% |
| $\Gamma_{50}$ | $\pi^+ \gamma$                    | [g] | < 2.3  | $\times 10^{-9}$  | CL=90% |

- [a] See the Particle Listings below for the energy limits used in this measurement.
- [b] Most of this radiative mode, the low-momentum  $\gamma$  part, is also included in the parent mode listed without  $\gamma$ 's.
- [c] Structure-dependent part.
- [d] See the review on "Form Factors for Radiative Pion and Kaon Decays" for definitions and details.
- [e] Direct-emission branching fraction.
- [f] Derived from an analysis of neutrino-oscillation experiments.
- [g] Violates angular-momentum conservation.

### CONSTRAINED FIT INFORMATION

An overall fit to the mean life, a decay rate, and 15 branching ratios uses 35 measurements and one constraint to determine 8 parameters. The overall fit has a  $\chi^2 = 53.4$  for 28 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients  $\langle \delta p_i \delta p_j \rangle / (\delta p_i \cdot \delta p_j)$ , in percent, from the fit to parameters  $p_i$ , including the branching fractions,  $x_i \equiv \Gamma_i / \Gamma_{\text{total}}$ . The fit constrains the  $x_i$  whose labels appear in this array to sum to one.

|          |     |       |       |       |       |       |          |
|----------|-----|-------|-------|-------|-------|-------|----------|
| $x_3$    | -66 |       |       |       |       |       |          |
| $x_4$    | -64 | 90    |       |       |       |       |          |
| $x_5$    | -12 | -5    | -5    |       |       |       |          |
| $x_9$    | -67 | 0     | -1    | -6    |       |       |          |
| $x_{10}$ | -13 | -6    | -5    | 91    | -6    |       |          |
| $x_{11}$ | -14 | -6    | -6    | 2     | -7    | 2     |          |
| $\Gamma$ | 3   | 1     | 1     | 0     | 2     | 0     | -24      |
|          |     | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_9$ | $x_{10}$ |

| Mode   | Rate ( $10^8 \text{ s}^{-1}$ )     | Scale factor |
|--|------------------------------------|--------------|
| $\Gamma_2$ $\mu^+ \nu_\mu$                               | $0.5134 \pm 0.0012$                | 1.5          |
| $\Gamma_3$ $\pi^0 e^+ \nu_e$<br>Called $K_{e3}^+$        | $0.0410 \pm 0.0004$                | 2.1          |
| $\Gamma_4$ $\pi^0 \mu^+ \nu_\mu$<br>Called $K_{\mu 3}^+$ | $0.02707 \pm 0.00027$              | 1.9          |
| $\Gamma_5$ $\pi^0 \pi^0 e^+ \nu_e$                       | $(2.059 \pm 0.029) \times 10^{-5}$ | 1.1          |
| $\Gamma_9$ $\pi^+ \pi^0$                                 | $0.1670 \pm 0.0007$                | 1.3          |
| $\Gamma_{10}$ $\pi^+ \pi^0 \pi^0$                        | $0.01421 \pm 0.00018$              | 1.1          |
| $\Gamma_{11}$ $\pi^+ \pi^+ \pi^-$                        | $0.04510 \pm 0.00019$              |              |

## $K^\pm$ DECAY RATES

$\Gamma(\mu^+ \nu_\mu)$   $\Gamma_2$

VALUE ( $10^6 \text{ s}^{-1}$ )      DOCUMENT ID      TECN      CHG

**51.34 ± 0.12 OUR FIT** Error includes scale factor of 1.5.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|            |      |    |      |   |
|------------|------|----|------|---|
| 51.2 ± 0.8 | FORD | 67 | CNTR | ± |
|------------|------|----|------|---|

$\Gamma(\pi^+ \pi^+ \pi^-)$   $\Gamma_{11}$

VALUE ( $10^6 \text{ s}^{-1}$ )      EVTS      DOCUMENT ID      TECN      CHG

**4.510 ± 0.019 OUR FIT**

**4.511 ± 0.024**      <sup>1</sup>FORD      70      ASPK

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |      |                   |    |      |
|---------------|------|-------------------|----|------|
| 4.529 ± 0.032 | 3.2M | <sup>1</sup> FORD | 70 | ASPK |
|---------------|------|-------------------|----|------|

|               |  |                   |    |        |
|---------------|--|-------------------|----|--------|
| 4.496 ± 0.030 |  | <sup>1</sup> FORD | 67 | CNTR ± |
|---------------|--|-------------------|----|--------|

<sup>1</sup>First FORD 70 value is second FORD 70 combined with FORD 67.

## $K^+$ BRANCHING RATIOS

### ———— Leptonic and semileptonic modes ————

$\Gamma(e^+ \nu_e) / \Gamma(\mu^+ \nu_\mu)$   $\Gamma_1 / \Gamma_2$

See the note on “Decay Constants of Charged Pseudoscalar Mesons” in the  $D_s^+$  Listings.

VALUE (units  $10^{-5}$ )      EVTS      DOCUMENT ID      TECN      CHG

**2.488 ± 0.009 OUR AVERAGE**

|                       |      |                        |    |      |   |
|-----------------------|------|------------------------|----|------|---|
| 2.488 ± 0.007 ± 0.007 | 150k | <sup>1</sup> LAZZERONI | 13 | NA62 | ± |
|-----------------------|------|------------------------|----|------|---|

|                       |       |                        |     |      |   |
|-----------------------|-------|------------------------|-----|------|---|
| 2.493 ± 0.025 ± 0.019 | 13.8k | <sup>2</sup> AMBROSINO | 09E | KLOE | ± |
|-----------------------|-------|------------------------|-----|------|---|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                       |     |                        |    |      |   |
|-----------------------|-----|------------------------|----|------|---|
| 2.487 ± 0.011 ± 0.007 | 60k | <sup>3</sup> LAZZERONI | 11 | NA62 | + |
|-----------------------|-----|------------------------|----|------|---|

|             |     |         |    |      |   |
|-------------|-----|---------|----|------|---|
| 2.51 ± 0.15 | 404 | HEINTZE | 76 | SPEC | + |
|-------------|-----|---------|----|------|---|

|             |     |       |     |      |   |
|-------------|-----|-------|-----|------|---|
| 2.37 ± 0.17 | 534 | HEARD | 75B | SPEC | + |
|-------------|-----|-------|-----|------|---|

|             |     |       |    |      |   |
|-------------|-----|-------|----|------|---|
| 2.42 ± 0.42 | 112 | CLARK | 72 | OSPK | + |
|-------------|-----|-------|----|------|---|

<sup>1</sup>LAZZERONI 13 uses full data sample collected from 2007 to 2008. This ratio is defined to be fully inclusive, including internal-bremsstrahlung.

<sup>2</sup>The ratio is defined to include internal-bremsstrahlung, ignoring direct-emission contributions. AMBROSINO 09E determined the ratio from the measurement of  $\Gamma(K \rightarrow e\nu(\gamma), E_\gamma < 10 \text{ MeV}) / \Gamma(K \rightarrow \mu\nu(\gamma))$ . 89.8% of  $K \rightarrow e\nu(\gamma)$  events had  $E_\gamma < 10 \text{ MeV}$ .

<sup>3</sup>This ratio is defined to be fully inclusive, including internal-bremsstrahlung.

$\Gamma(\mu^+ \nu_\mu) / \Gamma_{\text{total}}$   $\Gamma_2 / \Gamma$

See the note on “Decay Constants of Charged Pseudoscalar Mesons” in the  $D_s^+$  Listings.

VALUE (units  $10^{-2}$ )      EVTS      DOCUMENT ID      TECN      CHG      COMMENT

**63.56 ± 0.11 OUR FIT** Error includes scale factor of 1.2.

**63.60 ± 0.16 OUR AVERAGE**

|                     |      |                        |     |      |   |
|---------------------|------|------------------------|-----|------|---|
| 63.66 ± 0.09 ± 0.15 | 865k | <sup>1</sup> AMBROSINO | 06A | KLOE | + |
|---------------------|------|------------------------|-----|------|---|

|              |     |        |    |      |                         |
|--------------|-----|--------|----|------|-------------------------|
| 63.24 ± 0.44 | 62k | CHIANG | 72 | OSPK | +      1.84 GeV/c $K^+$ |
|--------------|-----|--------|----|------|-------------------------|

<sup>1</sup>Fully inclusive. Used tagged kaons from  $\phi$  decays.

$\Gamma(\pi^0 e^+ \nu_e)/\Gamma_{\text{total}}$   $\Gamma_3/\Gamma$

| VALUE (units $10^{-2}$ ) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|--------------------------|------|-------------|------|-----|---------|
|--------------------------|------|-------------|------|-----|---------|

**5.07 ± 0.04 OUR FIT** Error includes scale factor of 2.1.

**4.94 ± 0.05 OUR AVERAGE**

|                       |      |                            |      |      |                    |
|-----------------------|------|----------------------------|------|------|--------------------|
| 4.965 ± 0.038 ± 0.037 |      | <sup>1</sup> AMBROSINO 08A | KLOE | ±    |                    |
| 4.86 ± 0.10           | 3516 | CHIANG                     | 72   | OSPK | + 1.84 GeV/c $K^+$ |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|           |     |         |    |      |   |
|-----------|-----|---------|----|------|---|
| 4.7 ± 0.3 | 429 | SHAKLEE | 64 | HLBC | + |
| 5.0 ± 0.5 |     | ROE     | 61 | HLBC | + |

<sup>1</sup> Depends on  $K^+$  lifetime  $\tau$ . AMBROSINO 08A uses PDG 06 value of  $\tau = (1.2385 \pm 0.0024) \times 10^{-8}$  sec. The correlation between  $K_{e3}^+$  and  $K_{\mu 3}^+$  branching fraction measurements is 62.7%.

$\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\mu^+ \nu_\mu)$   $\Gamma_3/\Gamma_2$

| VALUE | EVTS | DOCUMENT ID | TECN | CHG |
|-------|------|-------------|------|-----|
|-------|------|-------------|------|-----|

**0.0798 ± 0.0008 OUR FIT** Error includes scale factor of 1.9.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                 |     |                       |     |      |   |
|-----------------|-----|-----------------------|-----|------|---|
| 0.069 ± 0.006   | 350 | ZELLER                | 69  | ASPK | + |
| 0.0775 ± 0.0033 | 960 | BOTTERILL             | 68C | ASPK | + |
| 0.069 ± 0.006   | 561 | GARLAND               | 68  | OSPK | + |
| 0.0791 ± 0.0054 | 295 | <sup>1</sup> AUERBACH | 67  | OSPK | + |

<sup>1</sup> AUERBACH 67 changed from 0.0797 ± 0.0054. See comment with ratio  $\Gamma(\pi^0 \mu^+ \nu_\mu)/\Gamma(\mu^+ \nu_\mu)$ . The value 0.0785 ± 0.0025 given in AUERBACH 67 is an average of AUERBACH 67  $\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\mu^+ \nu_\mu)$  and CESTER 66  $\Gamma(\pi^0 e^+ \nu_e)/[\Gamma(\mu^+ \nu_\mu) + \Gamma(\pi^+ \pi^0)]$ .

$\Gamma(\pi^0 e^+ \nu_e)/[\Gamma(\mu^+ \nu_\mu) + \Gamma(\pi^+ \pi^0)]$   $\Gamma_3/(\Gamma_2 + \Gamma_9)$

| VALUE (units $10^{-2}$ ) | EVTS | DOCUMENT ID | TECN | CHG |
|--------------------------|------|-------------|------|-----|
|--------------------------|------|-------------|------|-----|

**6.02 ± 0.06 OUR FIT** Error includes scale factor of 2.1.

**6.02 ± 0.15 OUR AVERAGE**

|             |      |            |    |      |   |
|-------------|------|------------|----|------|---|
| 6.16 ± 0.22 | 5110 | ESCHSTRUTH | 68 | OSPK | + |
| 5.89 ± 0.21 | 1679 | CESTER     | 66 | OSPK | + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |  |                           |    |      |   |
|-------------|--|---------------------------|----|------|---|
| 5.92 ± 0.65 |  | <sup>1</sup> WEISSENBE... | 76 | SPEC | + |
|-------------|--|---------------------------|----|------|---|

<sup>1</sup> Value calculated from WEISSENBERG 76 ( $\pi^0 e \nu$ ), ( $\mu \nu$ ), and ( $\pi \pi^0$ ) values to eliminate dependence on our 1974 ( $\pi 2\pi^0$ ) and ( $\pi \pi^+ \pi^-$ ) fractions.

$\Gamma(\pi^0 e^+ \nu_e)/[\Gamma(\pi^0 \mu^+ \nu_\mu) + \Gamma(\pi^+ \pi^0) + \Gamma(\pi^+ \pi^0 \pi^0)]$   $\Gamma_3/(\Gamma_4 + \Gamma_9 + \Gamma_{10})$

| VALUE | EVTS | DOCUMENT ID | TECN | CHG |
|-------|------|-------------|------|-----|
|-------|------|-------------|------|-----|

**0.1967 ± 0.0016 OUR FIT** Error includes scale factor of 2.5.

**0.1962 ± 0.0008 ± 0.0035** 71k SHER 03 B865 +

$\Gamma(\pi^0 e^+ \nu_e)/\Gamma(\pi^+ \pi^0)$   $\Gamma_3/\Gamma_9$

| VALUE | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|-------|------|-------------|------|-----|---------|
|-------|------|-------------|------|-----|---------|

**0.2454 ± 0.0023 OUR FIT** Error includes scale factor of 2.6.

**0.2467 ± 0.0011 OUR AVERAGE** Error includes scale factor of 1.1.

|                          |     |        |     |      |         |
|--------------------------|-----|--------|-----|------|---------|
| 0.2423 ± 0.0015 ± 0.0037 | 31k | UVAROV | 14  | ISTR | - ISTR+ |
| 0.2470 ± 0.0009 ± 0.0004 | 87k | BATLEY | 07A | NA48 | ±       |

• • • We do not use the following data for averages, fits, limits, etc. • • •

0.221 ± 0.012                      786            <sup>1</sup>LUCAS            73B HBC            –            Dalitz pairs only  
<sup>1</sup>LUCAS 73B gives  $N(K_{e3}) = 786 \pm 3.1\%$ ,  $N(2\pi) = 3564 \pm 3.1\%$ . We use these values to obtain quoted result.

**$\Gamma(\pi^0 e^+ \nu_e) / \Gamma(\pi^+ \pi^+ \pi^-)$   $\Gamma_3 / \Gamma_{11}$**

| <u>VALUE</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|--------------|-------------|--------------------|-------------|------------|
|--------------|-------------|--------------------|-------------|------------|

**0.908 ± 0.009 OUR FIT** Error includes scale factor of 1.6.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |      |                      |     |      |   |
|---------------|------|----------------------|-----|------|---|
| 0.867 ± 0.027 | 2768 | BARMIN               | 87  | XEBC | + |
| 0.856 ± 0.040 | 2827 | BRAUN                | 75  | HLBC | + |
| 0.850 ± 0.019 | 4385 | <sup>1</sup> HAIDT   | 71  | HLBC | + |
| 0.846 ± 0.021 | 4385 | <sup>1</sup> EICHTEN | 68  | HLBC | + |
| 0.94 ± 0.09   | 854  | BELLOTTI             | 67B | HLBC |   |
| 0.90 ± 0.06   | 230  | BORREANI             | 64  | HBC  | + |

<sup>1</sup>HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in  $\Gamma(\pi^0 \mu^+ \nu) / \Gamma(\pi^0 e^+ \nu)$  with more precise results.

**$\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma_{\text{total}}$   $\Gamma_4 / \Gamma$**

| <u>VALUE (units 10<sup>-2</sup>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|--------------------------------------|-------------|--------------------|-------------|------------|----------------|
|--------------------------------------|-------------|--------------------|-------------|------------|----------------|

**3.352 ± 0.033 OUR FIT** Error includes scale factor of 1.9.

**3.24 ± 0.04 OUR AVERAGE**

|                       |      |                        |     |      |                             |
|-----------------------|------|------------------------|-----|------|-----------------------------|
| 3.233 ± 0.029 ± 0.026 |      | <sup>1</sup> AMBROSINO | 08A | KLOE | ±                           |
| 3.33 ± 0.16           | 2345 | CHIANG                 | 72  | OSPK | + 1.84 GeV/c K <sup>+</sup> |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|           |  |                     |    |      |   |
|-----------|--|---------------------|----|------|---|
| 2.8 ± 0.4 |  | <sup>2</sup> TAYLOR | 59 | EMUL | + |
|-----------|--|---------------------|----|------|---|

<sup>1</sup>Depends on K<sup>+</sup> lifetime  $\tau$ . AMBROSINO 08A uses PDG 06 value of  $\tau = (1.2385 \pm 0.0024) \times 10^{-8}$  sec. The correlation between K<sub>e3</sub><sup>+</sup> and K <sub>$\mu$ 3</sub><sup>+</sup> branching fraction measurements is 62.7%.

<sup>2</sup>Earlier experiments not averaged.

**$\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma(\mu^+ \nu_\mu)$   $\Gamma_4 / \Gamma_2$**

| <u>VALUE</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|--------------|-------------|--------------------|-------------|------------|
|--------------|-------------|--------------------|-------------|------------|

**0.0527 ± 0.0006 OUR FIT** Error includes scale factor of 1.8.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                 |     |                       |    |      |   |
|-----------------|-----|-----------------------|----|------|---|
| 0.054 ± 0.009   | 240 | ZELLER                | 69 | ASPK | + |
| 0.0480 ± 0.0037 | 424 | <sup>1</sup> GARLAND  | 68 | OSPK | + |
| 0.0486 ± 0.0040 | 307 | <sup>2</sup> AUERBACH | 67 | OSPK | + |

<sup>1</sup>GARLAND 68 changed from 0.055 ± 0.004 in agreement with  $\mu$ -spectrum calculation of GAILLARD 70 appendix B. L.G.Pondrom, (private communication 73).

<sup>2</sup>AUERBACH 67 changed from 0.0602 ± 0.0046 by erratum which brings the  $\mu$ -spectrum calculation into agreement with GAILLARD 70 appendix B.

**$\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma(\pi^0 e^+ \nu_e)$   $\Gamma_4 / \Gamma_3$**

| <u>VALUE</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|--------------|-------------|--------------------|-------------|------------|----------------|
|--------------|-------------|--------------------|-------------|------------|----------------|

**0.6608 ± 0.0029 OUR FIT** Error includes scale factor of 1.1.

**0.6618 ± 0.0027 OUR AVERAGE**

|                       |      |                      |     |      |   |
|-----------------------|------|----------------------|-----|------|---|
| 0.663 ± 0.003 ± 0.001 | 77k  | BATLEY               | 07A | NA48 | ± |
| 0.671 ± 0.007 ± 0.008 | 24k  | HORIE                | 01  | SPEC |   |
| 0.670 ± 0.014         |      | <sup>1</sup> HEINTZE | 77  | SPEC | + |
| 0.667 ± 0.017         | 5601 | BOTTERILL            | 68B | ASPK | + |

• • • We use the following data for averages but not for fits. • • •

|                 |  |                            |      |   |  |
|-----------------|--|----------------------------|------|---|--|
| 0.6511 ± 0.0064 |  | <sup>2</sup> AMBROSINO 08A | KLOE | ± |  |
|-----------------|--|----------------------------|------|---|--|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |      |                      |     |      |                             |
|---------------|------|----------------------|-----|------|-----------------------------|
| 0.608 ± 0.014 | 1585 | <sup>3</sup> BRAUN   | 75  | HLBC | +                           |
| 0.705 ± 0.063 | 554  | <sup>4</sup> LUCAS   | 73B | HBC  | - Dalitz pairs only         |
| 0.698 ± 0.025 | 3480 | <sup>5</sup> CHIANG  | 72  | OSPK | + 1.84 GeV/c K <sup>+</sup> |
| 0.596 ± 0.025 |      | <sup>6</sup> HAIDT   | 71  | HLBC | +                           |
| 0.604 ± 0.022 | 1398 | <sup>6</sup> EICHTEN | 68  | HLBC |                             |
| 0.703 ± 0.056 | 1509 | CALLAHAN             | 66B | HLBC |                             |

<sup>1</sup> HEINTZE 77 value from fit to  $\lambda_0$ . Assumes  $\mu$ -e universality.

<sup>2</sup> Not used in the fit. This result enters the fit via correlation of  $K_{e3}^+$  and  $K_{\mu 3}^+$  branching fraction measurements of AMBROSINO 08A.

<sup>3</sup> BRAUN 75 value is from form factor fit. Assumes  $\mu$ -e universality.

<sup>4</sup> LUCAS 73B gives  $N(K_{\mu 3}) = 554 \pm 7.6\%$ ,  $N(K_{e3}) = 786 \pm 3.1\%$ . We divide.

<sup>5</sup> CHIANG 72  $\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma(\pi^0 e^+ \nu_e)$  is statistically independent of CHIANG 72  $\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma_{\text{total}}$  and  $\Gamma(\pi^0 e^+ \nu_e) / \Gamma_{\text{total}}$ .

<sup>6</sup> HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy with more precise results.

