19. Fragmentation Functions in e^+e^-, ep, and pp Collisions

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19.1 Introduction to fragmentation

Quarks and gluons produced in hard-scattering reactions will ultimately give rise to the colorless hadronic bound states that may be observed in the detector. The associated hadronization process is described by fragmentation functions $D_i^h(x,\mu^2)$ $(i=q,\bar{q},g)$ which are universal functions representing, in the simplest picture, a measure of the probability density that an outgoing parton produces a hadron h. Here, x is the fraction of the parton's momentum transferred to the hadron, and μ is a 'resolution' scale known as factorization scale. The $D_i^h(x,\mu^2)$ may be viewed as the final-state analogs of the initial-state parton distribution functions (PDFs) addressed in Section 18 of this *Review*. They are also sometimes referred to as *timelike* distributions since they are primarily accessed in e^+e^- annihilation via a timelike intermediate boson. (See Refs. [1, 2] for introductory reviews, and Refs. [3–5] for summaries of experimental and theoretical research in this field).

The cleanest laboratory for the study of fragmentation functions is provided by semi-inclusive electron-positron annihilation, $e^+e^- \to \gamma/Z \to h+X$. The cross section for this reaction may be expressed in terms of 'fragmentation structure functions' $F_{T,L,A}$ that are directly related to the fragmentation functions. At center-of-mass (CM) energy $\sqrt{s}=q^2$ we have

$$\frac{1}{\sigma_0}\frac{d^2\sigma^h}{dx\,d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)F_T^h(x,q^2)+\frac{3}{4}\sin^2\theta\,F_L^h(x,q^2)+\frac{3}{4}\cos\theta\,F_A^h(x,q^2)\,. \tag{19.1}$$

Here, q is the four-momentum of the intermediate photon or Z-boson, with $q^2>0$, and $x=2P_h{\cdot}q/q^2$ with the hadron's four-momentum P_h is the fragmentation counterpart of the familiar DIS Bjorken variable. (Note that $x=2E_h/\sqrt{s}\le 1$ in terms of the energy E_h of the produced hadron in the CM frame of the electron positron pair.) Furthermore, in the same frame, θ is the hadron's angle relative to the electron beam direction. Eq. (19.1) is the most general form for unpolarized inclusive single-particle production via vector bosons [6]. The fragmentation structure functions F_T and F_L represent the contributions from γ/Z polarizations transverse or longitudinal with respect to the direction of motion of the hadron. The parity-violating term with the asymmetric fragmentation function F_A arises from the interference between vector and axial-vector contributions. Various normalization factors σ_0 are used in the literature, ranging from the total cross section $\sigma_{\rm tot}$ for $e^+e^-\to$ hadrons, including all weak and QCD contributions, to $\sigma_0=4\pi\alpha^2N_c/3s$ with $N_c=3$, the lowest-order QED cross section for $e^+e^-\to\mu^+\mu^-$ times the number of colors N_c. LEP1 measurements of the three fragmentation structure functions are shown in Fig. 19.1.

Integration of Eq. (19.1) over all θ yields the total fragmentation structure function $F^h = F_T^h+F_L^h$:

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dx} = F^h(x,q^2) = \sum_i\int_x^1\frac{dz}{z}\,C_i\left(z,\alpha_{\rm s}(\mu),\frac{q^2}{\mu^2}\right)D_i^h\left(\frac{x}{z},\mu^2\right)\,. \tag{19.2}$$

On the right we have written the factorized expression for the structure function in terms of a sum over convolutions of the fragmentation functions D_i^h for partons $i=u,\bar{u},d,\bar{d},\ldots,g$ with perturbative coefficient functions C_i. Since photons and Z bosons do not distinguish between quarks and antiquarks, e^+e^- annihilation primarily constrains the combinations $D_q^h+D_{\bar{q}}^h$. Gluon fragmentation contributes only at higher order in perturbation theory or by scaling violations. Corrections to the factorized expression in Eq. (19.2) are suppressed by inverse powers of q^2. They

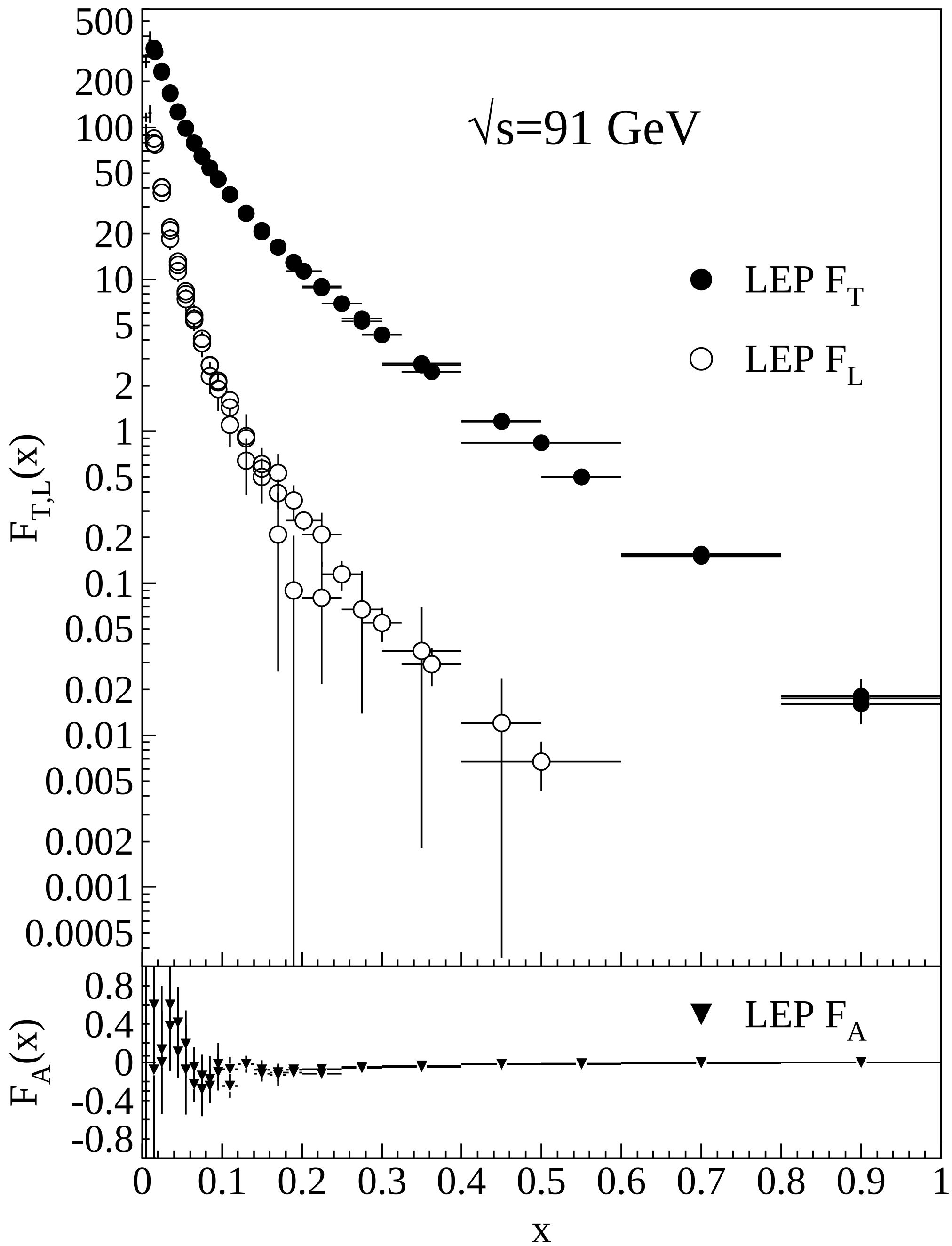

Figure 19.1: LEP1 measurements of total transverse (F_T), longitudinal (F_L), and asymmetric (F_A) fragmentation structure functions [7]. Data points with relative errors greater than 100% are omitted.

arise from quark and hadron mass terms and from non-perturbative effects. Analogous factorized expressions as in Eq. (19.2) may be written for each of the structure functions $F_{T,L,A}$ individually.