$[\Gamma(\pi^0 \mu^+ \nu_\mu) + \Gamma(\pi^+ \pi^0)] / \Gamma_{\text{total}}$   $(\Gamma_4 + \Gamma_9) / \Gamma$

We combine these two modes for experiments measuring them in xenon bubble chamber because of difficulties of separating them there.

|                                      |             |                    |             |            |
|--------------------------------------|-------------|--------------------|-------------|------------|
| <u>VALUE (units 10<sup>-2</sup>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|--------------------------------------|-------------|--------------------|-------------|------------|

**24.02 ± 0.08 OUR FIT** Error includes scale factor of 1.2.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|            |     |         |    |      |   |
|------------|-----|---------|----|------|---|
| 25.4 ± 0.9 | 886 | SHAKLEE | 64 | HLBC | + |
| 23.4 ± 1.1 |     | ROE     | 61 | HLBC | + |

$\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma(\pi^+ \pi^0)$   $\Gamma_4 / \Gamma_9$

|              |             |                    |             |            |
|--------------|-------------|--------------------|-------------|------------|
| <u>VALUE</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|--------------|-------------|--------------------|-------------|------------|

**0.1637 ± 0.0006 ± 0.0003** 77k BATLEY 07A NA48 ±

$\Gamma(\pi^0 \mu^+ \nu_\mu) / \Gamma(\pi^+ \pi^+ \pi^-)$   $\Gamma_4 / \Gamma_{11}$

|              |             |                    |             |            |                |
|--------------|-------------|--------------------|-------------|------------|----------------|
| <u>VALUE</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|--------------|-------------|--------------------|-------------|------------|----------------|

**0.600 ± 0.007 OUR FIT** Error includes scale factor of 1.6.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |      |                      |     |      |            |
|---------------|------|----------------------|-----|------|------------|
| 0.503 ± 0.019 | 1505 | <sup>1</sup> HAIDT   | 71  | HLBC | +          |
| 0.510 ± 0.017 | 1505 | <sup>1</sup> EICHTEN | 68  | HLBC | +          |
| 0.63 ± 0.07   | 2845 | <sup>2</sup> BISI    | 65B | BC   | + HBC+HLBC |

<sup>1</sup> HAIDT 71 is a reanalysis of EICHTEN 68. Not included in average because of large discrepancy in  $\Gamma(\pi^0 \mu^+ \nu) / \Gamma(\pi^0 e^+ \nu)$  with more precise results.

<sup>2</sup> Error enlarged for background problems. See GAILLARD 70.

$\Gamma(\pi^0 \pi^0 e^+ \nu_e) / \Gamma_{\text{total}}$   $\Gamma_5 / \Gamma$

|                                      |             |                    |             |            |
|--------------------------------------|-------------|--------------------|-------------|------------|
| <u>VALUE (units 10<sup>-5</sup>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|--------------------------------------|-------------|--------------------|-------------|------------|

**2.55 ± 0.04 OUR FIT** Error includes scale factor of 1.1.

**2.54 ± 0.89** 10 BARMIN 88B HLBC +

$\Gamma(\pi^0 \pi^0 e^+ \nu_e) / \Gamma(\pi^+ \pi^0 \pi^0)$   $\Gamma_5 / \Gamma_{10}$

| <u>VALUE (units <math>10^{-3}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> |
|---|-------------|---------------------|-------------|------------|
| <b>1.449 ± 0.008 OUR FIT</b>              |             |                     |             |            |
| <b>1.449 ± 0.006 ± 0.006</b>              | 65.2k       | <sup>1</sup> BATLEY | 14A NA48    | ±          |

<sup>1</sup> Data collected in 2003–2004. This leads to the scalar form factor  $(1 + \delta_{EM}) f_5 = 6.079 \pm 0.012 \pm 0.027 \pm 0.046$  where the last error is due to the normalizing decay mode uncertainty.

$\Gamma(\pi^0 \pi^0 e^+ \nu_e) / \Gamma(\pi^0 e^+ \nu_e)$   $\Gamma_5 / \Gamma_3$

| <u>VALUE (units <math>10^{-4}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u>                          |
|---|-------------|--------------------|-------------|-------------------------------------|
| <b>5.03 ± 0.09 OUR FIT</b>                |             |                    |             |                                     |
|   |             |                    |             | Error includes scale factor of 1.2. |

**4.1  $\begin{smallmatrix} +1.0 \\ -0.7 \end{smallmatrix}$  OUR AVERAGE**

|  |    |         |          |   |
|--|----|---------|----------|---|
| 4.2 $\begin{smallmatrix} +1.0 \\ -0.9 \end{smallmatrix}$ | 25 | BOLOTOV | 86B CALO | – |
| 3.8 $\begin{smallmatrix} +5.0 \\ -1.2 \end{smallmatrix}$ | 2  | LJUNG   | 73 HLBC  | + |

$\Gamma(\pi^+ \pi^- e^+ \nu_e) / \Gamma(\pi^+ \pi^+ \pi^-)$   $\Gamma_6 / \Gamma_{11}$

| <u>VALUE (units <math>10^{-4}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> |
|---|-------------|---------------------|-------------|------------|
| <b>7.606 ± 0.029 OUR AVERAGE</b>          |             |                     |             |            |
| 7.615 ± 0.008 ± 0.028                     | 1.1M        | <sup>1</sup> BATLEY | 12 NA48     | ±          |
| 7.35 ± 0.01 ± 0.19                        | 388k        | <sup>2</sup> PISLAK | 01 B865     |            |
| 7.21 ± 0.32                               | 30k         | ROSSELET            | 77 SPEC     | +          |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |     |             |         |   |
|-------------|-----|-------------|---------|---|
| 7.36 ± 0.68 | 500 | BOURQUIN    | 71 ASPK |   |
| 7.0 ± 0.9   | 106 | SCHWEINB... | 71 HLBC | + |
| 5.83 ± 0.63 | 269 | ELY         | 69 HLBC | + |

<sup>1</sup> BATLEY 12 uses data collected in 2003–2004. The result is inclusive of  $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu \gamma$  decays. Using PDG 12 value for  $\Gamma(\pi^+ \pi^- \pi^+) / \Gamma = (5.59 \pm 0.04) \times 10^{-2}$ . BATLEY 12 obtains  $B(\pi^+ \pi^- e \nu) = (4.257 \pm 0.004 \pm 0.035) \times 10^{-5}$  where the syst. error is dominated by the error on the normalization mode.

<sup>2</sup> PISLAK 01 reports  $\Gamma(\pi^+ \pi^- e^+ \nu_e) / \Gamma_{\text{total}} = (4.109 \pm 0.008 \pm 0.110) \times 10^{-5}$  using the PDG 00 value  $\Gamma(\pi^+ \pi^+ \pi^-) / \Gamma_{\text{total}} = (5.59 \pm 0.05) \times 10^{-2}$ . We divide by the PDG value and unfold its error from the systematic error. PISLAK 03 and PISLAK 10A give additional details on the branching ratio measurement and give improved errors on the S-wave  $\pi$ - $\pi$  scattering length:  $a_0^0 = 0.235 \pm 0.013$  and  $a_0^2 = -0.0410 \pm 0.0027$ .

$\Gamma(\pi^+ \pi^- \mu^+ \nu_\mu) / \Gamma_{\text{total}}$   $\Gamma_7 / \Gamma$

| <u>VALUE (units <math>10^{-5}</math>)</u>                                     | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
| <b>0.77 <math>\begin{smallmatrix} +0.54 \\ -0.50 \end{smallmatrix}</math></b> |             |                    |             |            |
|   | 1           | CLINE              | 65 FBC      | +          |

$\Gamma(\pi^+ \pi^- \mu^+ \nu_\mu) / \Gamma(\pi^+ \pi^+ \pi^-)$   $\Gamma_7 / \Gamma_{11}$

| <u>VALUE (units <math>10^{-4}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
| <b>2.57 ± 1.55</b>                        | 7           | BISI               | 67 DBC      | +          |
| ~ 2.5                                     | 1           | GREINER            | 64 EMUL     | +          |

$\Gamma(\pi^0\pi^0\pi^0e^+\nu_e)/\Gamma_{\text{total}}$   $\Gamma_8/\Gamma$

| VALUE (units $10^{-6}$ )  | CL% | EVTS | DOCUMENT ID | TECN | CHG    |
|---|-----|------|-------------|------|--------|
| <b>&lt;3.5</b>  | 90  | 0    | BOLOTOV     | 88   | SPEC - |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |     |      |             |      |        |
| <9  | 90  | 0    | BARMIN      | 92   | XEBC + |

————— **Hadronic modes** —————

$\Gamma(\pi^+\pi^0)/\Gamma_{\text{total}}$   $\Gamma_9/\Gamma$

| VALUE (units $10^{-2}$ )  | EVTS                                | DOCUMENT ID            | TECN | CHG    | COMMENT                    |
|---|-------------------------------------|------------------------|------|--------|----------------------------|
| <b>20.67±0.08 OUR FIT</b>   | Error includes scale factor of 1.2. |                        |      |        |                            |
| <b>20.70±0.16 OUR AVERAGE</b>   | Error includes scale factor of 1.8. |                        |      |        |                            |
| 20.65±0.05±0.08   | 1.4M                                | <sup>1</sup> AMBROSINO | 08E  | KLOE + | $\phi \rightarrow K^+K^-$  |
| 21.18±0.28  | 16k                                 | CHIANG                 | 72   | OSPK + | 1.84 GeV/c $K^+$           |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |                                     |                        |      |        |                            |
| 21.0 ±0.6   |                                     | CALLAHAN               | 65   | HLBC   | See $\Gamma_9/\Gamma_{11}$ |

<sup>1</sup> Fully inclusive of final-state radiation. The branching ratio is evaluated using  $K^+$  lifetime,  $\tau = 12.385$  ns.

$\Gamma(\pi^+\pi^0)/\Gamma(\mu^+\nu_\mu)$   $\Gamma_9/\Gamma_2$

| VALUE   | EVTS                                | DOCUMENT ID               | TECN | CHG    | COMMENT            |
|---|-------------------------------------|---------------------------|------|--------|--------------------|
| <b>0.3252±0.0016 OUR FIT</b>  | Error includes scale factor of 1.2. |                           |      |        |                    |
| <b>0.3325±0.0032 OUR AVERAGE</b>  |                                     |                           |      |        |                    |
| 0.3329±0.0047±0.0010  | 45k                                 | USHER                     | 92   | SPEC + | $p\bar{p}$ at rest |
| 0.3355±0.0057   |                                     | <sup>1</sup> WEISSENBE... | 76   | SPEC + |                    |
| 0.3277±0.0065   | 4517                                | <sup>2</sup> AUERBACH     | 67   | OSPK + |                    |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |                                     |                           |      |        |                    |
| 0.328 ±0.005  | 25k                                 | <sup>1</sup> WEISSENBE... | 74   | STRC + |                    |
| 0.305 ±0.018  | 1600                                | ZELLER                    | 69   | ASPK + |                    |

<sup>1</sup> WEISSENBERG 76 revises WEISSENBERG 74.

<sup>2</sup> AUERBACH 67 changed from  $0.3253 \pm 0.0065$ . See comment with ratio  $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma(\mu^+\nu_\mu)$ .

$\Gamma(\pi^+\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$   $\Gamma_9/\Gamma_{11}$

| VALUE   | EVTS                                | DOCUMENT ID | TECN | CHG   |
|---|-------------------------------------|-------------|------|-------|
| <b>3.702±0.022 OUR FIT</b>  | Error includes scale factor of 1.1. |             |      |       |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |                                     |             |      |       |
| 3.96 ±0.15  | 1045                                | CALLAHAN    | 66   | FBC + |

$\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{\text{total}}$   $\Gamma_{10}/\Gamma$

| VALUE (units $10^{-2}$ )  | EVTS                                | DOCUMENT ID            | TECN | CHG    | COMMENT          |
|---|-------------------------------------|------------------------|------|--------|------------------|
| <b>1.760±0.023 OUR FIT</b>  | Error includes scale factor of 1.1. |                        |      |        |                  |
| <b>1.775±0.028 OUR AVERAGE</b>  | Error includes scale factor of 1.2. |                        |      |        |                  |
| 1.763±0.013±0.022   |                                     | ALOISIO                | 04A  | KLOE ± |                  |
| 1.84 ±0.06  | 1307                                | CHIANG                 | 72   | OSPK + | 1.84 GeV/c $K^+$ |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |                                     |                        |      |        |                  |
| 1.53 ±0.11  | 198                                 | <sup>1</sup> PANDOULAS | 70   | EMUL + |                  |
| 1.8 ±0.2  | 108                                 | SHAKLEE                | 64   | HLBC + |                  |



|           |                     |    |      |   |
|-----------|---------------------|----|------|---|
| 1.7 ± 0.2 | ROE                 | 61 | HLBC | + |
| 1.5 ± 0.2 | <sup>2</sup> TAYLOR | 59 | EMUL | + |

<sup>1</sup> Includes events of TAYLOR 59.

<sup>2</sup> Earlier experiments not averaged.

### $\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^0)$ $\Gamma_{10}/\Gamma_9$

| VALUE | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|-------|------|-------------|------|-----|---------|
|-------|------|-------------|------|-----|---------|

**0.0851 ± 0.0012 OUR FIT** Error includes scale factor of 1.1.

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |     |                    |     |     |   |                   |
|---------------|-----|--------------------|-----|-----|---|-------------------|
| 0.081 ± 0.005 | 574 | <sup>1</sup> LUCAS | 73B | HBC | – | Dalitz pairs only |
|---------------|-----|--------------------|-----|-----|---|-------------------|

<sup>1</sup> LUCAS 73B gives  $N(\pi^+\pi^0\pi^0) = 574 \pm 5.9\%$ ,  $N(2\pi) = 3564 \pm 3.1\%$ . We quote  $0.5N(\pi^+\pi^0\pi^0)/N(2\pi)$  where 0.5 is because only Dalitz pair  $\pi^0$ 's were used.

### $\Gamma(\pi^+\pi^0\pi^0)/\Gamma(\pi^+\pi^+\pi^-)$ $\Gamma_{10}/\Gamma_{11}$

| VALUE | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|-------|------|-------------|------|-----|---------|
|-------|------|-------------|------|-----|---------|

**0.315 ± 0.004 OUR FIT** Error includes scale factor of 1.1.

**0.303 ± 0.009** 2027 BISI 65 BC + HBC+HLBC

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |    |       |    |      |   |
|---------------|----|-------|----|------|---|
| 0.393 ± 0.099 | 17 | YOUNG | 65 | EMUL | + |
|---------------|----|-------|----|------|---|

### $\Gamma(\pi^+\pi^+\pi^-)/\Gamma_{\text{total}}$ $\Gamma_{11}/\Gamma$

| VALUE (units $10^{-2}$ ) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|--------------------------|------|-------------|------|-----|---------|
|--------------------------|------|-------------|------|-----|---------|

**5.583 ± 0.024 OUR FIT**

**5.565 ± 0.031 ± 0.025** 68k <sup>1</sup> BABUSCI 14B KLOE +

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |      |                        |    |      |   |                  |
|-------------|------|------------------------|----|------|---|------------------|
| 5.56 ± 0.20 | 2330 | <sup>2</sup> CHIANG    | 72 | OSPK | + | 1.84 GeV/c $K^+$ |
| 5.34 ± 0.21 | 693  | <sup>3</sup> PANDOULAS | 70 | EMUL | + |                  |
| 5.71 ± 0.15 |      | DEMARCO                | 65 | HBC  |   |                  |
| 6.0 ± 0.4   | 44   | YOUNG                  | 65 | EMUL | + |                  |
| 5.54 ± 0.12 | 2332 | CALLAHAN               | 64 | HLBC | + |                  |
| 5.1 ± 0.2   | 540  | SHAKLEE                | 64 | HLBC | + |                  |
| 5.7 ± 0.3   |      | ROE                    | 61 | HLBC | + |                  |

<sup>1</sup> Inclusive of final-state radiation. Result obtained from averaging two branching ratios: one from a sample with  $K^- \rightarrow \mu\nu(\gamma)$  tagging and another with  $K^- \rightarrow \pi^-\pi^0(\gamma)$  tagging.

<sup>2</sup> Value is not independent of CHIANG 72  $\Gamma(\mu^+\nu_\mu)/\Gamma_{\text{total}}$ ,  $\Gamma(\pi^+\pi^0)/\Gamma_{\text{total}}$ ,  $\Gamma(\pi^+\pi^0\pi^0)/\Gamma_{\text{total}}$ ,  $\Gamma(\pi^0\mu^+\nu_\mu)/\Gamma_{\text{total}}$ , and  $\Gamma(\pi^0e^+\nu_e)/\Gamma_{\text{total}}$ .

<sup>3</sup> Includes events of TAYLOR 59.

## ———— Leptonic and semileptonic modes with photons ————

### $\Gamma(\mu^+\nu_\mu\gamma)/\Gamma_{\text{total}}$ $\Gamma_{12}/\Gamma$

| VALUE (units $10^{-3}$ ) | EVTS | DOCUMENT ID | TECN | CHG | COMMENT |
|--------------------------|------|-------------|------|-----|---------|
|--------------------------|------|-------------|------|-----|---------|

**6.2 ± 0.8 OUR AVERAGE**

|           |  |                        |    |      |   |                                |
|-----------|--|------------------------|----|------|---|--------------------------------|
| 6.6 ± 1.5 |  | <sup>1,2</sup> DEMIDOV | 90 | XEBC |   | $P(\mu) < 231.5 \text{ MeV}/c$ |
| 6.0 ± 0.9 |  | BARMIN                 | 88 | HLBC | + | $P(\mu) < 231.5 \text{ MeV}/c$ |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|           |    |                        |    |      |   |                              |
|-----------|----|------------------------|----|------|---|------------------------------|
| 3.5 ± 0.8 |    | <sup>2,3</sup> DEMIDOV | 90 | XEBC |   | $E(\gamma) > 20 \text{ MeV}$ |
| 3.2 ± 0.5 | 57 | <sup>4</sup> BARMIN    | 88 | HLBC | + | $E(\gamma) > 20 \text{ MeV}$ |

5.4 ± 0.3 <sup>5</sup> AKIBA 85 SPEC P(μ) < 231.5 MeV/c

<sup>1</sup> P(μ) cut given in DEMIDOV 90 paper, 235.1 MeV/c, is a misprint according to authors (private communication).

<sup>2</sup> DEMIDOV 90 quotes only inner bremsstrahlung (IB) part.

<sup>3</sup> Not independent of above DEMIDOV 90 value. Cuts differ.

<sup>4</sup> Not independent of above BARMIN 88 value. Cuts differ.

<sup>5</sup> Assumes μ-e universality and uses constraints from K → eνγ.

### $\Gamma(\mu^+ \nu_\mu \gamma(SD^+))/\Gamma_{\text{total}}$ $\Gamma_{13}/\Gamma$

Structure-dependent part with +γ helicity (SD<sup>+</sup> term). See the “Note on π<sup>±</sup> → ℓ<sup>±</sup>νγ and K<sup>±</sup> → ℓ<sup>±</sup>νγ Form Factors” in the π<sup>±</sup> section of the Particle Data Listings above.

| VALUE (units 10 <sup>-5</sup> ) | CL% | EVTS | DOCUMENT ID        | TECN     |
|---------------------------------|-----|------|--------------------|----------|
| <b>1.33 ± 0.12 ± 0.18</b>       |     | 2588 | <sup>1</sup> ADLER | 00B B787 |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|      |    |       |    |      |
|------|----|-------|----|------|
| <3.0 | 90 | AKIBA | 85 | SPEC |
|------|----|-------|----|------|

<sup>1</sup> ADLER 00B obtains the branching ratio by extrapolating the measurement in the kinematic region E<sub>μ</sub> > 137 MeV, E<sub>γ</sub> > 90 MeV to the full SD<sup>+</sup> phase-space. Also reports |F<sub>V</sub> + F<sub>A</sub>| = 0.165 ± 0.007 ± 0.011 and -0.04 < F<sub>V</sub> - F<sub>A</sub> < 0.24 at 90% CL.

### $\Gamma(\mu^+ \nu_\mu \gamma(SD^+INT))/\Gamma_{\text{total}}$ $\Gamma_{14}/\Gamma$

Interference term between internal Bremsstrahlung and SD<sup>+</sup> term. See the “Note on π<sup>±</sup> → ℓ<sup>±</sup>νγ and K<sup>±</sup> → ℓ<sup>±</sup>νγ Form Factors” in the π<sup>±</sup> section of the Particle Data Listings above.

| VALUE (units 10 <sup>-5</sup> ) | CL% | DOCUMENT ID | TECN    |
|---------------------------------|-----|-------------|---------|
| <b>&lt;2.7</b>                  | 90  | AKIBA       | 85 SPEC |

### $\Gamma(\mu^+ \nu_\mu \gamma(SD^- + SD^-INT))/\Gamma_{\text{total}}$ $\Gamma_{15}/\Gamma$

Sum of structure-dependent part with -γ helicity (SD<sup>-</sup> term) and interference term between internal Bremsstrahlung and SD<sup>-</sup> term. See the “Note on π<sup>±</sup> → ℓ<sup>±</sup>νγ and K<sup>±</sup> → ℓ<sup>±</sup>νγ Form Factors” in the π<sup>±</sup> section of the Particle Data Listings above.

| VALUE (units 10 <sup>-4</sup> ) | CL% | DOCUMENT ID        | TECN    |
|---------------------------------|-----|--------------------|---------|
| <b>&lt;2.6</b>                  | 90  | <sup>1</sup> AKIBA | 85 SPEC |

<sup>1</sup> Assumes μ-e universality and uses constraints from K → eνγ.

### $\Gamma(e^+ \nu_e \gamma)/\Gamma(\mu^+ \nu_\mu)$ $\Gamma_{16}/\Gamma_2$

| VALUE (units 10 <sup>-5</sup> ) | EVTS | DOCUMENT ID            | TECN     | CHG | COMMENT  |
|---------------------------------|------|------------------------|----------|-----|--|
| <b>1.483 ± 0.066 ± 0.013</b>    | 1.4k | <sup>1</sup> AMBROSINO | 09E KLOE | ±   | E <sub>γ</sub> in 10–250 MeV, p <sub>e</sub> > 200 MeV/c |

<sup>1</sup> AMBROSINO 09E measured the differential width dR<sub>γ</sub>/dE<sub>γ</sub> = (1/Γ(K → μν)) (dΓ(K → eνγ)/dE<sub>γ</sub>). Result obtained by integrating the differential width over E<sub>γ</sub> from 10 to 250 MeV.