The fragmentation functions obey the momentum sum rule constraint

$$\sum_h \int_0^1 dx\, x\, D_i^h(x,\mu^2) \;=\; 1\;, \tag{19.3}$$

separately for each flavor i. Note that the sum rule involves a sum over all possible produced hadrons. The dependence of the functions D_i^h on the factorization scale μ^2 will be discussed in the next section.

Measurements of hadron production in deeply-inelastic lepton-proton scattering and hadron-hadron scattering are complementary to those in e^+e^- annihilation. The former process, $\ell p \to \ell' + h + X$, is known as *semi-inclusive deep-inelastic scattering (SIDIS)*. Here, in analogy with Eq. (19.2), the high virtuality of the photon in DIS also permits factorization of the cross section in terms of fragmentation functions, PDFs for the incoming proton, and perturbative hard-scattering cross sections. Likewise, factorization also occurs for $pp \to h + X$ at large transverse momentum of the produced hadron, and for $pp \to \text{jet}(h) + X$, where the hadron is part of a fully reconstructed jet. The fragmentation functions contributing to $e^+e^- \to h + X$, $\ell p \to \ell' + h + X$, and $pp \to h + X$, $pp \to \text{jet}(h) + X$ are universal in the sense that the same functions appear in the factorized expressions for the three reactions. Modern QCD analyses of fragmentation functions "globally" take into account experimental data sets for all three types of processes in order to obtain optimal sets of fragmentation functions.

Electron-positron annihilation has the advantage that there is no hadronic initial state and hence no beam remnant. This is in contrast to $\ell p \to \ell' + h + X$ or $pp \to h + X$, which are affected by hadron remnant contributions associated with the partons of the initial-state hadron(s) which are collaterally involved in the hard lepton-parton or parton-parton collision. On the other hand, $e^+e^- \to h + X$ has little sensitivity to D_g^h and is insensitive to the charge asymmetries $D_q^h - D_{\bar{q}}^h$. These quantities are best constrained in proton–(anti-)proton and electron-proton scattering, respectively. Especially the latter provides an environment that allows the study of the influence of initial-state QCD radiation on the fragmentation process, of the partonic and spin structure of the hadron target, and of the target remnant system. (See Ref. [8] for a comprehensive review of the measurements and models of fragmentation in lepton-hadron scattering).

Moreover, unlike e^+e^- annihilation where $q^2 = s$ is fixed by the collider energy, lepton-hadron scattering has two independent scales, $Q^2 = -q^2$ and the invariant mass squared, $W^2 \approx Q^2(1-x)/x$, of the hadronic final state, which both can vary by several orders of magnitudes for a given CM energy, thus allowing the study of fragmentation in different environments by a single experiment. For example, in photoproduction the exchanged photon is quasi-real ($Q^2 \approx 0$), leading to processes akin to hadron-hadron scattering. In DIS ($Q^2 \gg 1$ GeV^2), using factorization, the hadronic fragments of the struck quark can be directly compared with quark fragmentation in e^+e^- in a suitable frame. Results from lepton-hadron experiments quoted in this report primarily concern fragmentation in the DIS regime. Studies performed by lepton-hadron experiments of fragmentation with photoproduction data containing high transverse momentum jets or particles are also reported, when these are directly comparable to DIS and e^+e^- results.

Fragmentation studies in lepton-hadron collisions are usually performed in one of two frames in which the target hadron and the exchanged boson are collinear. The hadronic center-of-mass frame (HCMS) is defined as the rest system of the exchanged boson and incoming hadron, with the z^*-axis defined along the direction of the exchanged boson. The positive z^* direction defines the so-called current fragmentation region. Fragmentation measurements performed in the HCMS often use the Feynman-x variable $x_F = 2p_z^*/W$, where p_z^* is the longitudinal momentum of the particle in this frame. As W is the invariant mass of the hadronic final state, x_F ranges between -1 and 1.

The Breit system [9, 10] is related to the HCMS by a longitudinal boost such that the time component of q vanishes, i.e, $q = (0, 0, 0, -Q)$. In the parton model, the struck parton then has the longitudinal momentum $Q/2$ which becomes $-Q/2$ after the collision. As compared with the

HCMS, the current fragmentation region of the Breit frame is more closely matched to the partonic scattering process, and is thus appropriate for direct comparisons of fragmentation functions in DIS with those from e^+e^- annihilation. The variable $x_p = 2p^*/Q$, where p^* is the particle's momentum in the current region of the Breit frame, is used at HERA for measurements in the Breit frame, enabling rather direct comparisons of DIS and e^+e^- results.

19.2 Scaling violations and QCD corrections

As mentioned, the coefficient functions for the fragmentation structure functions in $e^+e^- \to h + X$ are amenable to QCD perturbation theory. For each of the structure functions $F_{T,L,A}(x, q^2)$ in Eq. (19.1) (and hence for the total structure function F^h in Eq. (19.2)) the coefficient function has an expansion of the form

$$\begin{aligned} C_{a,i}\left(z, \alpha_{\mathrm{s}}(\mu), \frac{q^2}{\mu^2}\right) \;=\; & (1-\delta_{aL})\,\delta_{iq}\,\delta(1-z) \\ & + \frac{\alpha_{\mathrm{s}}(\mu)}{2\pi}\, c_{a,i}^{(1)}\left(z, \frac{q^2}{\mu^2}\right) + \left(\frac{\alpha_{\mathrm{s}}(\mu)}{2\pi}\right)^2 c_{a,i}^{(2)}\left(z, \frac{q^2}{\mu^2}\right) + \ldots\;, \end{aligned} \tag{19.4}$$

where $a = T, L, A$. At the zeroth order in the strong coupling α_{s} the coefficient functions C_g for gluons vanish, while for (anti-) quarks $C_i = g_i(s)\,\delta(1-z)$ (except for F_L for which the leading contribution is of order α_{s}, as indicated in Eq. (19.4)). Here $g_i(s)$ is the appropriate electroweak coupling. In particular, $g_i(s)$ is proportional to the squared charge of the quark i at $s \ll M_Z^2$, when weak effects can be neglected. The full electroweak prefactors $g_i(s)$ can be found in Ref. [6]. The first-order QCD corrections to the coefficient functions have been calculated in Refs. [11,12], and the second-order terms in [13–15]. Thus, the coefficient functions are known to NNLO, except for F_L. We note that beyond the leading order the coefficient functions, and hence the fragmentation functions, start to depend on the choice of factorization scheme. The standard choice in the literature is the $\overline{\mathrm{MS}}$ scheme.

The simplest parton-model approach would predict scale-independent ('scaling') x-distributions for both the structure function F^h and the parton fragmentation functions D_i^h. Perturbative QCD corrections lead to logarithmic scaling violations via the evolution equations [35]

$$\frac{\partial}{\partial \ln \mu^2}\, D_i^h(x, \mu^2) \;=\; \sum_j \int_x^1 \frac{dz}{z}\, P_{ji}(z, \alpha_{\mathrm{s}}(\mu^2))\, D_j^h\left(\frac{x}{z}, \mu^2\right)\;, \tag{19.5}$$

where the functions $P_{ij}(z, \alpha_{\mathrm{s}}(\mu^2))$ describe the splitting process $i \to j + X$, where parton j carries the longitudinal momentum fraction z of parton i. Note that for fragmentation the relevant splitting functions are P_{ji} (rather than P_{ij} as for the PDFs) since D_j^h represents the fragmentation of the final parton. Usually the system of evolution equations is decomposed into a 2×2 flavor-singlet sector comprising the gluon and the sum of all quark and antiquark fragmentation functions, and scalar ('non-singlet') equations for quark-antiquark and flavor differences.