### $\Gamma(\pi^0 e^+ \nu_e \gamma)/\Gamma(\pi^0 e^+ \nu_e)$ $\Gamma_{17}/\Gamma_3$

| VALUE (units 10 <sup>-2</sup> )  | EVTS | DOCUMENT ID   | TECN    | CHG | COMMENT  |
|----------------------------------|------|---|---------|-----|--|
| <b>0.505 ± 0.032 OUR AVERAGE</b> |      | Error includes scale factor of 1.3. See the ideogram below. |         |     |  |
| 0.47 ± 0.02 ± 0.03               | 4476 | <sup>1</sup> AKIMENKO                                       | 07 ISTR | -   | E <sub>γ</sub> > 10 MeV, 0.6 < cos(θ <sub>eγ</sub> ) < 0.9 |
| 0.46 ± 0.08                      | 82   | <sup>2</sup> BARMIN   | 91 XEBC |     | E <sub>γ</sub> > 10 MeV, 0.6 < cos(θ <sub>eγ</sub> ) < 0.9 |

|   |      |                       |     |      |   |  |
|---|------|-----------------------|-----|------|---|--|
| 0.56 ±0.04  | 192  | <sup>3</sup> BOLOTOV  | 86B | CALO | – | $E_\gamma > 10$ MeV                                  |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |      |                       |     |      |   |  |
| 1.81 ±0.03 ±0.07  | 4476 | <sup>1</sup> AKIMENKO | 07  | ISTR | – | $E_\gamma > 10$ MeV, $\theta_{e\gamma} > 10^\circ$   |
| 0.63 ±0.02 ±0.03  | 4476 | <sup>1</sup> AKIMENKO | 07  | ISTR | – | $E_\gamma > 30$ MeV, $\theta_{e\gamma} > 20^\circ$   |
| 1.51 ±0.25  | 82   | <sup>2</sup> BARMIN   | 91  | XEBC |   | $E_\gamma > 10$ MeV, $\cos(\theta_{e\gamma}) < 0.98$ |
| 0.48 ±0.20  | 16   | <sup>4</sup> LJUNG    | 73  | HLBC | + | $E_\gamma > 30$ MeV                                  |
| 0.22 <sup>+0.15</sup> <sub>–0.10</sub>  |      | <sup>4</sup> LJUNG    | 73  | HLBC | + | $E_\gamma > 30$ MeV                                  |
| 0.76 ±0.28  | 13   | <sup>5</sup> ROMANO   | 71  | HLBC |   | $E_\gamma > 10$ MeV                                  |
| 0.53 ±0.22  |      | <sup>5</sup> ROMANO   | 71  | HLBC | + | $E_\gamma > 30$ MeV                                  |
| 1.2 ±0.8  |      | BELLOTTI              | 67  | HLBC |   | $E_\gamma > 30$ MeV                                  |

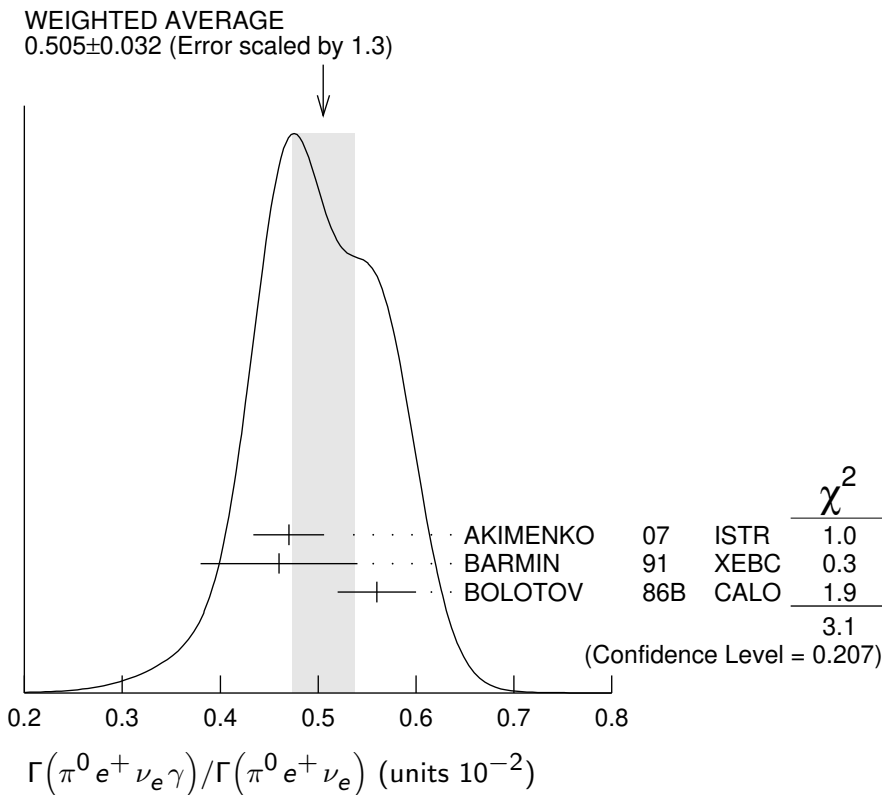
<sup>1</sup> AKIMENKO 07 provides values for three kinematic regions. For averaging, we use value with  $E_\gamma > 10$  MeV and  $0.6 < \cos(\theta_{e\gamma}) < 0.9$ .

<sup>2</sup> BARMIN 91 quotes branching ratio  $\Gamma(K \rightarrow e\pi^0\nu\gamma)/\Gamma_{\text{all}}$ . The measured normalization is  $[\Gamma(K \rightarrow e\pi^0\nu) + \Gamma(K \rightarrow \pi^+\pi^+\pi^-)]$ . For comparison with other experiments we used  $\Gamma(K \rightarrow e\pi^0\nu)/\Gamma_{\text{all}} = 0.0482$  to calculate the values quoted here.

<sup>3</sup>  $\cos(\theta_{e\gamma})$  between 0.6 and 0.9.

<sup>4</sup> First LJUNG 73 value is for  $\cos(\theta_{e\gamma}) < 0.9$ , second value is for  $\cos(\theta_{e\gamma})$  between 0.6 and 0.9 for comparison with ROMANO 71.

<sup>5</sup> Both ROMANO 71 values are for  $\cos(\theta_{e\gamma})$  between 0.6 and 0.9. Second value is for comparison with second LJUNG 73 value. We use lowest  $E_\gamma$  cut for Summary Table value. See ROMANO 71 for  $E_\gamma$  dependence.



$\Gamma(\pi^0 e^+ \nu_e \gamma(\text{SD}))/\Gamma_{\text{total}}$   $\Gamma_{18}/\Gamma$   
 Structure-dependent part.

| VALUE (units $10^{-5}$ ) | CL% | DOCUMENT ID | TECN | CHG    |
|--------------------------|-----|-------------|------|--------|
| <b>&lt;5.3</b>           | 90  | BOLOTOV     | 86B  | CALO - |

$\Gamma(\pi^0 \mu^+ \nu_\mu \gamma)/\Gamma_{\text{total}}$   $\Gamma_{19}/\Gamma$

| VALUE (units $10^{-5}$ )       | CL% | EVTS | DOCUMENT ID            | TECN | CHG    | COMMENT                  |
|--------------------------------|-----|------|------------------------|------|--------|--------------------------|
| <b>1.25 ± 0.25 OUR AVERAGE</b> |     |      |                        |      |        |                          |
| 1.10 ± 0.32 ± 0.05             |     | 23   | <sup>1</sup> ADLER     | 10   | B787   | 30 < $E_\gamma$ < 60 MeV |
| 1.46 ± 0.22 ± 0.32             |     | 153  | <sup>2</sup> TCHIKILEV | 07   | ISTR - | 30 < $E_\gamma$ < 60 MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                 |  |     |         |    |        |   |
|-----------------|--|-----|---------|----|--------|---|
| 2.4 ± 0.5 ± 0.6 |  | 125 | SHIMIZU | 06 | K470 + | $E_\gamma > 30$ MeV;<br>$\Theta_{\mu\gamma} > 20^\circ$ |
| <6.1            |  | 90  | LJUNG   | 73 | HLBC + | $E(\gamma) > 30$ MeV                                    |

<sup>1</sup> Value obtained from  $B(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu \gamma) = (2.51 \pm 0.74 \pm 0.12) \times 10^{-5}$  obtained in the kinematic region  $E_\gamma > 20$  MeV, and then theoretical  $K_{\mu 3\gamma}$  spectrum has been used. Also  $B(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu \gamma) = (1.58 \pm 0.46 \pm 0.08) \times 10^{-5}$ , for  $E_\gamma > 30$  MeV and  $\theta_{\mu\gamma} > 20^\circ$ , was determined.

<sup>2</sup> Obtained from measuring  $B(K_{\mu 3\gamma}) / B(K_{\mu 3})$  and using PDG 02 value  $B(K_{\mu 3}) = 3.27\%$ .  $B(K_{\mu 3\gamma}) = (8.82 \pm 0.94 \pm 0.86) \times 10^{-5}$  is obtained for  $5 \text{ MeV} < E_\gamma < 30 \text{ MeV}$ .

$\Gamma(\pi^0 \pi^0 e^+ \nu_e \gamma)/\Gamma_{\text{total}}$   $\Gamma_{20}/\Gamma$

| VALUE (units $10^{-6}$ ) | CL% | EVTS | DOCUMENT ID | TECN | CHG    | COMMENT             |
|--------------------------|-----|------|-------------|------|--------|---------------------|
| <b>&lt;5</b>             | 90  | 0    | BARMIN      | 92   | XEBC + | $E_\gamma > 10$ MeV |

————— **Hadronic modes with photons** —————

$\Gamma(\pi^+ \pi^0 \gamma(\text{INT}))/\Gamma_{\text{total}}$   $\Gamma_{21}/\Gamma$

The  $K^+ \rightarrow \pi^+ \pi^0 \gamma$  differential decay rate can be described in terms of  $T_{\pi^+}$ , the charged pion kinetic energy, and  $W^2 = (P_K \cdot P_\gamma)(P_{\pi^+} \cdot P_\gamma) / (m_K m_{\pi^+})^2$ ; then we can write  $d^2\Gamma(K^+ \rightarrow \pi^+ \pi^0 \gamma) / (dT_{\pi^+} dW^2) = d^2\Gamma(K^+ \rightarrow \pi^+ \pi^0 \gamma)_{IB} / (dT_{\pi^+} dW^2) [1 + 2 \cos(\pm\phi + \delta_1^1 - \delta_0^2) m_\pi^2 m_K^2 W^2 X_E + m_\pi^4 m_K^4 (X_E^2 + X_M^2) W^4]$ . The IB differential and total branching ratios are expressed in terms of the non-radiative experimental width  $\Gamma(K^+ \rightarrow \pi^+ \pi^0)$  by Low's theorem. Using PDG 10  $B(K^+ \rightarrow \pi^+ \pi^0) = 0.2066 \pm 0.0008$ , one obtains respectively  $B(K^+ \rightarrow \pi^+ \pi^0 \gamma)_{IB} (55 < T_{\pi^+} < 90 \text{ MeV}) = 2.55 \times 10^{-4}$  and  $B(K^+ \rightarrow \pi^+ \pi^0 \gamma)_{IB} (0 < T_{\pi^+} < 80 \text{ MeV}) = 1.80 \times 10^{-4}$ . Fitting respectively the piece proportional to  $W^2$  and the piece proportional to  $W^4$ , the interference contribution (INT), proportional to  $X_E$ , and the direct contribution (DE) proportional to  $X_E^2 + X_M^2$  are extracted.

| VALUE (units $10^{-6}$ )   | EVTS | DOCUMENT ID         | TECN | CHG    | COMMENT              |
|----------------------------|------|---------------------|------|--------|----------------------|
| <b>-4.24 ± 0.63 ± 0.70</b> | 600k | <sup>1</sup> BATLEY | 10A  | NA48 ± | $T_{\pi^+}$ 0–80 MeV |

<sup>1</sup> The cut on the photon energy implies  $W^2 > 0.2$ . BATLEY 10A obtains the INT and DE fractional branchings with respect to IB from a simultaneous kinematical fit of INT and DE and then we use the PDG 10 value for  $B(K^+ \rightarrow \pi^+ \pi^0) = 20.66 \pm 0.08$  to determine the IB. The INT and DE correlation coefficients  $-0.83$ . Assuming a constant electric amplitude,  $X_E$ , this INT value implies  $X_E = -24 \pm 6 \text{ GeV}^{-4}$ .

$\Gamma(\pi^+\pi^0\gamma(\text{DE}))/\Gamma_{\text{total}}$   $\Gamma_{22}/\Gamma$

Direct emission (DE) part of  $\Gamma(\pi^+\pi^0\gamma)/\Gamma_{\text{total}}$ , assuming that interference (INT) component is zero.

| <u>VALUE (units <math>10^{-6}</math>)</u>                                     | <u>EVTS</u> | <u>DOCUMENT ID</u>      | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u>          |
|---|-------------|-------------------------|-------------|------------|-------------------------|
| <b>5.99±0.27±0.25</b>   | 600k        | <sup>1</sup> BATLEY 10A | NA48        | ±          | $T_{\pi^+}$ 0–80 MeV    |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |             |                         |             |            |                         |
| 3.8 ±0.8 ±0.7   | 10k         | ALIEV 06                | K470        | +          | $T_{\pi^+}$ 55–90 MeV   |
| 3.7 ±3.9 ±1.0   | 930         | UVAROV 06               | ISTR        | –          | $T_{\pi^-}$ 55–90 MeV   |
| 3.2 ±1.3 ±1.0   | 4k          | ALIEV 03                | K470        | +          | $T_{\pi^+}$ 55–90 MeV   |
| 6.1 ±2.5 ±1.9   | 4k          | ALIEV 03                | K470        | +          | $T_{\pi^+}$ full range  |
| 4.7 ±0.8 ±0.3   | 20k         | <sup>2</sup> ADLER 00C  | B787        | +          | $T_{\pi^+}$ 55–90 MeV   |
| 20.5 ±4.6 <sup>+3.9</sup> <sub>–2.3</sub>                                     |             | BOLOTOV 87              | WIRE        | –          | $T_{\pi^-}$ 55–90 MeV   |
| 15.6 ±3.5 ±5.0  |             | ABRAMS 72               | ASPK        | ±          | $T_{\pi^\pm}$ 55–90 MeV |

<sup>1</sup> The cut on the photon energy implies  $W^2 > 0.2$ . BATLEY 10A obtains the INT and DE fractional branchings with respect to IB from a simultaneous kinematical fit of INT and DE and then we use the PDG 10 value for  $B(K^+ \rightarrow \pi^+\pi^0) = 20.66 \pm 0.08$  to determine the IB. The INT and DE correlation coefficients  $-0.93$ . Assuming constant electric and magnetic amplitudes,  $X_E$  and  $X_M$ , these INT and DE values imply  $X_E = -24 \pm 6 \text{ GeV}^{-4}$  and  $X_M = -254 \pm 9 \text{ GeV}^{-4}$ .

<sup>2</sup> ADLER 00C measures the INT component to be  $(-0.4 \pm 1.6)\%$  of the inner bremsstrahlung (IB) component.

$\Gamma(\pi^+\pi^0e^+e^-)/\Gamma_{\text{total}}$   $\Gamma_{23}/\Gamma$

| <u>VALUE (units <math>10^{-6}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>     | <u>TECN</u> |
|---|-------------|------------------------|-------------|
| <b>4.24±0.14</b>                          | 4.9k        | <sup>1</sup> BATLEY 19 | NA48        |

<sup>1</sup> BATLEY 19 result is obtained from an exposure of  $1.7 \times 10^{11}$  charged kaon decays recorded in 2003–2004. The study of the kinematic space shows evidence for a structure dependent contribution consistent with predictions from chiral perturbation theory.

$\Gamma(\pi^+\pi^0\pi^0\gamma)/\Gamma(\pi^+\pi^0\pi^0)$   $\Gamma_{24}/\Gamma_{10}$

| <u>VALUE (units <math>10^{-4}</math>)</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u>               |
|---|--------------------|-------------|------------|------------------------------|
| <b>4.3<sup>+3.2</sup><sub>–1.7</sub></b>  | BOLOTOV 85         | SPEC        | –          | $E(\gamma) > 10 \text{ MeV}$ |

$\Gamma(\pi^+\pi^+\pi^-\gamma)/\Gamma_{\text{total}}$   $\Gamma_{25}/\Gamma$

| <u>VALUE (units <math>10^{-4}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u>               |
|---|-------------|--------------------|-------------|------------|------------------------------|
| <b>0.071±0.005 OUR AVERAGE</b>            |             |                    |             |            |                              |
| 0.071±0.005                               | 450         | SHAPKIN 19         | OKA         | +          | $E(\gamma) > 30 \text{ MeV}$ |
| 1.10 ±0.48                                | 7           | BARMIN 89          | XEBC        |            | $E(\gamma) > 5 \text{ MeV}$  |
| 1.0 ±0.4                                  |             | STAMER 65          | EMUL        | +          | $E(\gamma) > 11 \text{ MeV}$ |

$\Gamma(\pi^+\gamma\gamma)/\Gamma_{\text{total}}$   $\Gamma_{26}/\Gamma$

| <u>VALUE (units <math>10^{-7}</math>)</u> | <u>CL%</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>        | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|---|------------|-------------|---------------------------|-------------|------------|----------------|
| <b>10.1 ±0.6 OUR AVERAGE</b>              |            |             |                           |             |            |                |
| 10.03±0.51±0.24                           |            | 215         | <sup>1</sup> LAZZERONI 14 | NA62        | ±          |                |
| 11 ±3 ±1                                  |            | 31          | <sup>2</sup> KITCHING 97  | B787        | +          |                |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                          |     |                        |     |      |       |  |
|--------------------------|-----|------------------------|-----|------|-------|--|
| $9.10 \pm 0.72 \pm 0.22$ | 149 | <sup>3</sup> BATLEY    | 14  | NA48 | $\pm$ |  |
| $< 0.083$                | 90  | <sup>4</sup> ARTAMONOV | 05  | B949 | $+$   | $P_\pi > 213 \text{ MeV}/c$                  |
| $< 10$                   | 90  | ATIYA                  | 90B | B787 | $+$   | $T_\pi 117\text{--}127 \text{ MeV}$          |
| $< 84$                   | 90  | ASANO                  | 82  | CNTR | $+$   | $T_\pi 117\text{--}127 \text{ MeV}$          |
| $-420 \pm 520$           |     | ABRAMS                 | 77  | SPEC | $+$   | $T_\pi < 92 \text{ MeV}$                     |
| $< 350$                  | 90  | LJUNG                  | 73  | HLBC | $+$   | $6\text{--}102, 114\text{--}127 \text{ MeV}$ |
| $< 500$                  | 90  | KLEMS                  | 71  | OSPK | $+$   | $T_\pi < 117 \text{ MeV}$                    |
| $-100 \pm 600$           |     | CHEN                   | 68  | OSPK | $+$   | $T_\pi 60\text{--}90 \text{ MeV}$            |

<sup>1</sup> LAZZERONI 14 combines NA62 and NA48/2 results. The result for the full kinematic range is extrapolated from the model-independent branching fraction  $(9.65 \pm 0.61 \pm 0.14) \times 10^{-7}$  for  $(m_{\gamma\gamma}/m_K)^2 > 0.2$ . The measured ChPT parameter  $\hat{c} = 1.86 \pm 0.25$ .

<sup>2</sup> KITCHING 97 is extrapolated from their model-independent branching fraction  $(6.0 \pm 1.5 \pm 0.7) \times 10^{-7}$  for  $100 \text{ MeV}/c < P_{\pi^+} < 180 \text{ MeV}/c$  using Chiral Perturbation Theory.

<sup>3</sup> BATLEY 14 uses data collected in 2003 and 2004. Branching ratio is obtained by determining the parameter  $\hat{c} = 1.41 \pm 0.38 \pm 0.11$  and integrating the  $\mathcal{O}(p^6)$  chiral spectrum. A model independent value for the branching ratio is also obtained  $(8.77 \pm 0.87 \pm 0.17) \times 10^{-7}$  for kinematic range  $(m_{\gamma\gamma}/m_K)^2 > 0.2$ .

<sup>4</sup> ARTAMONOV 05 limit assumes ChPT with  $\hat{c} = 1.8$  with unitarity corrections. With  $\hat{c} = 1.6$  and no unitarity corrections they obtain  $< 2.3 \times 10^{-8}$  at 90% CL. This partial branching ratio is predicted to be  $6.10 \times 10^{-9}$  and  $0.49 \times 10^{-9}$  for the cases with and without unitarity correction.

### $\Gamma(\pi^+ 3\gamma)/\Gamma_{\text{total}}$ $\Gamma_{27}/\Gamma$

Values given here assume a phase space pion energy spectrum.

| VALUE (units $10^{-4}$ )     | CL% | DOCUMENT ID | TECN | CHG  | COMMENT                                  |
|------------------------------|-----|-------------|------|------|--|
| <b><math>&lt; 1.0</math></b> | 90  | ASANO       | 82   | CNTR | $+$ $T(\pi) 117\text{--}127 \text{ MeV}$ |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|         |    |       |    |      |                                |
|---------|----|-------|----|------|--------------------------------|
| $< 3.0$ | 90 | KLEMS | 71 | OSPK | $+$ $T(\pi) > 117 \text{ MeV}$ |
|---------|----|-------|----|------|--------------------------------|

### $\Gamma(\pi^+ e^+ e^- \gamma)/\Gamma_{\text{total}}$ $\Gamma_{28}/\Gamma$

| VALUE (units $10^{-8}$ )                   | EVTS | DOCUMENT ID         | TECN | COMMENT                               |
|--|------|---------------------|------|---------------------------------------|
| <b><math>1.19 \pm 0.12 \pm 0.04</math></b> | 113  | <sup>1</sup> BATLEY | 08   | NA48 $m_{ee\gamma} > 260 \text{ MeV}$ |

<sup>1</sup> BATLEY 08 also reports the Chiral Perturbation Theory parameter  $\hat{c} = 0.9 \pm 0.45$  obtained using the shape of the  $e^+ e^- \gamma$  invariant mass spectrum. By extrapolating the theoretical amplitude to  $m_{ee\gamma} < 260 \text{ MeV}$ , it obtains the inclusive  $B(K^+ \rightarrow \pi^+ e^+ e^- \gamma) = (1.29 \pm 0.13 \pm 0.03) \times 10^{-8}$ , where the first error is the combined statistical and systematic errors and the second error is from the uncertainty in  $\hat{c}$ .

## ———— Leptonic modes with $\ell\bar{\ell}$ pairs ————

### $\Gamma(e^+ \nu_e \nu \bar{\nu})/\Gamma(e^+ \nu_e)$ $\Gamma_{29}/\Gamma_1$

| VALUE                        | CL% | EVTS | DOCUMENT ID | TECN | CHG      |
|------------------------------|-----|------|-------------|------|----------|
| <b><math>&lt; 3.8</math></b> | 90  | 0    | HEINTZE     | 79   | SPEC $+$ |

### $\Gamma(\mu^+ \nu_\mu \nu \bar{\nu})/\Gamma_{\text{total}}$ $\Gamma_{30}/\Gamma$

| VALUE                                       | CL% | DOCUMENT ID            | TECN | CHG      |
|---|-----|------------------------|------|----------|
| <b><math>&lt; 2.4 \times 10^{-6}</math></b> | 90  | <sup>1</sup> ARTAMONOV | 16   | B949 $+$ |

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<6.0 \times 10^{-6}$       90      <sup>2</sup> PANG      73      CNTR +

<sup>1</sup> ARTAMONOV 16 assumes Standard model  $\mu$  spectrum. The search is performed in the muon momentum region between 130 and 175 MeV/c.

<sup>2</sup> PANG 73 assumes  $\mu$  spectrum from  $\nu$ - $\nu$  interaction of BARDIN 70.

$\Gamma(e^+ \nu_e e^+ e^-) / \Gamma_{\text{total}}$   $\Gamma_{31} / \Gamma$

| VALUE (units $10^{-8}$ )  | EVTS | DOCUMENT ID  | TECN | CHG | COMMENT            |
|---------------------------|------|--------------|------|-----|--------------------|
| <b>2.48 ± 0.14 ± 0.14</b> | 410  | POBLAGUEV 02 | B865 | +   | $m_{ee} > 150$ MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

20 ± 20      4      DIAMANT-... 76      SPEC +       $m_{e^+e^-} > 140$  MeV

$\Gamma(\mu^+ \nu_\mu e^+ e^-) / \Gamma_{\text{total}}$   $\Gamma_{32} / \Gamma$

| VALUE (units $10^{-8}$ )  | EVTS | DOCUMENT ID  | TECN | CHG | COMMENT            |
|---------------------------|------|--------------|------|-----|--------------------|
| <b>7.06 ± 0.16 ± 0.26</b> | 2.7k | POBLAGUEV 02 | B865 | +   | $m_{ee} > 145$ MeV |

• • • We do not use the following data for averages, fits, limits, etc. • • •

100 ± 30      14      DIAMANT-... 76      SPEC +       $m_{e^+e^-} > 140$  MeV

$\Gamma(e^+ \nu_e \mu^+ \mu^-) / \Gamma_{\text{total}}$   $\Gamma_{33} / \Gamma$

| VALUE (units $10^{-8}$ ) | CL% | DOCUMENT ID | TECN    |
|--------------------------|-----|-------------|---------|
| <b>1.72 ± 0.45</b>       |     | MA          | 06 B865 |

• • • We do not use the following data for averages, fits, limits, etc. • • •

$<50$       90      ADLER      98      B787

$\Gamma(\mu^+ \nu_\mu \mu^+ \mu^-) / \Gamma_{\text{total}}$   $\Gamma_{34} / \Gamma$

| VALUE (units $10^{-7}$ ) | CL% | DOCUMENT ID | TECN    | CHG |
|--------------------------|-----|-------------|---------|-----|
| <b>&lt;4.1</b>           | 90  | ATIYA       | 89 B787 | +   |

————— Lepton Family number ( $LF$ ), Lepton number ( $L$ ),  $\Delta S = \Delta Q$  ( $SQ$ ) —————

————— violating modes, or  $\Delta S = 1$  weak neutral current ( $S1$ ) modes —————

$\Gamma(\pi^+ \pi^+ e^- \bar{\nu}_e) / \Gamma_{\text{total}}$   $\Gamma_{35} / \Gamma$

Test of  $\Delta S = \Delta Q$  rule.

| VALUE (units $10^{-7}$ ) | CL% | EVTS | DOCUMENT ID | TECN | CHG |
|--------------------------|-----|------|-------------|------|-----|
|--------------------------|-----|------|-------------|------|-----|

• • • We do not use the following data for averages, fits, limits, etc. • • •

$< 9.0$       95      0      SCHWEINB... 71      HLBC +

$< 6.9$       95      0      ELY      69      HLBC +

$< 20.$       95      BIRGE      65      FBC +

$\Gamma(\pi^+ \pi^+ e^- \bar{\nu}_e) / \Gamma(\pi^+ \pi^- e^+ \nu_e)$   $\Gamma_{35} / \Gamma_6$

Test of  $\Delta S = \Delta Q$  rule.

| VALUE (units $10^{-4}$ ) | CL% | EVTS | DOCUMENT ID | TECN |
|--------------------------|-----|------|-------------|------|
|--------------------------|-----|------|-------------|------|

**< 3**      90      3      <sup>1</sup> BLOCH      76      SPEC

• • • We do not use the following data for averages, fits, limits, etc. • • •

$< 130.$       95      0      BOURQUIN      71      ASPK

<sup>1</sup> BLOCH 76 quotes  $3.6 \times 10^{-4}$  at CL = 95%, we convert.

$\Gamma(\pi^+ \pi^+ \mu^- \bar{\nu}_\mu) / \Gamma_{\text{total}}$   $\Gamma_{36} / \Gamma$   
 Test of  $\Delta S = \Delta Q$  rule.

| VALUE (units $10^{-6}$ ) | CL% | EVTS | DOCUMENT ID | TECN | CHG   |
|--------------------------|-----|------|-------------|------|-------|
| <b>&lt;3.0</b>           | 95  | 0    | BIRGE       | 65   | FBC + |

$\Gamma(\pi^+ e^+ e^-) / \Gamma_{\text{total}}$   $\Gamma_{37} / \Gamma$   
 Test for  $\Delta S = 1$  weak neutral current. Allowed by combined first-order weak and electromagnetic interactions.

| VALUE (units $10^{-7}$ )       | EVTS  | DOCUMENT ID              | TECN | CHG |
|--------------------------------|-------|--------------------------|------|-----|
| <b>3.00 ± 0.09 OUR AVERAGE</b> |       |                          |      |     |
| 3.11 ± 0.04 ± 0.12             | 7253  | <sup>1</sup> BATLEY 09   | NA48 | ±   |
| 2.94 ± 0.05 ± 0.14             | 10300 | <sup>2</sup> APPEL 99    | SPEC | +   |
| 2.75 ± 0.23 ± 0.13             | 500   | <sup>3</sup> ALLIEGRO 92 | SPEC | +   |
| 2.7 ± 0.5                      | 41    | <sup>4</sup> BLOCH 75    | SPEC | +   |

- <sup>1</sup> Value extrapolated from a measurement in the region  $z = (m_{ee}/m_K)^2 > 0.08$ . BATLEY 09 also evaluated the shape of the form factor using four different theoretical models.
- <sup>2</sup> APPEL 99 establishes vector nature of this decay and determines form factor  $f(Z) = f_0 (1 + \delta Z)$ ,  $Z = M_{ee}^2/m_K^2$ ,  $\delta = 2.14 \pm 0.13 \pm 0.15$ . Also reports constraints on scalar and tensor interactions:  $|f_S| < 6.6 \times 10^{-5}$ ,  $|f_T| < 3.7 \times 10^{-4}$  at 90% C.L.
- <sup>3</sup> ALLIEGRO 92 assumes a vector interaction with a form factor given by  $\lambda = 0.105 \pm 0.035 \pm 0.015$  and a correlation coefficient of  $-0.82$ .
- <sup>4</sup> BLOCH 75 assumes a vector interaction.

$\Gamma(\pi^+ \mu^+ \mu^-) / \Gamma_{\text{total}}$   $\Gamma_{38} / \Gamma$   
 Test for  $\Delta S = 1$  weak neutral current. Allowed by higher-order electroweak interactions.

| VALUE (units $10^{-8}$ )     | CL% | EVTS | DOCUMENT ID | TECN | CHG | COMMENT   |
|------------------------------|-----|------|-------------|------|-----|---|
| <b>9.4 ± 0.6 OUR AVERAGE</b> |     |      |             |      |     | Error includes scale factor of 2.6. See the ideogram below. |

|                    |      |                         |      |   |              |
|--------------------|------|-------------------------|------|---|--------------|
| 9.62 ± 0.21 ± 0.13 | 3120 | <sup>1</sup> BATLEY 11A | NA48 | ± | 2003-04 data |
| 9.8 ± 1.0 ± 0.5    | 110  | <sup>2</sup> PARK 02    | HYCP | ± |              |
| 9.22 ± 0.60 ± 0.49 | 402  | <sup>3</sup> MA 00      | B865 | + |              |
| 5.0 ± 0.4 ± 0.9    | 207  | <sup>4</sup> ADLER 97C  | B787 | + |              |

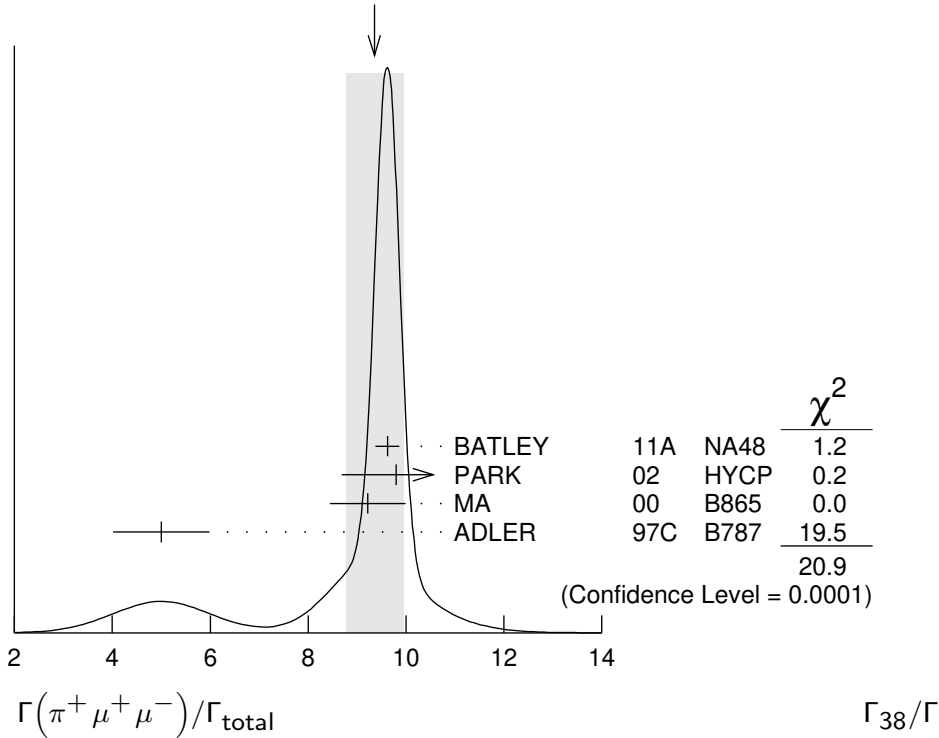
• • • We do not use the following data for averages, fits, limits, etc. • • •

|                  |    |          |      |   |  |
|------------------|----|----------|------|---|--|
| 9.7 ± 1.2 ± 0.4  | 65 | PARK 02  | HYCP | + |  |
| 10.0 ± 1.9 ± 0.7 | 35 | PARK 02  | HYCP | - |  |
| <23              | 90 | ATIYA 89 | B787 | + |  |

- <sup>1</sup> BATLEY 11A also studies the form factor  $f(z)$  dependence of the decay, described via single photon exchange: i) assuming a linear form factor,  $f(z) = f_0 (1 + \delta z)$ ,  $z = (M_{\mu\mu}/m_K)^2$ , finding  $f_0 = 0.470 \pm 0.040$  and  $\delta = 3.11 \pm 0.57$  and ii) assuming a linear form factor including  $\pi$ - $\pi$  rescattering,  $W_{\pi\pi}$ , as in DAMBROSIO 98A, finding  $f(z) = G_F m_K^2 (a_+ + b_+ z) + W_{\pi\pi}(z)$ ,  $a_+ = -0.575 \pm 0.039$ ,  $b_+ = -0.813 \pm 0.145$ .
- <sup>2</sup> PARK 02 "±" result comes from combining  $K^+ \rightarrow \pi^+ \mu^+ \mu^-$  and  $K^- \rightarrow \pi^- \mu^+ \mu^-$ , assuming  $CP$  is conserved.
- <sup>3</sup> MA 00 establishes vector nature of this decay and determines form factor  $f(z) = f_0 (1 + \delta z)$ ,  $z = (M_{\mu\mu}/m_K)^2$ ,  $\delta = 2.45^{+1.30}_{-0.95}$ .
- <sup>4</sup> ADLER 97C gives systematic error  $0.7 \times 10^{-8}$  and theoretical uncertainty  $0.6 \times 10^{-8}$ , which we combine in quadrature to obtain our second error.



WEIGHTED AVERAGE  
 $9.4 \pm 0.6$  (Error scaled by 2.6)



$\Gamma(\pi^+ \nu \bar{\nu})/\Gamma_{\text{total}}$

$\Gamma_{38}/\Gamma$

$\Gamma_{39}/\Gamma$

Test for  $\Delta S = 1$  weak neutral current. Allowed by higher-order electroweak interactions. Branching ratio values are extrapolated from the momentum or energy regions shown in the comments assuming Standard Model phase space except for those labeled "Scalar" or "Tensor" to indicate the assumed non-Standard-Model interaction.

VALUE (units  $10^{-10}$ )   CL%   EVTS   DOCUMENT ID   TECN   CHG   COMMENT

**0.8  $^{+0.6}_{-0.4}$  OUR AVERAGE**

|                         |   |  |                        |   |   |  |
|-------------------------|---|--|------------------------|---|---|--|
| 0.48 $^{+0.72}_{-0.48}$ |   |  | 1 CORTINA-GIL 20C NA62 | + |   |  |
| 1.73 $^{+1.15}_{-1.05}$ | 7 |  | 2 ARTAMONOV 08 B949    | + | 140 < $P_\pi$ < 199 MeV,<br>211 < $P_\pi$ < 229 MeV |  |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                         |    |   |                        |   |                         |
|-------------------------|----|---|------------------------|---|-------------------------|
| < 11                    | 90 | 1 | 3 CORTINA-GIL 19B NA62 | + | decay-in-flight         |
| 7.89 $^{+9.26}_{-5.10}$ |    | 3 | 4 ARTAMONOV 08 B949    | + | 140 < $P_\pi$ < 199 MeV |
| < 22                    | 90 | 1 | 5 ADLER 04 B787        | + | 211 < $P_\pi$ < 229 MeV |
| < 27                    | 90 |   | ADLER 04 B787          | + | Scalar                  |
| < 18                    | 90 |   | ADLER 04 B787          | + | Tensor                  |
| 1.47 $^{+1.30}_{-0.89}$ |    | 3 | 6 ANISIMOVSK..04 B949  | + | 211 < $P_\pi$ < 229 MeV |
| 1.57 $^{+1.75}_{-0.82}$ |    | 2 | ADLER 02 B787          | + | $P_\pi > 211$ MeV/c     |
| < 42                    | 90 | 1 | ADLER 02C B787         | + | 140 < $P_\pi$ < 195 MeV |
| < 47                    | 90 | 7 | ADLER 02C B787         | + | Scalar                  |

|       |  |                    |          |   |                      |
|-------|--|--------------------|----------|---|----------------------|
| < 25  | 90   | <sup>7</sup> ADLER | 02C B787 | + | Tensor               |
| 1.5   | $\begin{smallmatrix} +3.4 \\ -1.2 \end{smallmatrix}$ | 1 ADLER            | 00 B787  |   | In ADLER 02          |
| 4.2   | $\begin{smallmatrix} +9.7 \\ -3.5 \end{smallmatrix}$ | 1 ADLER            | 97 B787  |   |                      |
| < 24  | 90   | ADLER              | 96 B787  |   |                      |
| < 75  | 90   | ATIYA              | 93 B787  | + | $T(\pi)$ 115–127 MeV |
| < 52  | 90   | <sup>8</sup> ATIYA | 93 B787  | + |                      |
| < 170 | 90   | 0 ATIYA            | 93B B787 | + | $T(\pi)$ 60–100 MeV  |
| < 340 | 90   | ATIYA              | 90 B787  | + |                      |
| <1400 | 90   | ASANO              | 81B CNTR | + | $T(\pi)$ 116–127 MeV |

<sup>1</sup> CORTINA-GIL 20C analysis of the NA62 data taken in 2017 observed two candidate events with an expected background of 1.5 events. The achieved single event sensitivity is  $(0.389 \pm 0.024) \times 10^{-10}$ . The quoted branching ratio includes result from CORTINA-GIL 19B which observed one candidate event. The corresponding upper limit  $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 1.78 \times 10^{-10}$  at 90% C.L.

<sup>2</sup> Value obtained combining ANISIMOVSKY 04, ADLER 04, and the present ARTAMONOV 08 results.

<sup>3</sup> Based on a sample of  $1.21 \times 10^{11}$   $K^+$  decays collected in 2016. One signal candidate is observed while the expected background is 0.152 events. The single-event-sensitivity is estimated to be  $3.15 \times 10^{-10}$ .

<sup>4</sup> Observed 3 events with an estimated background of  $0.93 \pm 0.17^{+0.32}_{-0.24}$ . Signal-to-background ratio for each of these 3 events is 0.20, 0.42, and 0.47.

<sup>5</sup> Value obtained combining the previous result ADLER 02C with 1 event and the present result with 0 events to obtain an expected background  $1.22 \pm 0.24$  events and 1 event observed.

<sup>6</sup> Value obtained combining the previous E787 result ADLER 02 with 2 events and the present E949 with 1 event. The additional event has a signal-to-background ratio 0.9. Superseded by ARTAMONOV 08.

<sup>7</sup> Superseded by ADLER 04.

<sup>8</sup> Combining ATIYA 93 and ATIYA 93B results. Superseded by ADLER 96.

**$\Gamma(\pi^+ \pi^0 \nu \bar{\nu})/\Gamma_{\text{total}}$**   **$\Gamma_{40}/\Gamma$**   
 Test for  $\Delta S = 1$  weak neutral current. Allowed by higher-order electroweak interactions.

| VALUE (units $10^{-5}$ ) | CL% | DOCUMENT ID        | TECN    |
|--------------------------|-----|--------------------|---------|
| <b>&lt;4.3</b>           | 90  | <sup>1</sup> ADLER | 01 SPEC |

<sup>1</sup> Search region defined by  $90 \text{ MeV}/c < P_{\pi^+} < 188 \text{ MeV}/c$  and  $135 \text{ MeV} < E_{\pi^0} < 180 \text{ MeV}$ .

**$\Gamma(\mu^- \nu e^+ e^-)/\Gamma(\pi^+ \pi^- e^+ \nu_e)$**   **$\Gamma_{41}/\Gamma_6$**   
 Test of lepton family number conservation.

| VALUE (units $10^{-3}$ ) | CL% | EVTS | DOCUMENT ID              | TECN    | CHG |
|--------------------------|-----|------|--------------------------|---------|-----|
| <b>&lt;0.5</b>           | 90  | 0    | <sup>1</sup> DIAMANT-... | 76 SPEC | +   |

<sup>1</sup> DIAMANT-BERGER 76 quotes this result times our 1975  $\pi^+ \pi^- e \nu$  BR ratio.

**$\Gamma(\mu^+ \nu_e)/\Gamma_{\text{total}}$**   **$\Gamma_{42}/\Gamma$**   
 Forbidden by lepton family number conservation.

| VALUE            | CL% | EVTS | DOCUMENT ID        | TECN    | COMMENT                              |
|------------------|-----|------|--------------------|---------|--------------------------------------|
| <b>&lt;0.004</b> | 90  | 0    | <sup>1</sup> LYONS | 81 HLBC | 200 GeV $K^+$ narrow band $\nu$ beam |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|        |    |                     |         |                     |
|--------|----|---------------------|---------|---------------------|
| <0.012 | 90 | <sup>1</sup> COOPER | 82 HLBC | Wideband $\nu$ beam |
|--------|----|---------------------|---------|---------------------|

<sup>1</sup> COOPER 82 and LYONS 81 limits on  $\nu_e$  observation are here interpreted as limits on lepton family number violation in the absence of mixing.