The splitting functions in Eq. (19.5) have the perturbative expansion

$$P_{ji}(z, \alpha_{\mathrm{s}}) \;=\; \frac{\alpha_{\mathrm{s}}}{2\pi}\, P_{ji}^{(0)}(z) + \left(\frac{\alpha_{\mathrm{s}}}{2\pi}\right)^2 P_{ji}^{(1)}(z) + \left(\frac{\alpha_{\mathrm{s}}}{2\pi}\right)^3 P_{ji}^{(2)}(z) + \ldots\;, \tag{19.6}$$

where the leading-order (LO) functions $P^{(0)}(z)$ [35, 36] are the same as those for the initial-state parton distributions. The next-to-leading order (NLO) corrections $P^{(1)}(z)$ have been calculated in Refs. [37–41] (there are well-known misprints in the journal version of Ref. [38]). Ref. [41] also includes the spin-dependent case. The timelike functions are different from, but related to, their

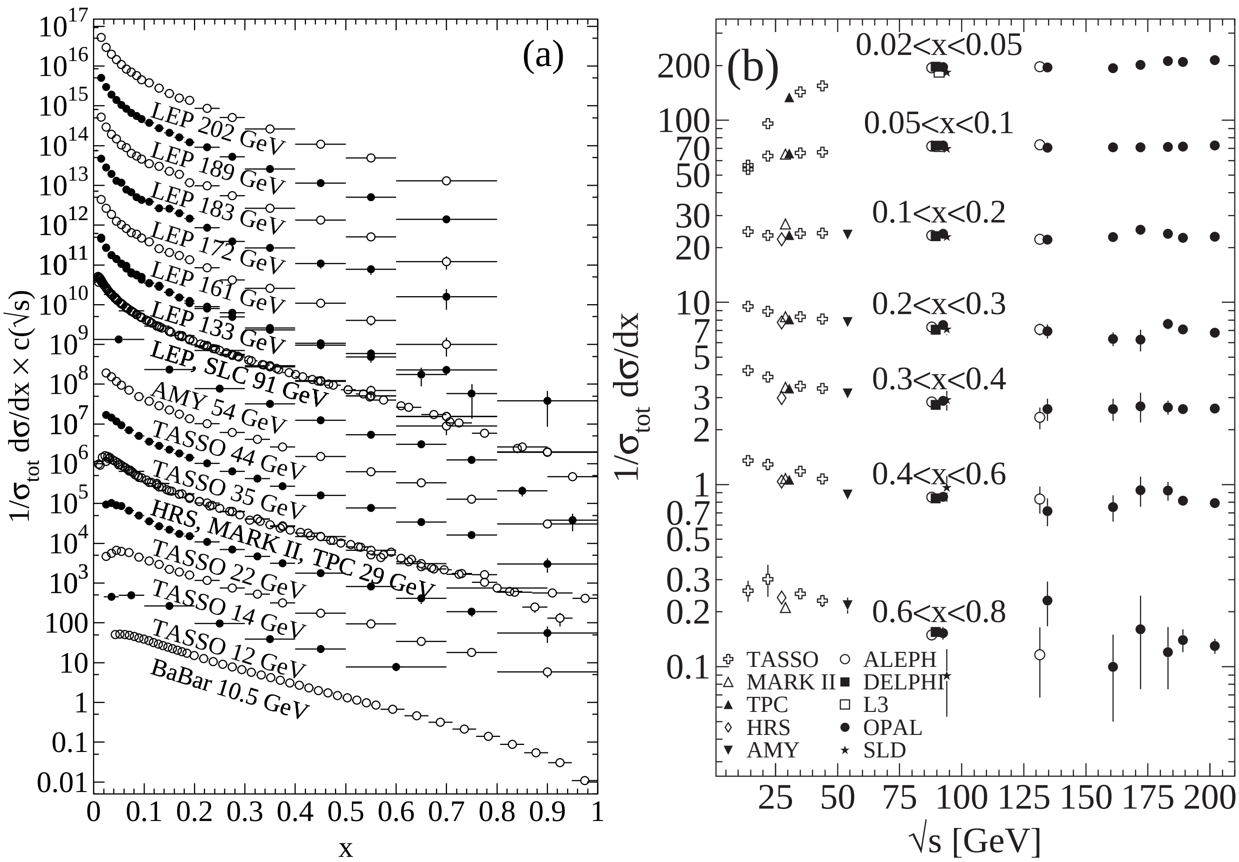

Figure 19.2: Cross section for $e^+e^- \to h + X$ for all charged hadrons [16–34], (a) for different CM energies $\sqrt{s}$ versus x, and (b) for various ranges of x versus $\sqrt{s}$. For (a) the distributions have been scaled by $c(\sqrt{s}) = 10^i$ with i ranging from $i = 0$ ($\sqrt{s} = 12$ GeV) to $i = 13$ ($\sqrt{s} = 202$ GeV).

spacelike counterparts, see also Ref. [42]. The connections between the two sets of functions has facilitated recent calculations of the next-to-next-to-leading order (NNLO) quantities $P_{qq}^{(2)}(z)$ and $P_{gg}^{(2)}(z)$ in Eq. (19.6) [14, 43]. In the same way, the corresponding off-diagonal quantities $P_{qg}^{(2)}$ and $P_{gq}^{(2)}$ were recently obtained in Ref. [44] with the help of constraints from the momentum sum rule Eq. (19.3) [43] and of the limit of $C_A = C_F = n_f$ for which QCD becomes supersymmetric. An uncertainty still remains for the $P_{qg}^{(2)}$ kernel, which however does not affect the logarithmic behavior at small and large momentum fractions. With the exception of Ref. [40], all these higher-order results refer to the standard $\overline{\text{MS}}$ scheme with a fixed number n_f of light flavors. When the threshold for the production of a heavier quark flavor is crossed in the course of the scale evolution, fragmentation functions change. The NLO treatment of these flavor thresholds in the evolution has been addressed in Ref. [45].

The phenomenological effect of scale evolution is similar in the timelike and spacelike cases: As the scale increases, one observes a scaling violation in which the x-distribution is shifted towards lower values. This can be seen from Fig. 19.2 where a set of measurements of the total fragmentation structure function in e^+e^- annihilation are shown. In particular, the figure on the right exhibits the dependence on $\sqrt{q^2} = \sqrt{s}$ at fixed values of x. QCD analyses of these data are discussed in Section 19.5 below.

The NLO coefficient functions for SIDIS, $ep \to e + h + X$, have been presented in Refs. [11, 12]

Corresponding results have also been obtained for the case that a non-vanishing hadron transverse momentum is required in the HCMS frame [46, 47].

Scaling violations in DIS are shown in Fig. 19.3 for both the HCMS and the Breit frames. In Fig. 1.3(a) the distribution in terms of $x_F = 2p_z^*/W$ shows a steeper slope in ep data than for the lower-energy μp data for $x_F > 0.15$, indicating the scaling violations. At smaller values of x_F in the current jet region, the multiplicity of particles substantially increases with W, owing to the increased phase space available for the fragmentation process. The EMC data access both the current region and the region of the fragmenting target remnant system. At higher values of $|x_F|$, due to the extended nature of the remnant, the multiplicity in the target region far exceeds that in the current region. For acceptance reasons the remnant hemisphere of the HCMS is only accessible by the lower-energy fixed-target experiments.