$\Gamma(\pi^+ \mu^+ e^-)/\Gamma_{\text{total}}$

$\Gamma_{43}/\Gamma$

Test of lepton family number conservation.

| VALUE (units $10^{-10}$ ) | CL% | DOCUMENT ID       | TECN | CHG    |
|---------------------------|-----|-------------------|------|--------|
| <b>&lt;0.13</b>           | 90  | <sup>1</sup> SHER | 05   | RVUE + |
| <0.21                     | 90  | SHER              | 05   | B865 + |
| <0.39                     | 90  | APPEL             | 00   | B865 + |
| <2.1                      | 90  | LEE               | 90   | SPEC + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup>This result combines SHER 05 1998 data, APPEL 00 1996 data, and data from BERGMAN 97 and PISLAK 97 theses, all from BNL-E865, with LEE 90 BNL-E777 data.

$\Gamma(\pi^+ \mu^- e^+)/\Gamma_{\text{total}}$

$\Gamma_{44}/\Gamma$

Test of lepton family number conservation.

| VALUE (units $10^{-10}$ ) | CL% | EVTS | DOCUMENT ID              | TECN | CHG    |
|---------------------------|-----|------|--------------------------|------|--------|
| <b>&lt; 5.2</b>           | 90  | 0    | APPEL                    | 00B  | B865 + |
| <70                       | 90  | 0    | <sup>1</sup> DIAMANT-... | 76   | SPEC + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup>Measurement actually applies to the sum of the  $\pi^+ \mu^- e^+$  and  $\pi^- \mu^+ e^+$  modes.

$\Gamma(\pi^- \mu^+ e^+)/\Gamma_{\text{total}}$

$\Gamma_{45}/\Gamma$

Test of total lepton number conservation.

| VALUE (units $10^{-10}$ ) | CL% | EVTS | DOCUMENT ID              | TECN | CHG    |
|---------------------------|-----|------|--------------------------|------|--------|
| <b>&lt; 5.0</b>           | 90  | 0    | APPEL                    | 00B  | B865 + |
| <70                       | 90  | 0    | <sup>1</sup> DIAMANT-... | 76   | SPEC + |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup>Measurement actually applies to the sum of the  $\pi^+ \mu^- e^+$  and  $\pi^- \mu^+ e^+$  modes.

$\Gamma(\pi^- e^+ e^+)/\Gamma_{\text{total}}$

$\Gamma_{46}/\Gamma$

Test of total lepton number conservation.

| VALUE                             | CL% | DOCUMENT ID                  | TECN | CHG  | COMMENT         |
|-----------------------------------|-----|------------------------------|------|------|-----------------|
| <b>&lt;2.2 × 10<sup>-10</sup></b> | 90  | <sup>1</sup> CORTINA-GIL 19A | NA62 | +    | decay-in-flight |
| <6.4 × 10 <sup>-10</sup>          | 90  | APPEL                        | 00B  | B865 | +               |
| <9.2 × 10 <sup>-9</sup>           | 90  | DIAMANT-...                  | 76   | SPEC | +               |
| <1.5 × 10 <sup>-5</sup>           |     | CHANG                        | 68   | HBC  | -               |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup>CORTINA-GIL 19A results are obtained with 2017 data.

$\Gamma(\pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$

$\Gamma_{47}/\Gamma$

Forbidden by total lepton number conservation.

| VALUE                             | CL% | DOCUMENT ID                  | TECN | CHG  | COMMENT         |
|-----------------------------------|-----|------------------------------|------|------|-----------------|
| <b>&lt;4.2 × 10<sup>-11</sup></b> | 90  | <sup>1</sup> CORTINA-GIL 19A | NA62 | +    | decay-in-flight |
| <8.6 × 10 <sup>-11</sup>          | 90  | <sup>2</sup> BATLEY          | 17   | NA48 | ±               |
| <1.1 × 10 <sup>-9</sup>           | 90  | BATLEY                       | 11A  | NA48 | ±               |
| <3.0 × 10 <sup>-9</sup>           | 90  | APPEL                        | 00B  | B865 | +               |
| <1.5 × 10 <sup>-4</sup>           | 90  | <sup>3</sup> LITTENBERG      | 92   | HBC  |                 |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup>CORTINA-GIL 19A results are obtained with 2017 data.

<sup>2</sup>BATLEY 17 result is based on data taken in 2003 to 2004. Limits for two-body resonance

$X$  in  $K^\pm \rightarrow \pi \mu \mu$  decays are also reported.

<sup>3</sup>LITTENBERG 92 is from retroactive data analysis of CHANG 68 bubble chamber data.

$\Gamma(\mu^+\bar{\nu}_e)/\Gamma_{\text{total}}$   $\Gamma_{48}/\Gamma$

Forbidden by total lepton number conservation.

| VALUE (units $10^{-3}$ ) | CL% | DOCUMENT ID         | TECN | COMMENT                  |
|--------------------------|-----|---------------------|------|--------------------------|
| <b>&lt;3.3</b>           | 90  | <sup>1</sup> COOPER | 82   | HLBC Wideband $\nu$ beam |

<sup>1</sup> COOPER 82 limit on  $\bar{\nu}_e$  observation is here interpreted as a limit on lepton number violation in the absence of mixing.

$\Gamma(\pi^0 e^+\bar{\nu}_e)/\Gamma_{\text{total}}$   $\Gamma_{49}/\Gamma$

Forbidden by total lepton number conservation.

| VALUE            | CL% | DOCUMENT ID         | TECN | COMMENT                  |
|------------------|-----|---------------------|------|--------------------------|
| <b>&lt;0.003</b> | 90  | <sup>1</sup> COOPER | 82   | HLBC Wideband $\nu$ beam |

<sup>1</sup> COOPER 82 limit on  $\bar{\nu}_e$  observation is here interpreted as a limit on lepton number violation in the absence of mixing.

$\Gamma(\pi^+\gamma)/\Gamma_{\text{total}}$   $\Gamma_{50}/\Gamma$

Violates angular momentum conservation and gauge invariance. Current interest in this decay is as a search for non-commutative space-time effects as discussed in ARTAMONOV 05 and for exotic physics such as a vacuum expectation value of a new vector field, non-local Superstring effects, or departures from Lorentz invariance, as discussed in ADLER 02B.

| VALUE (units $10^{-9}$ ) | CL% | DOCUMENT ID  | TECN | CHG |
|--------------------------|-----|--------------|------|-----|
| <b>&lt; 2.3</b>          | 90  | ARTAMONOV 05 | B949 | +   |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|       |    |                    |     |      |   |
|-------|----|--------------------|-----|------|---|
| < 360 | 90 | ADLER              | 02B | B787 | + |
| <1400 | 90 | ASANO              | 82  | CNTR | + |
| <4000 | 90 | <sup>1</sup> KLEMS | 71  | OSPK | + |

<sup>1</sup> Test of model of Selleri, Nuovo Cimento **60A** 291 (1969).

### CPT VIOLATION TESTS IN $K^\pm$ DECAYS

$$\Delta = (\Gamma(K^+) - \Gamma(K^-)) / (\Gamma(K^+) + \Gamma(K^-))$$

$\Delta(K^\pm \rightarrow \mu^\pm \nu_\mu)$  RATE DIFFERENCE/SUM

| VALUE (%)                          | DOCUMENT ID | TECN    |
|------------------------------------|-------------|---------|
| <b><math>-0.27 \pm 0.21</math></b> | FORD        | 67 CNTR |

$\Delta(K^\pm \rightarrow \pi^\pm \pi^0)$  RATE DIFFERENCE/SUM

| VALUE (%)                       | DOCUMENT ID | TECN    |
|---------------------------------|-------------|---------|
| <b><math>0.4 \pm 0.6</math></b> | HERZO       | 69 OSPK |

### CP VIOLATION TESTS IN $K^\pm$ DECAYS

$$\Delta = (\Gamma(K^+) - \Gamma(K^-)) / (\Gamma(K^+) + \Gamma(K^-))$$

$\Delta(K^\pm \rightarrow \pi^\pm e^+ e^-)$  RATE DIFFERENCE/SUM

| VALUE (units $10^{-2}$ )                 | DOCUMENT ID         | TECN    |
|--|---------------------|---------|
| <b><math>-2.2 \pm 1.5 \pm 0.6</math></b> | <sup>1</sup> BATLEY | 09 NA48 |

<sup>1</sup> This implies an upper limit of  $2.1 \times 10^{-2}$  at 90% CL.

**$\Delta(K^\pm \rightarrow \pi^\pm \mu^+ \mu^-)$  RATE DIFFERENCE/SUM**

| <u>VALUE</u>                   | <u>DOCUMENT ID</u>  | <u>TECN</u> |
|--------------------------------|---------------------|-------------|
| <b>0.010±0.023 OUR AVERAGE</b> |                     |             |
| 0.011±0.023                    | <sup>1</sup> BATLEY | 11A NA48    |
| -0.02 ±0.11 ±0.04              | PARK                | 02 HYCP     |

<sup>1</sup>This corresponds to the asymmetry upper limit of  $< 2.9 \times 10^{-2}$  at 90% CL.

 **$\Delta(K^\pm \rightarrow \pi^\pm \pi^0 \gamma)$  RATE DIFFERENCE/SUM**

| <u>VALUE (units <math>10^{-3}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u>       |
|---|-------------|---------------------|-------------|------------|----------------------|
| <b>0.0± 1.2 OUR AVERAGE</b>               |             |                     |             |            |                      |
| 0.0± 1.0±0.6                              | 1M          | <sup>1</sup> BATLEY | 10A         | NA48       |                      |
| 4 ±29                                     | 2461        | SMITH               | 76          | WIRE       | ± $E_\pi$ 55–90 MeV  |
| 5 ±20                                     | 4000        | ABRAMS              | 73B         | ASPK       | ± $E_\pi$ 51–100 MeV |

<sup>1</sup>This value implies the upper bound for this asymmetry  $1.5 \times 10^{-3}$  at 90% CL.

 **$\Delta(K^\pm \rightarrow \pi^\pm \pi^+ \pi^-)$  RATE DIFFERENCE/SUM**

| <u>VALUE (%)</u>  | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
| <b>0.04±0.06</b>  |             | <sup>1</sup> FORD  | 70          | ASPK       |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |             |                    |             |            |
| -0.01±0.08  |             | <sup>2</sup> SMITH | 73          | ASPK ±     |
| 0.05±0.07   | 3.2M        | <sup>1</sup> FORD  | 70          | ASPK       |
| -0.25±0.45  |             | FLETCHER           | 67          | OSPK       |
| -0.02±0.11  |             | <sup>1</sup> FORD  | 67          | CNTR       |

<sup>1</sup>First FORD 70 value is second FORD 70 combined with FORD 67.

<sup>2</sup>SMITH 73 value of  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  rate difference is derived from SMITH 73 value of  $K^\pm \rightarrow \pi^\pm 2\pi^0$  rate difference.

 **$\Delta(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0)$  RATE DIFFERENCE/SUM**

| <u>VALUE (%)</u>              | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|-------------------------------|-------------|--------------------|-------------|------------|
| <b>-0.02±0.28 OUR AVERAGE</b> |             |                    |             |            |
| 0.04±0.29                     |             | SMITH              | 73          | ASPK ±     |
| -0.6 ±0.9                     | 1802        | HERZO              | 69          | OSPK       |

**T VIOLATION TESTS IN  $K^+$  AND  $K^-$  DECAYS** **$P_T$  in  $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$** 

T-violating muon polarization. Sensitive to new sources of  $CP$  violation beyond the Standard Model.

| <u>VALUE (units <math>10^{-3}</math>)</u>                                     | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
| <b>-1.7±2.3±1.1</b>   |             | <sup>1</sup> ABE   | 04F         | K246 +     |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |             |                    |             |            |
| -4.2±4.9±0.9  | 3.9M        | ABE                | 99S         | K246 +     |

<sup>1</sup>Includes three sets of data: 96-97 (ABE 99S), 98, and 99-00 totaling about three times the ABE 99S data sample. Corresponds to  $P_T < 5.0 \times 10^{-3}$  at 90% CL.

 **$P_T$  in  $K^+ \rightarrow \mu^+ \nu_\mu \gamma$** 

T-violating muon polarization. Sensitive to new sources of  $CP$  violation beyond the Standard Model.

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>          | <u>TECN</u> | <u>CHG</u> |
|---|-------------|-----------------------------|-------------|------------|
| <b>-0.64±1.85±0.10</b>                    | 114k        | <sup>1</sup> ANISIMOVSK..03 | K246        | +          |

<sup>1</sup>Muons stopped and polarization measured from decay to positrons.

### Im( $\xi$ ) in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ DECAY (from transverse $\mu$ pol.)

Test of  $T$  reversal invariance.

| VALUE   | EVT5 | DOCUMENT ID      | TECN     | CHG | COMMENT           |
|---|------|------------------|----------|-----|-------------------|
| <b>-0.006 ± 0.008 OUR AVERAGE</b>   |      |                  |          |     |                   |
| -0.0053 ± 0.0071 ± 0.0036   |      | <sup>1</sup> ABE | 04F K246 | +   |                   |
| -0.016 ± 0.025  | 20M  | CAMPBELL         | 81 CNTR  | +   | Pol.              |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |      |                  |          |     |                   |
| -0.013 ± 0.016 ± 0.003  | 3.9M | ABE              | 99S CNTR | +   | $p_T K^+$ at rest |

<sup>1</sup> Includes three sets of data: 96-97 (ABE 99S), 98, and 99-00 totaling about three times the ABE 99S data sample. Corresponds to Im( $\xi$ ) < 0.016 at 90% CL.

## DALITZ PLOT PARAMETERS FOR $K \rightarrow 3\pi$ DECAYS

Revised 1999 by T.G. Trippe (LBNL).

The Dalitz plot distribution for  $K^\pm \rightarrow \pi^\pm \pi^\pm \pi^\mp$ ,  $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ , and  $K_L^0 \rightarrow \pi^+ \pi^- \pi^0$  can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$\begin{aligned}
 |M|^2 \propto & 1 + g \frac{(s_3 - s_0)}{m_{\pi^+}^2} + h \left[ \frac{s_3 - s_0}{m_{\pi^+}^2} \right]^2 \\
 & + j \frac{(s_2 - s_1)}{m_{\pi^+}^2} + k \left[ \frac{s_2 - s_1}{m_{\pi^+}^2} \right]^2 \\
 & + f \frac{(s_2 - s_1)(s_3 - s_0)}{m_{\pi^+}^2} + \dots, \tag{1}
 \end{aligned}$$

where  $m_{\pi^+}^2$  has been introduced to make the coefficients  $g$ ,  $h$ ,  $j$ , and  $k$  dimensionless, and

$$\begin{aligned}
 s_i &= (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_K T_i, \quad i = 1, 2, 3, \\
 s_0 &= \frac{1}{3} \sum_i s_i = \frac{1}{3} (m_K^2 + m_1^2 + m_2^2 + m_3^2).
 \end{aligned}$$

Here the  $P_i$  are four-vectors,  $m_i$  and  $T_i$  are the mass and kinetic energy of the  $i^{th}$  pion, and the index 3 is used for the odd pion.

The coefficient  $g$  is a measure of the slope in the variable  $s_3$  (or  $T_3$ ) of the Dalitz plot, while  $h$  and  $k$  measure the quadratic dependence on  $s_3$  and  $(s_2 - s_1)$ , respectively. The coefficient  $j$  is related to the asymmetry of the plot and must be zero if  $CP$  invariance holds. Note also that if  $CP$  is good,  $g$ ,  $h$ , and  $k$  must be the same for  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  as for  $K^- \rightarrow \pi^- \pi^- \pi^+$ .

Since different experiments use different forms for  $|M|^2$ , in order to compare the experiments we have converted to  $g$ ,  $h$ ,  $j$ , and  $k$  whatever coefficients have been measured. Where such conversions have been done, the measured coefficient  $a_y$ ,  $a_t$ ,  $a_u$ , or  $a_v$  is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

## References

1. S. Weinberg, Phys. Rev. Lett. **4**, 87 (1960).
2. Particle Data Group, Phys. Lett. **111B**, 69 (1982).

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### ENERGY DEPENDENCE OF $K^\pm$ DALITZ PLOT

$$|\text{matrix element}|^2 = 1 + gu + hu^2 + kv^2$$

where  $u = (s_3 - s_0) / m_\pi^2$  and  $v = (s_2 - s_1) / m_\pi^2$

### LINEAR COEFFICIENT $g$ FOR $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

Some experiments use Dalitz variables  $x$  and  $y$ . In the comments we give  $a_y =$  coefficient of  $y$  term. See note above on "Dalitz Plot Parameters for  $K \rightarrow 3\pi$  Decays." For discussion of the conversion of  $a_y$  to  $g$ , see the earlier version of the same note in the *Review* published in Physics Letters **111B** 70 (1982).

| <u>VALUE</u>  | <u>EVTS</u> | <u>DOCUMENT ID</u>      | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u>            |
|---|-------------|-------------------------|-------------|------------|---------------------------|
| <b>-0.21134 ± 0.00017</b>   | 471M        | <sup>1</sup> BATLEY     | 07B         | NA48       | ±                         |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |             |                         |             |            |                           |
| -0.2221 ± 0.0065  | 225k        | DEVAUX                  | 77          | SPEC       | + $a_y = .2814 \pm .0082$ |
| -0.199 ± 0.008  | 81k         | <sup>2</sup> LUCAS      | 73          | HBC        | - $a_y = 0.252 \pm 0.011$ |
| -0.2157 ± 0.0028  | 750k        | FORD                    | 72          | ASPK       | + $a_y = .2734 \pm .0035$ |
| -0.2186 ± 0.0028  | 750k        | FORD                    | 72          | ASPK       | - $a_y = .2770 \pm .0035$ |
| -0.200 ± 0.009  | 39819       | <sup>3</sup> HOFFMASTER | 72          | HLBC       | +                         |
| -0.196 ± 0.012  | 17898       | <sup>4</sup> GRAUMAN    | 70          | HLBC       | + $a_y = 0.228 \pm 0.030$ |
| -0.193 ± 0.010  | 50919       | MAST                    | 69          | HBC        | - $a_y = 0.244 \pm 0.013$ |
| -0.218 ± 0.016  | 9994        | <sup>5</sup> BUTLER     | 68          | HBC        | + $a_y = 0.277 \pm 0.020$ |

|               |      |                          |    |     |   |                       |
|---------------|------|--------------------------|----|-----|---|-----------------------|
| -0.190 ±0.023 | 5778 | <sup>5,6</sup> MOSCOSO   | 68 | HBC | - | $a_y=0.242 \pm 0.029$ |
| -0.22 ±0.024  | 5428 | <sup>5,6</sup> ZINCHENKO | 67 | HBC | + | $a_y=0.28 \pm 0.03$   |
| -0.220 ±0.035 | 1347 | <sup>7</sup> FERRO-LUZZI | 61 | HBC | - | $a_y=0.28 \pm 0.045$  |

<sup>1</sup> Final state strong interaction and radiative corrections not included in the fit.

<sup>2</sup> Quadratic dependence is required by  $K_L^0$  experiments.

<sup>3</sup> HOFFMASTER 72 includes GRAUMAN 70 data.

<sup>4</sup> Emulsion data added — all events included by HOFFMASTER 72.

<sup>5</sup> Experiments with large errors not included in average.

<sup>6</sup> Also includes DBC events.

<sup>7</sup> No radiative corrections included.

### QUADRATIC COEFFICIENT $h$ FOR $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

| VALUE (units $10^{-2}$ ) | EVTS | DOCUMENT ID         | TECN     | CHG |
|--------------------------|------|---------------------|----------|-----|
| <b>1.848 ± 0.040</b>     | 471M | <sup>1</sup> BATLEY | 07B NA48 | ±   |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |       |              |      |      |   |
|-------------|-------|--------------|------|------|---|
| -0.06 ±1.43 | 225k  | DEVAUX       | 77   | SPEC | + |
| 1.87 ±0.62  | 750k  | FORD         | 72   | ASPK | + |
| 1.25 ±0.62  | 750k  | FORD         | 72   | ASPK | - |
| -0.9 ±1.4   | 39819 | HOFFMASTER72 | HLBC | +    |   |
| -0.1 ±1.2   | 50919 | MAST         | 69   | HBC  | - |

<sup>1</sup> Final state strong interaction and radiative corrections not included in the fit.

### QUADRATIC COEFFICIENT $k$ FOR $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

| VALUE (units $10^{-3}$ ) | EVTS | DOCUMENT ID         | TECN     | CHG |
|--------------------------|------|---------------------|----------|-----|
| <b>- 4.63 ± 0.14</b>     | 471M | <sup>1</sup> BATLEY | 07B NA48 | ±   |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |       |              |      |      |   |
|-------------|-------|--------------|------|------|---|
| -20.5 ± 3.9 | 225k  | DEVAUX       | 77   | SPEC | + |
| - 7.5 ± 1.9 | 750k  | FORD         | 72   | ASPK | + |
| - 8.3 ± 1.9 | 750k  | FORD         | 72   | ASPK | - |
| -10.5 ± 4.5 | 39819 | HOFFMASTER72 | HLBC | +    |   |
| -14 ±12     | 50919 | MAST         | 69   | HBC  | - |

<sup>1</sup> Final state strong interaction and radiative corrections not included in the fit.

### $(g_+ - g_-) / (g_+ + g_-)$ FOR $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$

This is a  $CP$  violating asymmetry between linear coefficients  $g_+$  for  $K^+ \rightarrow \pi^+ \pi^+ \pi^-$  decay and  $g_-$  for  $K^- \rightarrow \pi^- \pi^+ \pi^-$  decay.

| VALUE (units $10^{-4}$ ) | EVTS | DOCUMENT ID         | TECN     |
|--------------------------|------|---------------------|----------|
| <b>- 1.5 ± 1.5 ± 1.6</b> | 3.1G | <sup>1</sup> BATLEY | 07E NA48 |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                 |      |                     |         |
|-----------------|------|---------------------|---------|
| 1.7 ± 2.1 ± 2.0 | 1.7G | <sup>2</sup> BATLEY | 06 NA48 |
| -70.0 ± 53      | 3.2M | FORD                | 70 ASPK |

<sup>1</sup> BATLEY 07E includes data from BATLEY 06. Uses quadratic parametrization and value  $g_+ + g_- = 2g$  from BATLEY 07B. This measurement neglects any possible charge asymmetries in higher order slope parameters  $h$  or  $k$ .

<sup>2</sup> This measurement neglects any possible charge asymmetries in higher order slope parameters  $h$  or  $k$ .



**LINEAR COEFFICIENT  $g$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

Unless otherwise stated, all experiments include terms quadratic in  $(s_3 - s_0) / m_{\pi^+}^2$ . See note above on "Dalitz Plot Parameters for  $K \rightarrow 3\pi$  Decays."

See BATUSOV 98 for a discussion of the discrepancy between their result and others, especially BOLOTOV 86. At this time we have no way to resolve the discrepancy so we depend on the large scale factor as a warning.

| <u>VALUE</u>  | <u>EVTS</u> | <u>DOCUMENT ID</u>      | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|---|-------------|-------------------------|-------------|------------|----------------|
| <b>0.626 ± 0.007 OUR AVERAGE</b>  |             |                         |             |            |                |
| 0.6259 ± 0.0043 ± 0.0093  | 493k        | AKOPDZHAN..05B          | TNF         | ±          |                |
| 0.627 ± 0.004 ± 0.010   | 252k        | <sup>1,2</sup> AJINENKO | 03B         | ISTR       | –              |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |             |                         |             |            |                |
| 0.736 ± 0.014 ± 0.012   | 33k         | BATUSOV                 | 98          | SPEC       | +              |
| 0.582 ± 0.021   | 43k         | BOLOTOV                 | 86          | CALO       | –              |
| 0.670 ± 0.054   | 3263        | BRAUN                   | 76B         | HLBC       | +              |
| 0.630 ± 0.038   | 5635        | SHEAFF                  | 75          | HLBC       | +              |
| 0.510 ± 0.060   | 27k         | SMITH                   | 75          | WIRE       | +              |
| 0.67 ± 0.06   | 1365        | AUBERT                  | 72          | HLBC       | +              |
| 0.544 ± 0.048   | 4048        | DAVISON                 | 69          | HLBC       | +              |

<sup>1</sup> Measured using in-flight decays of the 25 GeV negative secondary beam.

<sup>2</sup> They form new world averages  $g_- = (0.617 \pm 0.018)$  and  $g_+ = (0.684 \pm 0.033)$  which give  $\Delta g_{\tau'} = 0.051 \pm 0.028$ .

**QUADRATIC COEFFICIENT  $h$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

| <u>VALUE</u>  | <u>EVTS</u> | <u>DOCUMENT ID</u>    | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|---|-------------|-----------------------|-------------|------------|----------------|
| <b>0.052 ± 0.008 OUR AVERAGE</b>  |             |                       |             |            |                |
| 0.0551 ± 0.0044 ± 0.0086  | 493k        | AKOPDZHAN..05B        | TNF         | ±          |                |
| 0.046 ± 0.004 ± 0.012   | 252k        | <sup>1</sup> AJINENKO | 03B         | ISTR       | –              |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |             |                       |             |            |                |
| 0.128 ± 0.015 ± 0.024   | 33k         | BATUSOV               | 98          | SPEC       | +              |
| 0.037 ± 0.024   | 43k         | BOLOTOV               | 86          | CALO       | –              |
| 0.152 ± 0.082   | 3263        | BRAUN                 | 76B         | HLBC       | +              |
| 0.041 ± 0.030   | 5635        | SHEAFF                | 75          | HLBC       | +              |
| 0.009 ± 0.040   | 27k         | SMITH                 | 75          | WIRE       | +              |
| –0.01 ± 0.08  | 1365        | AUBERT                | 72          | HLBC       | +              |
| 0.026 ± 0.050   | 4048        | DAVISON               | 69          | HLBC       | +              |

<sup>1</sup> Measured using in-flight decays of the 25 GeV negative secondary beam.

**QUADRATIC COEFFICIENT  $k$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

| <u>VALUE</u>  | <u>EVTS</u> | <u>DOCUMENT ID</u>    | <u>TECN</u> | <u>CHG</u> |                                     |
|---|-------------|-----------------------|-------------|------------|-------------------------------------|
| <b>0.0054 ± 0.0035 OUR AVERAGE</b>  |             |                       |             |            | Error includes scale factor of 2.5. |
| 0.0082 ± 0.0011 ± 0.0014  | 493k        | AKOPDZHAN..05B        | TNF         | ±          |                                     |
| 0.001 ± 0.001 ± 0.002   | 252k        | <sup>1</sup> AJINENKO | 03B         | ISTR       | –                                   |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |             |                       |             |            |                                     |
| 0.0197 ± 0.0045 ± 0.0029  | 33k         | BATUSOV               | 98          | SPEC       | +                                   |

<sup>1</sup> Measured using in-flight decays of the 25 GeV negative secondary beam.

### $(g_+ - g_-) / (g_+ + g_-)$ FOR $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

A nonzero value for this quantity indicates *CP* violation.

| VALUE (units $10^{-4}$ ) | EVTS | DOCUMENT ID | TECN |
|--------------------------|------|-------------|------|
|--------------------------|------|-------------|------|

#### 1.8± 1.8 OUR AVERAGE

|              |       |                         |      |
|--------------|-------|-------------------------|------|
| 1.8± 1.7±0.6 | 91.3M | <sup>1</sup> BATLEY 07E | NA48 |
|--------------|-------|-------------------------|------|

|          |      |                            |     |
|----------|------|----------------------------|-----|
| 2 ±18 ±5 | 619k | <sup>2</sup> AKOPDZHAN..05 | TNF |
|----------|------|----------------------------|-----|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|              |     |                         |      |
|--------------|-----|-------------------------|------|
| 1.8± 2.2±1.3 | 47M | <sup>3</sup> BATLEY 06A | NA48 |
|--------------|-----|-------------------------|------|

<sup>1</sup> BATLEY 07E includes data from BATLEY 06A. Uses quadratic parametrization and PDG 06 value  $g = 0.626 \pm 0.007$  to obtain  $g_+ - g_- = (2.2 \pm 2.1 \pm 0.7) \times 10^{-4}$ . Neglects any possible charge asymmetries in higher order slope parameters  $h$  or  $k$ .

<sup>2</sup> Asymmetry obtained assuming that  $g_+ + g_- = 2 \times 0.652$  (PDG 02) and that asymmetries in  $h$  and  $k$  are zero.

<sup>3</sup> Linear and quadratic slopes from PDG 04 are used. Any possible charge asymmetries in higher order slope parameters  $h$  or  $k$  are neglected.

### ALTERNATIVE PARAMETRIZATIONS OF $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ DALITZ PLOT

The following functional form for the matrix element suggested by  $\pi\pi$  rescattering in  $K^+ \rightarrow \pi^+ \pi^+ \pi^- \rightarrow \pi^+ \pi^0 \pi^0$  is used for this fit (CABIBBO 04A, CABIBBO 05): Matrix element =  $M_0 + M_1$  where  $M_0 = 1 + (1/2)g_0 u + (1/2)h' u^2 + (1/2)k_0 v^2$  with  $u = (s_3 - s_0)/(m_{\pi^+})^2$ ,  $v = (s_2 - s_1)/(m_{\pi^+})^2$  and where  $M_1$  takes into account the non-analytic piece due to  $\pi\pi$  rescattering amplitudes  $a_0$  and  $a_2$ ; The parameters  $g_0$  and  $h'$  are related to the parameters  $g$  and  $h$  of the matrix element squared given in the previous section by the approximations  $g_0 \sim g^{PDG}$  and  $h' \sim h^{PDG} - (g/2)^2$  and  $k_0 \sim k^{PDG}$ .

In addition, we also consider the effective field theory framework of COLANGELO 06A and BISSEGGER 09 to extract  $g_{BB}$  and  $h'_{BB}$ .

### LINEAR COEFFICIENT $g_0$ FOR $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

| VALUE | EVTS | DOCUMENT ID | TECN | CHG |
|-------|------|-------------|------|-----|
|-------|------|-------------|------|-----|

|                             |     |                         |      |   |
|-----------------------------|-----|-------------------------|------|---|
| <b>0.6525±0.0009±0.0033</b> | 60M | <sup>1</sup> BATLEY 09A | NA48 | ± |
|-----------------------------|-----|-------------------------|------|---|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                     |     |                         |      |   |
|---------------------|-----|-------------------------|------|---|
| 0.645 ±0.004 ±0.009 | 23M | <sup>2</sup> BATLEY 06B | NA48 | ± |
|---------------------|-----|-------------------------|------|---|

<sup>1</sup> This fit is obtained with the CABIBBO 05 matrix element in the  $2\pi^0$  invariant mass squared range  $0.074094 < m_{2\pi^0}^2 < 0.104244 \text{ GeV}^2$ . Electromagnetic corrections and CHPT constraints for  $\pi\pi$  phase shifts ( $a_0$  and  $a_2$ ) have been used. Also measured  $(a_0 - a_2) m_{\pi^+} = 0.2646 \pm 0.0021 \pm 0.0023$ , where  $k_0$  was kept fixed in the fit at  $-0.0099$ .

<sup>2</sup> Superseded by BATLEY 09A. This fit is obtained with the CABIBBO 05 matrix element in the  $2\pi^0$  invariant mass squared range  $0.074 \text{ GeV}^2 < m_{2\pi^0}^2 < 0.097 \text{ GeV}^2$ , assuming  $k = 0$  (no term proportional to  $(s_2 - s_1)^2$ ) and excluding the kinematic region around the cusp ( $m_{2\pi^0}^2 = (2m_{\pi^+})^2 \pm 0.000525 \text{ GeV}^2$ ). Also  $\pi\pi$  phase shifts  $a_0$  and  $a_2$  are measured:  $(a_0 - a_2) m_{\pi^+} = 0.268 \pm 0.010 \pm 0.004 \pm 0.013(\text{external})$  and  $a_2 m_{\pi^+} = -0.041 \pm 0.022 \pm 0.014$ .

**QUADRATIC COEFFICIENT  $h'$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

| <u>VALUE</u>                                      | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> |
|---|-------------|---------------------|-------------|------------|
| <b><math>-0.0433 \pm 0.0008 \pm 0.0026</math></b> | 60M         | <sup>1</sup> BATLEY | 09A NA48    | $\pm$      |
| $-0.047 \pm 0.012 \pm 0.011$                      | 23M         | <sup>2</sup> BATLEY | 06B NA48    | $\pm$      |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup> This fit is obtained with the CABIBBO 05 matrix element in the  $2\pi^0$  invariant mass squared range  $0.074094 < m_{2\pi^0}^2 < 0.104244 \text{ GeV}^2$ . Electromagnetic corrections and CHPT constraints for  $\pi\pi$  phase shifts ( $a_0$  and  $a_2$ ) have been used. Also measured  $(a_0 - a_2) m_{\pi^+} = 0.2646 \pm 0.0021 \pm 0.0023$ , where  $k_0$  was kept fixed in the fit at  $-0.0099$ .

<sup>2</sup> Superseded by BATLEY 09A. This fit is obtained with the CABIBBO 05 matrix element in the  $2\pi^0$  invariant mass squared range  $0.074 \text{ GeV}^2 < m_{2\pi^0}^2 < 0.097 \text{ GeV}^2$ , assuming  $k = 0$  (no term proportional to  $(s_2 - s_1)^2$ ) and excluding the kinematic region around the cusp ( $m_{2\pi^0}^2 = (2m_{\pi^+})^2 \pm 0.000525 \text{ GeV}^2$ ). Also  $\pi\text{-}\pi$  phase shifts  $a_0$  and  $a_2$  are measured:  $(a_0 - a_2) m_{\pi^+} = 0.268 \pm 0.010 \pm 0.004 \pm 0.013(\text{external})$  and  $a_2 m_{\pi^+} = -0.041 \pm 0.022 \pm 0.014$ .

**QUADRATIC COEFFICIENT  $k_0$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

| <u>VALUE</u>                                       | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> |
|--|-------------|---------------------|-------------|------------|
| <b><math>0.0095 \pm 0.00017 \pm 0.00048</math></b> | 60M         | <sup>1</sup> BATLEY | 09A NA48    | $\pm$      |

<sup>1</sup> Assumed  $a_2 m_{\pi^+} = -0.0044$  in the fit.

**LINEAR COEFFICIENT  $g_{BB}$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

| <u>VALUE</u>                                     | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> |
|--|-------------|---------------------|-------------|------------|
| <b><math>0.6219 \pm 0.0009 \pm 0.0033</math></b> | 60M         | <sup>1</sup> BATLEY | 09A NA48    | $\pm$      |

<sup>1</sup> This fit is obtained using parametrizations of COLANGELO 06A and BISSEGGER 09 in the  $2\pi^0$  invariant mass squared range  $0.074094 < m_{2\pi^0}^2 < 0.104244 \text{ GeV}^2$ . Electromagnetic corrections and CHPT constraints for  $\pi\pi$  phase shifts ( $a_0$  and  $a_2$ ) have been used. Also measured  $(a_0 - a_2) m_{\pi^+} = 0.2633 \pm 0.0024 \pm 0.0024$ , where  $k_0$  was kept fixed in the fit at 0.0085.

**QUADRATIC COEFFICIENT  $h'_{BB}$  FOR  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$** 

| <u>VALUE</u>                                      | <u>EVTS</u> | <u>DOCUMENT ID</u>  | <u>TECN</u> | <u>CHG</u> |
|---|-------------|---------------------|-------------|------------|
| <b><math>-0.0520 \pm 0.0009 \pm 0.0026</math></b> | 60M         | <sup>1</sup> BATLEY | 09A NA48    | $\pm$      |

<sup>1</sup> This fit is obtained using parametrizations of COLANGELO 06A and BISSEGGER 09 in the  $2\pi^0$  invariant mass squared range  $0.074094 < m_{2\pi^0}^2 < 0.104244 \text{ GeV}^2$ . Electromagnetic corrections and CHPT constraints for  $\pi\pi$  phase shifts ( $a_0$  and  $a_2$ ) have been used. Also measured  $(a_0 - a_2) m_{\pi^+} = 0.2633 \pm 0.0024 \pm 0.0024$ , where  $k_0$  was kept fixed in the fit at 0.0085.

 **$K_{\ell 3}^\pm$  FORM FACTORS**

In the form factor comments, the following symbols are used.

$f_+$  and  $f_-$  are form factors for the vector matrix element.

$f_S$  and  $f_T$  refer to the scalar and tensor term.

$f_0 = f_+ + f_- t / (m_{K^+}^2 - m_{\pi^0}^2)$ .

$t$  = momentum transfer to the  $\pi$ .

$\lambda_+$  and  $\lambda_0$  are the linear expansion coefficients of  $f_+$  and  $f_0$ :

$$f_+(t) = f_+(0) (1 + \lambda_+ t / m_{\pi^+}^2)$$

For quadratic expansion

$$f_+(t) = f_+(0) (1 + \lambda'_+ t / m_{\pi^+}^2 + \frac{\lambda''_+}{2} t^2 / m_{\pi^+}^4)$$

as used by KTeV. If there is a non-vanishing quadratic term, then  $\lambda_+$  represents an average slope, which is then different from  $\lambda'_+$ .

NA48/2 and OKA quadratic expansion coefficients are converted with

$$\lambda'_+{}^{PDG} = \lambda'_+{}^{NA48/2} \text{ and } \lambda''_+{}^{PDG} = 2 \lambda''_+{}^{NA48/2}$$

$$\lambda'_+{}^{PDG} = \left(\frac{m_{\pi^+}}{m_{\pi^0}}\right)^2 \lambda'_+{}^{OKA} \text{ and}$$

$$\lambda''_+{}^{PDG} = 2 \left(\frac{m_{\pi^+}}{m_{\pi^0}}\right)^4 \lambda''_+{}^{OKA}$$

OKA linear expansion coefficients are converted with

$$\lambda_+{}^{PDG} = \left(\frac{m_{\pi^+}}{m_{\pi^0}}\right)^2 \lambda_+{}^{OKA} \text{ and } \lambda_0{}^{PDG} = \left(\frac{m_{\pi^+}}{m_{\pi^0}}\right)^2 \lambda_0{}^{OKA}$$

The pole parametrization is

$$f_+(t) = f_+(0) \left(\frac{M_V^2}{M_V^2 - t}\right)$$

$$f_0(t) = f_0(0) \left(\frac{M_S^2}{M_S^2 - t}\right)$$

where  $M_V$  and  $M_S$  are the vector and scalar pole masses.

The following abbreviations are used:

DP = Dalitz plot analysis.

PI =  $\pi$  spectrum analysis.

MU =  $\mu$  spectrum analysis.

POL =  $\mu$  polarization analysis.

BR =  $K_{\mu 3}^{\pm} / K_{e 3}^{\pm}$  branching ratio analysis.

E = positron or electron spectrum analysis.

RC = radiative corrections.

For previous  $\lambda'_+$  and  $\lambda''_+$  parametrizations used by NA48 (e.g. LAI 07A) and ISTRA (e.g. YUSHCHENKO 04B) see PDG 18.

## $\lambda_+$ (LINEAR ENERGY DEPENDENCE OF $f_+$ IN $K_{e3}^{\pm}$ DECAY)

These results are for a linear expansion only. See the next section for fits including a quadratic term. For radiative correction of the  $K_{e3}^{\pm}$  Dalitz plot, see GINSBERG 67, BECHERRAWY 70, CIRIGLIANO 02, CIRIGLIANO 04, and ANDRE 07. Results labeled OUR FIT are discussed in the review “ $K_{\ell 3}^{\pm}$  and  $K_{\ell 3}^0$  Form Factors” above. For earlier, lower statistics results, see the 2004 edition of this review, Physics Letters **B592** 1 (2004).

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u>                    | <u>DOCUMENT ID</u>          | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|---|--------------------------------|-----------------------------|-------------|------------|----------------|
| <b>2.959 ± 0.025 OUR FIT</b>              | Assuming $\mu$ -e universality |                             |             |            |                |
| <b>2.956 ± 0.025 OUR AVERAGE</b>          |                                |                             |             |            |                |
| 2.95 ± 0.022 ± 0.018                      | 5.25M                          | YUSHCHENKO 18               | OKA         | +          |                |
| 3.044 ± 0.083 ± 0.074                     | 1.1M                           | AKOPDZANOV 09               | TNF         | ±          |                |
| 2.966 ± 0.050 ± 0.034                     | 919k                           | <sup>1</sup> YUSHCHENKO 04B | ISTR        | -          | DP             |

|                  |     |                       |    |      |   |           |
|------------------|-----|-----------------------|----|------|---|-----------|
| 2.78 ±0.26 ±0.30 | 41k | SHIMIZU               | 00 | SPEC | + | DP        |
| 2.84 ±0.27 ±0.20 | 32k | <sup>2</sup> AKIMENKO | 91 | SPEC |   | PI, no RC |
| 2.9 ±0.4         | 62k | <sup>3</sup> BOLOTOV  | 88 | SPEC |   | PI, no RC |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|                  |      |                         |     |      |   |    |
|------------------|------|-------------------------|-----|------|---|----|
| 3.06 ±0.09 ±0.06 | 550k | <sup>1,4</sup> AJINENKO | 03C | ISTR | – | DP |
| 2.93 ±0.15 ±0.2  | 130k | <sup>4</sup> AJINENKO   | 02  | SPEC |   | DP |

<sup>1</sup> Rescaled to agree with our conventions as noted above.

<sup>2</sup> AKIMENKO 91 state that radiative corrections would raise  $\lambda_+$  by 0.0013.

<sup>3</sup> BOLOTOV 88 state radiative corrections of GINSBERG 67 would raise  $\lambda_+$  by 0.002.

<sup>4</sup> Superseded by YUSHCHENKO 04B.

### $\lambda_+$ (LINEAR ENERGY DEPENDENCE OF $f_+$ IN $K_{\mu 3}^{\pm}$ DECAY)

Results labeled OUR FIT are discussed in the review “ $K_{\ell 3}^{\pm}$  and  $K_{\ell 3}^0$  Form Factors” above. For earlier, lower statistics results, see the 2004 edition of this review, Physics Letters **B592** 1 (2004).

| VALUE (units $10^{-2}$ )    | EVTS   | DOCUMENT ID               | TECN | CHG | COMMENT |
|-----------------------------|--|---------------------------|------|-----|---------|
| <b>2.959 ±0.025 OUR FIT</b> | Assuming $\mu$ -e universality   |                           |      |     |         |
| <b>3.09 ±0.25 OUR FIT</b>   | Error includes scale factor of 1.5. Not assuming $\mu$ -e universality |                           |      |     |         |
| 2.96 ±0.14 ±0.10            | 540k   | <sup>1</sup> YUSHCHENKO04 | ISTR | –   | DP      |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|            |      |                       |    |      |   |    |
|------------|------|-----------------------|----|------|---|----|
| 3.21 ±0.45 | 112k | <sup>2</sup> AJINENKO | 03 | ISTR | – | DP |
|------------|------|-----------------------|----|------|---|----|

<sup>1</sup> Rescaled to agree with our conventions as noted above.

<sup>2</sup> Superseded by YUSHCHENKO 04.