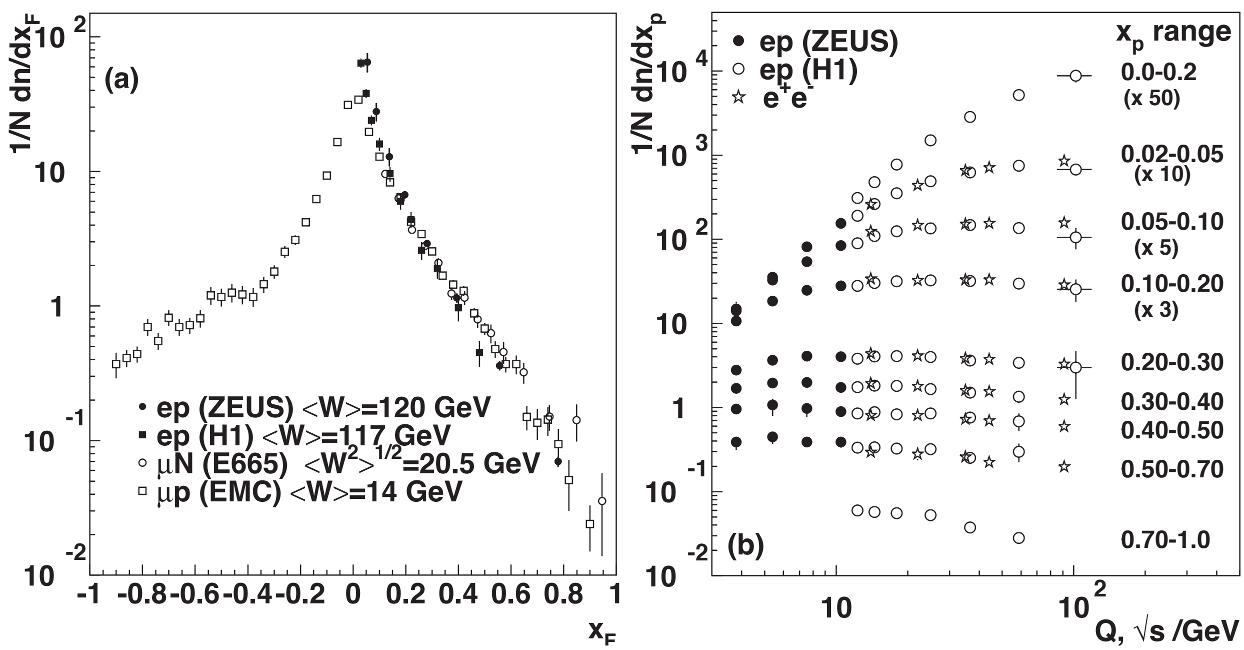

Figure 19.3: (a) The distribution $1/N \cdot dN/dx_F$ for all charged particles in DIS lepton-hadron experiments at different values of W, measured in the HCMS [48–51]. (b) Scaling violations of the fragmentation structure function for all charged particles in the current region of the Breit frame of DIS [52, 53] and in e^+e^- interactions [27, 54]. The data are shown as a function of $\sqrt{s}$ for e^+e^- results, and as a function of Q for the DIS results, each within the same indicated intervals of the scaled momentum x_p. The data for the four lowest intervals of x_p are multiplied by factors 50, 10, 5, and 3, respectively for better visibility.