### $\lambda_0$ (LINEAR ENERGY DEPENDENCE OF $f_0$ IN $K_{\mu 3}^{\pm}$ DECAY)

Results labeled OUR FIT are discussed in the review “ $K_{\ell 3}^{\pm}$  and  $K_{\ell 3}^0$  Form Factors” above. For earlier, lower statistics results, see the 2004 edition of this review, Physics Letters **B592** 1 (2004).

| VALUE (units $10^{-2}$ )  | $d\lambda_0/d\lambda_+$  | EVTS | DOCUMENT ID               | TECN | CHG  | COMMENT |    |
|---------------------------|--|------|---------------------------|------|------|---------|----|
| <b>1.76 ±0.25 OUR FIT</b> | Error includes scale factor of 2.7. Assuming $\mu$ -e universality     |      |                           |      |      |         |    |
| <b>1.73 ±0.27 OUR FIT</b> | Error includes scale factor of 2.6. Not assuming $\mu$ -e universality |      |                           |      |      |         |    |
| 1.420 ±0.114 ±0.107       |  | 2.3M | <sup>1</sup> BATLEY       | 18   | NA48 | ±       |    |
| 1.96 ±0.12 ±0.06          | –0.348   | 540k | <sup>2</sup> YUSHCHENKO04 | ISTR | –    | DP      |    |
| 2.09 ±0.45                | –0.46  | 112k | <sup>3</sup> AJINENKO     | 03   | ISTR | –       | DP |
| 1.9 ±0.64                 |  | 24k  | <sup>4</sup> HORIE        | 01   | SPEC | +       | BR |
| 1.9 ±1.0                  | +0.03  | 55k  | <sup>5</sup> HEINTZE      | 77   | SPEC | +       | BR |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup> Data collected in 2004 by NA48/2. Obtained from a fit with a quadratic vector form factor. Correlation coefficient with linear slope is 0.511, with quadratic slope is –0.513.

$\chi^2/NDF = 409.9/381$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu 3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $(14.47 \pm 0.63 \pm 1.17) \times 10^{-3}$ .

<sup>2</sup> Rescaled to agree with our conventions as noted above.

<sup>3</sup> Superseded by YUSHCHENKO 04.

<sup>4</sup> HORIE 01 assumes  $\mu$ -e universality in  $K_{\ell 3}^+$  decay and uses SHIMIZU 00 value  $\lambda = 0.0278 \pm 0.0040$  from  $K_{e3}^{\pm}$  decay.

<sup>5</sup> HEINTZE 77 uses  $\lambda_+ = 0.029 \pm 0.003$ .  $d\lambda_0/d\lambda_+$  estimated by us.

$\lambda'_+$  (LINEAR  $K_{e3}^\pm$  FORM FACTOR FROM QUADRATIC FIT)

| <u>VALUE (units <math>10^{-2}</math>)</u>                                     | <u>EVTS</u>        | <u>DOCUMENT ID</u>           | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|---|--------------------|------------------------------|-------------|------------|----------------|
| <b>2.59 ± 0.04</b>  | <b>OUR AVERAGE</b> |                              |             |            |                |
| 2.426 ± 0.078 ± 0.130   | 4.4M               | <sup>1</sup> BATLEY 18       | NA48        | ±          |                |
| 2.611 ± 0.035 ± 0.028   | 5.25M              | YUSHCHENKO18                 | OKA         | +          |                |
| 2.485 ± 0.163 ± 0.034   | 919k               | <sup>2,3</sup> YUSHCHENKO04B | ISTR        | –          | DP             |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |                    |                              |             |            |                |
| 3.07 ± 0.21   | 550k               | <sup>2,4</sup> AJINENKO 03C  | ISTR        | –          | DP             |

<sup>1</sup> Data collected in 2004 by NA48/2. Correlation coefficient with quadratic slope is  $-0.929$ .  $\chi^2/NDF = 569.1/687$ . BATLEY 18 also performed a combined  $K_{e3}^\pm$  and  $K_{\mu 3}^\pm$  fit assuming  $\mu - e$  universality and obtained  $(24.24 \pm 0.75 \pm 1.3) \times 10^{-3}$ .

<sup>2</sup> Rescaled to agree with our conventions as noted above.

<sup>3</sup> YUSHCHENKO 04B  $\lambda'_+$  and  $\lambda''_+$  are strongly correlated with coefficient  $\rho(\lambda'_+, \lambda''_+) = -0.95$ .

<sup>4</sup> Superseded by YUSHCHENKO 04B.

 $\lambda''_+$  (QUADRATIC  $K_{e3}^\pm$  FORM FACTOR)

| <u>VALUE (units <math>10^{-2}</math>)</u>                                     | <u>EVTS</u>        | <u>DOCUMENT ID</u>           | <u>TECN</u> | <u>CHG</u> | <u>COMMENT</u> |
|---|--------------------|------------------------------|-------------|------------|----------------|
| <b>0.186 ± 0.021</b>  | <b>OUR AVERAGE</b> |                              |             |            |                |
| 0.164 ± 0.030 ± 0.039   | 4.4M               | <sup>1</sup> BATLEY 18       | NA48        | ±          |                |
| 0.191 ± 0.019 ± 0.014   | 5.25M              | YUSHCHENKO18                 | OKA         | +          |                |
| 0.192 ± 0.062 ± 0.071   | 919k               | <sup>2,3</sup> YUSHCHENKO04B | ISTR        | –          | DP             |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |                    |                              |             |            |                |
| -0.5 ± 0.7 ± 1.5  | 550k               | <sup>2,4</sup> AJINENKO 03C  | ISTR        | –          | DP             |

<sup>1</sup> Data collected in 2004 by NA48/2. Correlation coefficient with quadratic slope is  $-0.929$ .  $\chi^2/NDF = 569.1/687$ . BATLEY 18 also performed a combined  $K_{e3}^\pm$  and  $K_{\mu 3}^\pm$  fit assuming  $\mu - e$  universality and obtained  $(1.67 \pm 0.29 \pm 0.41) \times 10^{-3}$ .

<sup>2</sup> Rescaled to agree with our conventions as noted above.

<sup>3</sup> YUSHCHENKO 04B  $\lambda'_+$  and  $\lambda''_+$  are strongly correlated with coefficient  $\rho(\lambda'_+, \lambda''_+) = -0.95$ .

<sup>4</sup> Superseded by YUSHCHENKO 04B.

 $\lambda'_+$  (LINEAR  $K_{\mu 3}^\pm$  FORM FACTOR FROM QUADRATIC FIT)

| <u>VALUE (units <math>10^{-3}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>     | <u>TECN</u> | <u>CHG</u> |
|---|-------------|------------------------|-------------|------------|
| <b>24.27 ± 2.88 ± 2.89</b>                | 2.3M        | <sup>1</sup> BATLEY 18 | NA48        | ±          |

<sup>1</sup> Data collected in 2004 by NA48/2. Correlation coefficient with quadratic slope is  $-0.974$ , with scalar slope is  $0.511$ .  $\chi^2/NDF = 409.9/381$ . BATLEY 18 also performed a combined  $K_{e3}^\pm$  and  $K_{\mu 3}^\pm$  fit assuming  $\mu - e$  universality and obtained  $(24.24 \pm 0.75 \pm 1.3) \times 10^{-3}$ .

 $\lambda''_+$  (QUADRATIC  $K_{\mu 3}^\pm$  FORM FACTOR)

| <u>VALUE (units <math>10^{-3}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>     | <u>TECN</u> | <u>CHG</u> |
|---|-------------|------------------------|-------------|------------|
| <b>1.83 ± 1.05 ± 1.09</b>                 | 2.3M        | <sup>1</sup> BATLEY 18 | NA48        | ±          |

<sup>1</sup> Data collected in 2004 by NA48/2. Correlation coefficient with linear slope is  $-0.974$ , with scalar slope is  $0.513$ .  $\chi^2/NDF = 409.9/381$ . BATLEY 18 also performed a combined  $K_{e3}^\pm$  and  $K_{\mu 3}^\pm$  fit assuming  $\mu - e$  universality and obtained  $(1.67 \pm 0.29 \pm 0.41) \times 10^{-3}$ .

**$M_V$  (VECTOR POLE MASS FOR  $K_{e3}^{\pm}$  DECAY)**See the review on  $K_{J3}^{\pm}$  and  $K_{J3}^0$  Form Factors for details.

| <u>VALUE (MeV)</u>           | <u>EVTS</u> | <u>DOCUMENT ID</u>        | <u>TECN</u> | <u>CHG</u> |
|------------------------------|-------------|---------------------------|-------------|------------|
| <b>890.3±2.8 OUR AVERAGE</b> |             |                           |             |            |
| 885.2±3.3±7.2                | 4.4M        | <sup>1</sup> BATLEY 18    | NA48        | ±          |
| 891 ±3                       | 5.25M       | <sup>2</sup> YUSHCHENKO18 | OKA         | +          |

<sup>1</sup> Data collected in 2004 by NA48/2.  $\chi^2/NDF = 568.9/688$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $884.4 \pm 3.1 \pm 6.7$  MeV.

<sup>2</sup> Assumed no scalar or tensor contributions to the form factor.

 **$M_V$  (VECTOR POLE MASS FOR  $K_{\mu3}^{\pm}$  DECAY)**

| <u>VALUE (MeV)</u>   | <u>EVTS</u> | <u>DOCUMENT ID</u>     | <u>TECN</u> | <u>CHG</u> |
|----------------------|-------------|------------------------|-------------|------------|
| <b>878.4±8.8±8.3</b> | 2.3M        | <sup>1</sup> BATLEY 18 | NA48        | ±          |

<sup>1</sup> Data collected in 2004 by NA48/2.  $\chi^2/NDF = 409.9/382$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $884.4 \pm 3.1 \pm 6.7$  MeV.

 **$M_S$  (SCALAR POLE MASS FOR  $K_{\mu3}^{\pm}$  DECAY)**

| <u>VALUE (MeV)</u>      | <u>EVTS</u> | <u>DOCUMENT ID</u>     | <u>TECN</u> | <u>CHG</u> |
|-------------------------|-------------|------------------------|-------------|------------|
| <b>1214.8±23.5±49.2</b> | 2.3M        | <sup>1</sup> BATLEY 18 | NA48        | ±          |

<sup>1</sup> Data collected in 2004 by NA48/2.  $\chi^2/NDF = 409.9/382$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $1208.3 \pm 21.2 \pm 47.5$  MeV.

 **$\Lambda_+$  (DISPERSIVE VECTOR FORM FACTOR IN  $K_{e3}^{\pm}$  DECAY)**See the review on  $K_{J3}^{\pm}$  and  $K_{J3}^0$  Form Factors for details.

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>        | <u>TECN</u> | <u>CHG</u> |
|---|-------------|---------------------------|-------------|------------|
| <b>2.460±0.017 OUR AVERAGE</b>            |             |                           |             |            |
| 2.494±0.021±0.064                         | 4.4M        | <sup>1</sup> BATLEY 18    | NA48        | ±          |
| 2.458±0.018                               | 5.25M       | <sup>2</sup> YUSHCHENKO18 | OKA         | +          |

<sup>1</sup> Data collected in 2004 by NA48/2.  $\chi^2/NDF = 569.0/688$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $(24.99 \pm 0.20 \pm 0.62) \times 10^{-3}$ .

<sup>2</sup> Assumed no scalar or tensor contributions to the form factor.

 **$\Lambda_+$  (DISPERSIVE VECTOR FORM FACTOR IN  $K_{\mu3}^{\pm}$  DECAY)**

| <u>VALUE (units <math>10^{-3}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u>     | <u>TECN</u> | <u>CHG</u> |
|---|-------------|------------------------|-------------|------------|
| <b>25.36±0.58±0.72</b>                    | 2.3M        | <sup>1</sup> BATLEY 18 | NA48        | ±          |

<sup>1</sup> Data collected in 2004 by NA48/2.  $\chi^2/NDF = 410.3/382$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $(24.99 \pm 0.20 \pm 0.62) \times 10^{-3}$ .

## In(C) (DISPERSIVE SCALAR FORM FACTOR in $K_{\mu 3}^{\pm}$ decays )

| VALUE (units $10^{-3}$ )                      | EVTS | DOCUMENT ID         | TECN    | CHG   |
|---|------|---------------------|---------|-------|
| <b><math>182.17 \pm 6.31 \pm 14.45</math></b> | 2.3M | <sup>1</sup> BATLEY | 18 NA48 | $\pm$ |

<sup>1</sup>Data collected in 2004 by NA48/2. Combined fit with dispersive vector form factor  $\Lambda_+ = 25.36 \pm 0.58 \pm 0.72$ . Correlation coefficient is 0.104.  $\chi^2/NDF = 410.3/382$ . BATLEY 18 also performed a combined  $K_{e3}^{\pm}$  and  $K_{\mu 3}^{\pm}$  fit assuming  $\mu - e$  universality and obtained  $(183.65 \pm 5.92 \pm 14.25) \times 10^{-3}$ .

### $|f_S/f_+|$ FOR $K_{e3}^{\pm}$ DECAY

Ratio of scalar to  $f_+$  couplings.

| VALUE (units $10^{-2}$ )  | CL% | EVTS  | DOCUMENT ID           | TECN | CHG  | COMMENT                            |                                 |
|---|-----|-------|-----------------------|------|------|------------------------------------|---------------------------------|
| <b><math>-0.08^{+0.34}_{-0.40}</math></b>                                     |     |       |                       |      |      | <b>OUR AVERAGE</b>                 |                                 |
| $0.01^{+0.38}_{-0.46}$  |     | 5.25M | YUSHCHENKO18          | OKA  | +    | $\lambda'_+, \lambda''_+, f_S$ fit |                                 |
| $-0.37^{+0.66}_{-0.56} \pm 0.41$  |     | 919k  | YUSHCHENKO04B         | ISTR | -    | $\lambda'_+, \lambda''_+, f_S$ fit |                                 |
| $0.2 \pm 2.6 \pm 1.4$   |     | 41k   | SHIMIZU               | 00   | SPEC | +                                  | $\lambda_+, f_S, f_T$ fit       |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |     |       |                       |      |      |                                    |                                 |
| $0.2^{+2.0}_{-2.2} \pm 0.3$   |     | 550k  | <sup>1</sup> AJINENKO | 03C  | ISTR | -                                  | $\lambda_+, f_S, f_T$ fit       |
| $-1.9^{+2.5}_{-1.6}$  |     | 130k  | <sup>1</sup> AJINENKO | 02   | SPEC |                                    | $\lambda_+, f_S$ fit            |
| $7.0 \pm 1.6 \pm 1.6$   |     | 32k   | AKIMENKO              | 91   | SPEC |                                    | $\lambda_+, f_S, f_T, \phi$ fit |
| $0 \pm 10$  |     | 2827  | <sup>2</sup> BRAUN    | 75   | HLBC | +                                  |                                 |
| $< 13$  | 90  | 4017  | CHIANG                | 72   | OSPK | +                                  |                                 |
| $14^{+3}_{-4}$  |     | 2707  | <sup>2</sup> STEINER  | 71   | HLBC | +                                  | $\lambda_+, f_S, f_T, \phi$ fit |
| $< 23$  | 90  |       | BOTTERILL             | 68C  | ASPK |                                    |                                 |
| $< 18$  | 90  |       | BELLOTTI              | 67B  | HLBC |                                    |                                 |
| $< 30$  | 95  |       | KALMUS                | 67   | HLBC | +                                  |                                 |

<sup>1</sup>Superseded by YUSHCHENKO 04B.

<sup>2</sup>Statistical errors only.

### $|f_T/f_+|$ FOR $K_{e3}^{\pm}$ DECAY

Ratio of tensor to  $f_+$  couplings.

| VALUE (units $10^{-2}$ )  | EVTS | DOCUMENT ID | TECN                  | CHG  | COMMENT            |                                    |                                 |
|---|------|-------------|-----------------------|------|--------------------|------------------------------------|---------------------------------|
| <b><math>-1.2^{+1.3}_{-1.1}</math></b>  |      |             |                       |      | <b>OUR AVERAGE</b> |                                    |                                 |
| $-1.24^{+1.6}_{-1.3}$   |      | 5.25M       | YUSHCHENKO18          | OKA  | +                  | $\lambda'_+, \lambda''_+, f_T$ fit |                                 |
| $-1.2 \pm 2.1 \pm 1.1$  |      | 919k        | YUSHCHENKO04B         | ISTR | -                  | $\lambda'_+, \lambda''_+, f_T$ fit |                                 |
| $1 \pm 14 \pm 9$  |      | 41k         | SHIMIZU               | 00   | SPEC               | +                                  | $\lambda_+, f_S, f_T$ fit       |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |      |             |                       |      |                    |                                    |                                 |
| $2.1^{+6.4}_{-7.5} \pm 2.6$   |      | 550k        | <sup>1</sup> AJINENKO | 03C  | ISTR               | -                                  | $\lambda_+, f_S, f_T$ fit       |
| $-4.5^{+6.0}_{-5.7}$  |      | 130k        | <sup>1</sup> AJINENKO | 02   | SPEC               |                                    | $\lambda_+, f_T$ fit            |
| $53^{+9}_{-10} \pm 10$  |      | 32k         | AKIMENKO              | 91   | SPEC               |                                    | $\lambda_+, f_S, f_T, \phi$ fit |

<sup>1</sup>Superseded by YUSHCHENKO 04B.



### $f_S/f_+$ FOR $K_{\mu 3}^{\pm}$ DECAY

Ratio of scalar to  $f_+$  couplings.

| VALUE (units $10^{-2}$ )                   | EVTS | DOCUMENT ID               | TECN | CHG  | COMMENT |
|--|------|---------------------------|------|------|---------|
| <b><math>0.17 \pm 0.14 \pm 0.54</math></b> | 540k | <sup>1</sup> YUSHCHENKO04 | ISTR | –    | DP      |
| $0.4 \pm 0.5 \pm 0.5$                      | 112k | <sup>2</sup> AJINENKO     | 03   | ISTR | – DP    |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup> The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for  $\lambda_0$ ,  $\pm 0.0053$ , combined in quadrature with the systematic error  $\pm 0.0009$ .

<sup>2</sup> The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for  $\lambda_0$ . Superseded by YUSHCHENKO 04.

### $f_T/f_+$ FOR $K_{\mu 3}^{\pm}$ DECAY

Ratio of tensor to  $f_+$  couplings.

| VALUE (units $10^{-2}$ )                    | EVTS | DOCUMENT ID           | TECN | CHG  | COMMENT |
|---|------|-----------------------|------|------|---------|
| <b><math>-0.07 \pm 0.71 \pm 0.20</math></b> | 540k | YUSHCHENKO04          | ISTR | –    | DP      |
| $-2.1 \pm 2.8 \pm 1.4$                      | 112k | <sup>1</sup> AJINENKO | 03   | ISTR | – DP    |
| $2 \pm 12$                                  | 1585 | BRAUN                 | 75   | HLBC |         |

• • • We do not use the following data for averages, fits, limits, etc. • • •

<sup>1</sup> The second error is the theoretical error from the uncertainty in the chiral perturbation theory prediction for  $\lambda_0$ . Superseded by YUSHCHENKO 04.

## $K_{\ell 4}^{\pm}$ FORM FACTORS

Based on the parametrizations of AMOROS 99, the  $K_{\ell 4}^{\pm}$  form factors can be expressed as

$$F_S = f_s + f'_s q^2 + f''_s q^4 + f'_e S_e / 4m_\pi^2$$

$$F_p = f_p$$

$$G_p = g_p + g'_p q^2$$

$$H_p = h_p$$

where  $q^2 = (S_\pi / 4m_\pi^2) - 1$ ,  $S_\pi$  is the invariant mass squared of the dipion, and  $S_e$  is the invariant mass squared of the dilepton.

### $f_S$ FOR $K^{\pm} \rightarrow \pi^+ \pi^- e^{\pm} \nu$ DECAY

| VALUE   | EVTS | DOCUMENT ID         | TECN | CHG    |
|---|------|---------------------|------|--------|
| <b><math>5.712 \pm 0.032</math> OUR AVERAGE</b> |      |                     |      |        |
| $5.705 \pm 0.003 \pm 0.035$                     | 1.1M | <sup>1</sup> BATLEY | 12   | NA48 ± |
| $5.75 \pm 0.02 \pm 0.08$                        | 400k | <sup>2</sup> PISLAK | 03   | B865 + |

<sup>1</sup> BATLEY 12 uses data collected in 2003–2004. The result is obtained from a measurement of  $\Gamma(\pi^+ \pi^- e \nu) / \Gamma(\pi^+ \pi^- \pi^+)$  and assumed PDG 12 value of  $\Gamma(\pi^+ \pi^- \pi^+) / \Gamma = (5.59 \pm 0.04) \times 10^{-2}$ .

<sup>2</sup> Radiative corrections included. Using Roy equations and not including isospin breaking, PISLAK 03 obtains the following  $\pi\pi$  scattering lengths  $a_0^0 = 0.228 \pm 0.012 \pm 0.004^{+0.012}_{-0.016}$  (theor.) and  $a_0^2 = -0.0365 \pm 0.0023 \pm 0.0008^{+0.0031}_{-0.0026}$  (theor.).

### $f'_s/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
|---|-------------|--------------------|-------------|------------|

|                     |       |                     |     |        |
|---------------------|-------|---------------------|-----|--------|
| <b>15.2±0.7±0.5</b> | 1.13M | <sup>1</sup> BATLEY | 10C | NA48 ± |
|---------------------|-------|---------------------|-----|--------|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|              |      |                     |     |        |
|--------------|------|---------------------|-----|--------|
| 17.2±0.9±0.6 | 670k | <sup>2</sup> BATLEY | 08A | NA48 ± |
|--------------|------|---------------------|-----|--------|

<sup>1</sup>Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). The correlation with  $f''_s/f_s = -0.954$  and with  $f'_e/f_s = 0.080$ . Supersedes BATLEY 08A.

<sup>2</sup>Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$ ,  $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .

### $f''_s/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
|---|-------------|--------------------|-------------|------------|

|                     |       |                     |     |        |
|---------------------|-------|---------------------|-----|--------|
| <b>-7.3±0.7±0.6</b> | 1.13M | <sup>1</sup> BATLEY | 10C | NA48 ± |
|---------------------|-------|---------------------|-----|--------|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|              |      |                     |     |        |
|--------------|------|---------------------|-----|--------|
| -9.0±0.9±0.7 | 670k | <sup>2</sup> BATLEY | 08A | NA48 ± |
|--------------|------|---------------------|-----|--------|

<sup>1</sup>Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). The correlation with  $f'_s/f_s = -0.954$  and with  $f'_e/f_s = 0.019$ . Supersedes BATLEY 08A.

<sup>2</sup>Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$ ,  $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .

### $f'_e/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
|---|-------------|--------------------|-------------|------------|

|                    |       |                     |     |        |
|--------------------|-------|---------------------|-----|--------|
| <b>6.8±0.6±0.7</b> | 1.13M | <sup>1</sup> BATLEY | 10C | NA48 ± |
|--------------------|-------|---------------------|-----|--------|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |      |                     |     |        |
|-------------|------|---------------------|-----|--------|
| 8.1±0.8±0.9 | 670k | <sup>2</sup> BATLEY | 08A | NA48 ± |
|-------------|------|---------------------|-----|--------|

<sup>1</sup>Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). The correlation with  $f'_s/f_s = 0.080$  and with  $f''_s/f_s = 0.019$ . Supersedes BATLEY 08A.

<sup>2</sup>Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$ ,  $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .

### $f_p/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| <u>VALUE (units <math>10^{-2}</math>)</u> | <u>EVTS</u> | <u>DOCUMENT ID</u> | <u>TECN</u> | <u>CHG</u> |
|---|-------------|--------------------|-------------|------------|
|---|-------------|--------------------|-------------|------------|

|                     |       |                     |     |        |
|---------------------|-------|---------------------|-----|--------|
| <b>-4.8±0.3±0.4</b> | 1.13M | <sup>1</sup> BATLEY | 10C | NA48 ± |
|---------------------|-------|---------------------|-----|--------|

• • • We do not use the following data for averages, fits, limits, etc. • • •

|              |      |                     |     |        |
|--------------|------|---------------------|-----|--------|
| -4.8±0.4±0.4 | 670k | <sup>2</sup> BATLEY | 08A | NA48 ± |
|--------------|------|---------------------|-----|--------|

<sup>1</sup>Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). Supersedes BATLEY 08A.

<sup>2</sup>Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$   
 $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .

### $g_p/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| VALUE (units $10^{-2}$ )  | EVTS  | DOCUMENT ID             | TECN | CHG |
|---|-------|-------------------------|------|-----|
| <b>86.8±1.0±1.0</b>   | 1.13M | <sup>1</sup> BATLEY 10C | NA48 | ±   |
| ••• We do not use the following data for averages, fits, limits, etc. ••• |       |                         |      |     |
| 87.3±1.3±1.2  | 670k  | <sup>2</sup> BATLEY 08A | NA48 | ±   |
| 80.9±0.9±1.2  | 400k  | <sup>3</sup> PISLAK 03  | B865 | ±   |

<sup>1</sup>Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). Supersedes BATLEY 08A. The correlation with  $g'_p/f_s = -0.914$ . Supersedes BATLEY 08A.

<sup>2</sup>Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$   
 $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .

<sup>3</sup>Radiative corrections included. Using Roy equations PISLAK 03 obtains the following scattering lengths  $a_0^0 = 0.203 \pm 0.033 \pm 0.004$ ,  $a_0^2 = -0.055 \pm 0.023 \pm 0.003$ .

### $g'_p/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| VALUE (units $10^{-2}$ )  | EVTS  | DOCUMENT ID             | TECN | CHG |
|---|-------|-------------------------|------|-----|
| <b>8.9±1.7±1.3</b>  | 1.13M | <sup>1</sup> BATLEY 10C | NA48 | ±   |
| ••• We do not use the following data for averages, fits, limits, etc. ••• |       |                         |      |     |
| 8.1±2.2±1.5   | 670k  | <sup>2</sup> BATLEY 08A | NA48 | ±   |
| 12.0±1.9±0.7  | 400k  | <sup>3</sup> PISLAK 03  | B865 | ±   |

<sup>1</sup>Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). The correlation with  $g_p/f_s = -0.914$ . Supersedes BATLEY 08A.

<sup>2</sup>Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$   
 $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .

<sup>3</sup>Radiative corrections included. Using Roy equations PISLAK 03 obtains the following scattering lengths  $a_0^0 = 0.203 \pm 0.033 \pm 0.004$ ,  $a_0^2 = -0.055 \pm 0.023 \pm 0.003$ .

### $h_p/f_s$ FOR $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$ DECAY

| VALUE (units $10^{-2}$ )  | EVTS  | DOCUMENT ID             | TECN | CHG |
|---|-------|-------------------------|------|-----|
| <b>-39.8±1.5±0.8</b>  | 1.13M | <sup>1</sup> BATLEY 10C | NA48 | ±   |
| ••• We do not use the following data for averages, fits, limits, etc. ••• |       |                         |      |     |
| -41.1±1.9±0.8   | 670k  | <sup>2</sup> BATLEY 08A | NA48 | ±   |
| -51.3±3.3±3.5   | 400k  | <sup>3</sup> PISLAK 03  | B865 | ±   |

- <sup>1</sup> Radiative corrections included. Using Roy equations and including isospin breaking, BATLEY 10C obtains the following scattering lengths  $a_0^0 = 0.2220 \pm 0.0128 \pm 0.0050 \pm 0.0037$  (theor.),  $a_0^2 = -0.0432 \pm 0.0086 \pm 0.0034 \pm 0.0028$  (theor.). Supersedes BATLEY 08A.
- <sup>2</sup> Radiative corrections included. Using Roy equations and not including isospin breaking, BATLEY 08A obtains the following  $\pi\pi$  scattering length  $a_0^0 = 0.233 \pm 0.016 \pm 0.007$ ,  $a_0^2 = -0.0471 \pm 0.011 \pm 0.004$ .
- <sup>3</sup> Radiative corrections included. Using Roy equations PISLAK 03 obtains the following scattering lengths  $a_0^0 = 0.203 \pm 0.033 \pm 0.004$ ,  $a_0^2 = -0.055 \pm 0.023 \pm 0.003$ .

## DECAY FORM FACTOR FOR $K^\pm \rightarrow \pi^0 \pi^0 e^\pm \nu$

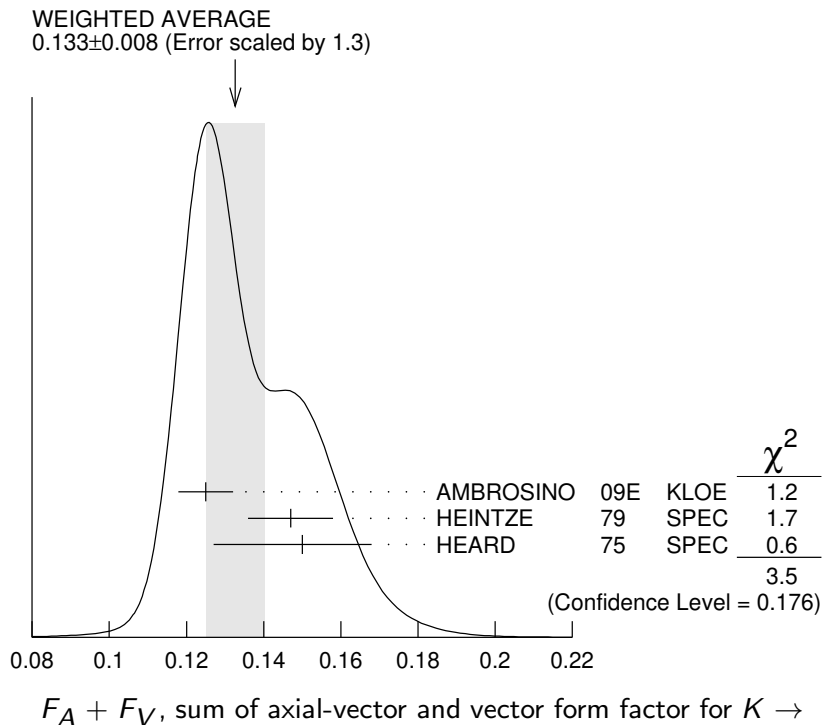
Given in BOLOTOV 86B, BARMIN 88B, and SHIMIZU 04.

### $K^\pm \rightarrow \ell^\pm \nu \gamma$ FORM FACTORS

For definitions of the axial-vector  $F_A$  and vector  $F_V$  form factor, see the "Note on  $\pi^\pm \rightarrow \ell^\pm \nu \gamma$  and  $K^\pm \rightarrow \ell^\pm \nu \gamma$  Form Factors" in the  $\pi^\pm$  section. In the kaon literature, often different definitions  $a_K = F_A/m_K$  and  $v_K = F_V/m_K$  are used.

### $F_A + F_V$ , SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow e \nu_e \gamma$

| VALUE                                     | EVTS | DOCUMENT ID                         | TECN     | COMMENT  |
|---|------|-------------------------------------|----------|--|
| <b>0.133±0.008 OUR AVERAGE</b>            |      | Error includes scale factor of 1.3. |          | See the ideogram below.                        |
| 0.125±0.007±0.001                         | 1.4k | <sup>1</sup> AMBROSINO              | 09E KLOE | $E_\gamma$ in 10–250 MeV,<br>$p_e > 200$ MeV/c |
| 0.147±0.011                               | 51   | <sup>2</sup> HEINTZE                | 79 SPEC  |  |
| 0.150 <sup>+0.018</sup> <sub>-0.023</sub> | 56   | <sup>3</sup> HEARD                  | 75 SPEC  |  |



<sup>1</sup> AMBROSINO 09E measures the absolute value  $|F_A + F_V|$  which is parametrized as  $|F_A + F_V| = F_V (1 + \lambda(1-x)) + F_A$ ,  $x = 2E_\gamma/m_K$ . ( $F_A + F_V$ ) and  $\lambda$  are fit parameters. The fitted value of  $\lambda = 0.38 \pm 0.20 \pm 0.02$  with a correlation of  $-0.93$  between ( $F_A + F_V$ ) and  $\lambda$ .

<sup>2</sup> HEINTZE 79 quotes absolute value of  $|F_A + F_V| \sin\theta_c$ . We use  $\sin\theta_c = V_{us} = 0.2205$ .

<sup>3</sup> HEARD 75 quotes absolute value of  $|F_A + F_V| \sin\theta_c$ . We use  $\sin\theta_c = V_{us} = 0.2205$ .

### $F_A + F_V$ , SUM OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow \mu\nu\mu\gamma$

| VALUE                    | CL% | EVTS | DOCUMENT ID        | TECN     | CHG |
|--------------------------|-----|------|--------------------|----------|-----|
| <b>0.165±0.007±0.011</b> |     | 2588 | <sup>1</sup> ADLER | 00B B787 | +   |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|             |    |  |                    |    |      |
|-------------|----|--|--------------------|----|------|
| -1.2 to 1.1 | 90 |  | DEMIDOV            | 90 | XEBC |
| < 0.23      | 90 |  | <sup>1</sup> AKIBA | 85 | SPEC |

<sup>1</sup> Quotes absolute value. Sign not determined.

### $F_A - F_V$ , DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow e\nu_e\gamma$

| VALUE           | CL% | DOCUMENT ID             | TECN |
|-----------------|-----|-------------------------|------|
| <b>&lt;0.49</b> | 90  | <sup>1</sup> HEINTZE 79 | SPEC |

<sup>1</sup> HEINTZE 79 quotes  $|F_A - F_V| < \sqrt{11} |F_A + F_V|$ .

### $F_A - F_V$ , DIFFERENCE OF AXIAL-VECTOR AND VECTOR FORM FACTOR FOR $K \rightarrow \mu\nu\mu\gamma$

| VALUE                           | CL% | EVTS | DOCUMENT ID                         | TECN | CHG |
|---------------------------------|-----|------|-------------------------------------|------|-----|
| <b>-0.153±0.033 OUR AVERAGE</b> |     |      | Error includes scale factor of 1.1. |      |     |

|                    |  |     |          |    |        |
|--------------------|--|-----|----------|----|--------|
| -0.134±0.021±0.027 |  | 95k | KRAVTSOV | 19 | OKA +  |
| -0.21 ±0.06        |  | 22k | DUK      | 11 | ISTR - |

• • • We do not use the following data for averages, fits, limits, etc. • • •

|               |    |      |         |          |      |
|---------------|----|------|---------|----------|------|
| -0.24 to 0.04 | 90 | 2588 | ADLER   | 00B B787 | +    |
| -2.2 to 0.6   | 90 |      | DEMIDOV | 90       | XEBC |
| -2.5 to 0.3   | 90 |      | AKIBA   | 85       | SPEC |

## $K^\pm$ CHARGE RADIUS

| VALUE (fm)  | DOCUMENT ID   | COMMENT                    |
|---|---------------|----------------------------|
| <b>0.560±0.031 OUR AVERAGE</b>  |               |                            |
| 0.580±0.040   | AMENDOLIA 86B | $K e \rightarrow K e$      |
| 0.530±0.050   | DALLY 80      | $K e \rightarrow K e$      |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |               |                            |
| 0.620±0.037   | BLATNIK 79    | VMD + dispersion relations |

## $K^+$ LONGITUDINAL POLARIZATION OF EMITTED $\mu^+$

| VALUE   | CL% | DOCUMENT ID              | TECN | CHG | COMMENT          |
|---|-----|--------------------------|------|-----|------------------|
| <b>&lt;-0.990</b>   | 90  | <sup>1</sup> AOKI 94     | SPEC | +   |                  |
| • • • We do not use the following data for averages, fits, limits, etc. • • • |     |                          |      |     |                  |
| <-0.990   | 90  | IMAZATO 92               | SPEC | +   | Repl. by AOKI 94 |
| -0.970±0.047  |     | <sup>2</sup> YAMANAKA 86 | SPEC | +   |                  |

−1.0 ± 0.1                                2 CUTTS                69    SPRK    +  
 −0.96 ± 0.12                              2 COOMBES            57    CNTR    +

<sup>1</sup> AOKI 94 measures  $\xi P_\mu = -0.9996 \pm 0.0030 \pm 0.0048$ . The above limit is obtained by summing the statistical and systematic errors in quadrature, normalizing to the physically significant region ( $|\xi P_\mu| < 1$ ) and assuming that  $\xi=1$ , its maximum value.

<sup>2</sup> Assumes  $\xi=1$ .

## FORWARD-BACKWARD ASYMMETRY IN $K^\pm$ DECAYS

$$A_{FB}(K^\pm_{\pi\mu\mu}) = \frac{\Gamma(\cos(\theta_{K\mu}) > 0) - \Gamma(\cos(\theta_{K\mu}) < 0)}{\Gamma(\cos(\theta_{K\mu}) > 0) + \Gamma(\cos(\theta_{K\mu}) < 0)}$$

| VALUE                            | CL% | DOCUMENT ID         | TECN     |
|----------------------------------|-----|---------------------|----------|
| <b>&lt;2.3 × 10<sup>-2</sup></b> | 90  | <sup>1</sup> BATLEY | 11A NA48 |

<sup>1</sup> BATLEY 11A gives a corresponding value of the asymmetry  $A_{FB} = (-2.4 \pm 1.8) \times 10^{-2}$ .

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| ATIYA      | 93B | PR D48 1                      | M.S. Atiya <i>et al.</i>         | (BNL E787 Collab.)       |
| ALLIEGRO   | 92  | PRL 68 278                    | C. Alliegro <i>et al.</i>        | (BNL, FNAL, PSI+)        |
| BARMIN     | 92  | SJNP 55 547                   | V.V. Barmin <i>et al.</i>        | (ITEP)                   |
|            |     | Translated from YAF 55 976.   |                                  |                          |
| IMAZATO    | 92  | PRL 69 877                    | J. Imazato <i>et al.</i>         | (KEK, INUS, TOKY+)       |
| IVANOV     | 92  | THESIS                        | Yu.M. Ivanov                     | (PNPI)                   |
| LITTENBERG | 92  | PRL 68 443                    | L.S. Littenberg, R.E. Shrock     | (BNL, STON)              |
| USHER      | 92  | PR D45 3961                   | T. Usher <i>et al.</i>           | (UCI)                    |
| AKIMENKO   | 91  | PL B259 225                   | S.A. Akimenko <i>et al.</i>      | (SERP, JINR, TBIL+)      |
| BARMIN     | 91  | SJNP 53 606                   | V.V. Barmin <i>et al.</i>        | (ITEP)                   |
|            |     | Translated from YAF 53 981.   |                                  |                          |
| DENISOV    | 91  | JETPL 54 558                  | A.S. Denisov <i>et al.</i>       | (PNPI)                   |
|            |     | Translated from ZETFP 54 557. |                                  |                          |
| Also       |     | THESIS                        | Yu.M. Ivanov                     | (PNPI)                   |
| ATIYA      | 90  | PRL 64 21                     | M.S. Atiya <i>et al.</i>         | (BNL E787 Collab.)       |
| ATIYA      | 90B | PRL 65 1188                   | M.S. Atiya <i>et al.</i>         | (BNL E787 Collab.)       |
| DEMIDOV    | 90  | SJNP 52 1006                  | V.S. Demidov <i>et al.</i>       | (ITEP)                   |
|            |     | Translated from YAF 52 1595.  |                                  |                          |
| LEE        | 90  | PRL 64 165                    | A.M. Lee <i>et al.</i>           | (BNL, FNAL, VILL, WASH+) |
| ATIYA      | 89  | PRL 63 2177                   | M.S. Atiya <i>et al.</i>         | (BNL E787 Collab.)       |
| BARMIN     | 89  | SJNP 50 421                   | V.V. Barmin <i>et al.</i>        | (ITEP)                   |
|            |     | Translated from YAF 50 679.   |                                  |                          |
| BARMIN     | 88  | SJNP 47 643                   | V.V. Barmin <i>et al.</i>        | (ITEP)                   |
|            |     | Translated from YAF 47 1011.  |                                  |                          |
| BARMIN     | 88B | SJNP 48 1032                  | V.V. Barmin <i>et al.</i>        | (ITEP)                   |
|            |     | Translated from YAF 48 1719.  |                                  |                          |
| BOLOTOV    | 88  | JETPL 47 7                    | V.N. Bolotov <i>et al.</i>       | (ASCI)                   |
|            |     | Translated from ZETFP 47 8.   |                                  |                          |
| GALL       | 88  | PRL 60 186                    | K.P. Gall <i>et al.</i>          | (BOST, MIT, WILL, CIT+)  |
| BARMIN     | 87  | SJNP 45 62                    | V.V. Barmin <i>et al.</i>        | (ITEP)                   |
|            |     | Translated from YAF 45 97.    |                                  |                          |
| BOLOTOV    | 87  | SJNP 45 1023                  | V.N. Bolotov <i>et al.</i>       | (INRM)                   |
|            |     | Translated from YAF 45 1652.  |                                  |                          |
| AMENDOLIA  | 86B | PL B178 435                   | S.R. Amendolia <i>et al.</i>     | (CERN NA7 Collab.)       |
| BOLOTOV    | 86  | SJNP 44 73                    | V.N. Bolotov <i>et al.</i>       | (INRM)                   |
|            |     | Translated from YAF 44 117.   |                                  |                          |
| BOLOTOV    | 86B | SJNP 44 68                    | V.N. Bolotov <i>et al.</i>       | (INRM)                   |
|            |     | Translated from YAF 44 108.   |                                  |                          |
| YAMANAKA   | 86  | PR D34 85                     | T. Yamanaka <i>et al.</i>        | (KEK, TOKY)              |
| Also       |     | PRL 52 329                    | R.S. Hayano <i>et al.</i>        | (TOKY, KEK)              |
| AKIBA      | 85  | PR D32 2911                   | Y. Akiba <i>et al.</i>           | (TOKY, TINT, TSUK, KEK)  |
| BOLOTOV    | 85  | JETPL 42 481                  | V.N. Bolotov <i>et al.</i>       | (INRM)                   |
|            |     | Translated from ZETFP 42 390. |                                  |                          |
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| WEISSENBE... | 76  | NP B115 55       | A.O. Weissenberg <i>et al.</i>             | (ITEP, LEBD)                     |
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| LJUNG        | 73  | PR D8 1307       | D. Ljung, D. Cline                         | (WISC)                           |
| Also         |     | PRL 28 523       | D. Ljung                                   | (WISC)                           |
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| HAIDT        | 71  | PR D3 10         | D. Haidt                                   | (AACH, BARI, CERN, EPOL, NIJM+)  |
| Also         |     | PL 29B 691       | D. Haidt <i>et al.</i>                     | (AACH, BARI, CERN, EPOL+)        |
| KLEMS        | 71  | PR D4 66         | J.H. Klems, R.H. Hildebrand, R. Stiening   | (CHIC+)                          |
| Also         |     | PRL 24 1086      | J.H. Klems, R.H. Hildebrand, R. Stiening   | (LRL+)                           |
| Also         |     | PRL 25 473       | J.H. Klems, R.H. Hildebrand, R. Stiening   | (LRL+)                           |
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| Also         |     | PRL 17 548       | F. Lobkowicz <i>et al.</i>                 | (ROCH, BNL)                      |
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| Erratum.     |     |                  |  |                                  |
| BELLOTTI     | 67  | Heidelberg Conf. | E. Bellotti, A. Pullia                     | (MILA)                           |
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| See footnote 1 in AUERBACH 67. |     |                   |   |                   |
| Also                           |     | PR 155 1505       | L.B. Auerbach <i>et al.</i>             | (PENN, PRIN)      |
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