Using hadrons from the current hemisphere in the Breit frame, measurements of fragmentation functions and the production properties of particles in ep scattering have been reported in Refs. [52, 53, 55–58]. Fig. 19.3(b) compares results from ep scattering and e^+e^- experiments; the latter results have been divided by two as they cover both event hemispheres. The agreement between the DIS and e^+e^- results is fairly good. However, processes in DIS which are not present in e^+e^- annihilation, such as boson-gluon fusion and initial-state QCD radiation, can depopulate the current region. These effects become most prominent at low values of Q and x_p. Hence, when compared with e^+e^- annihilation data at $\sqrt{s} = 5.2$, 6.5 GeV [59] not shown here, the DIS particle rates tend to lie below those observed in e^+e^- annihilation. A ZEUS study [60] finds that the direct comparability of the ep data to e^+e^- results at low scales is improved if twice the energy in the current hemisphere of the Breit frame, $2E_B^{\rm cr}$, is used instead of $Q/2$ as the fragmentation scale. Choosing $2E_B^{\rm cr}$ for the fragmentation scale approximates QCD radiation effects relevant at low scales, as detailed in Ref. [10].

19.3 Fragmentation functions for small particle momenta

The higher-order timelike splitting functions in Eq. (19.6) are singular at small values of x. They show a double-logarithmic enhancement, with leading terms of the form $\alpha_{\rm s}^k(\ln^{2k-2}x)/x$ at the kth order of perturbation theory, corresponding to poles $\alpha_{\rm s}^k\,(N-1)^{1-2k}$ for the Mellin moments

$$P^{(k)}(N) \;=\; \int_0^1 dx\; x^{N-1}\; P^{(k)}(x)\;. \tag{19.7}$$

Despite large cancellations between leading and non-leading logarithms at non-asymptotic values of x, the resulting small-x rise in the timelike splitting functions dwarfs that of their spacelike counterparts for the evolution of the parton distributions in Section 18 of this *Review*, see Fig. 1 of Ref. [43]. Consequently, in fragmentation the fixed-order approximation to the evolution breaks down orders of magnitude earlier in x than in DIS.

The pattern of the known coefficients and other considerations suggest that the double-logarithmic terms sum to all-order expressions without any pole at $N=1$, such as [61, 62]

$$P_{gg}^{\rm LL}(N) \;=\; -\frac{1}{4}\Big(N-1-\sqrt{(N-1)^2\cdot 24\,\alpha_{\rm s}/\pi}\,\Big) \tag{19.8}$$

for the gluon-to-gluon splitting function at leading logarithmic order. Keeping the first three terms in the resulting expansion of Eq. (19.5) around $N=1$ and taking the Mellin inverse yields a Gaussian in the variable $\xi=\ln(1/x)$ for the small-x fragmentation functions,

$$xD(x,q^2=s)\;\propto\;\exp\left[-\frac{1}{2\sigma^2}(\xi-\xi_p)^2\right]\;, \tag{19.9}$$

with the peak position and width varying with the energy as [63] (see also Ref. [2])

$$\xi_p\;\simeq\;\frac{1}{4}\ln\left(\frac{s}{\Lambda^2}\right)\;,\quad \sigma\;\propto\;\left[\ln\left(\frac{s}{\Lambda^2}\right)\right]^{3/4}\;. \tag{19.10}$$

Next-to-leading logarithmic corrections to the above predictions have been calculated [64]. In the method of Ref. [65], see also Refs. [66, 67], the corrections are included in an analytical form known as the 'modified leading logarithmic approximation' (MLLA). Alternatively they can be used to compute higher-moment corrections to the shape in Eq. (19.9) [68]. The small-x resummation of the coefficient functions for semi-inclusive e^+e^- annihilation and of the timelike spitting functions in the standard $\overline{\rm MS}$ scheme was extended in Refs. [69–73] and has reached full next-to-next-to-leading logarithmic accuracy. Applications of these results to gluon and quark jet multiplicities have been presented in Refs. [74].

Fig. 19.4 shows the ξ distribution for charged particles produced in the current region of the Breit frame in DIS and in e^+e^- annihilation. Consistently with Eq. (19.9) (the 'hump backed plateau') and Eq. (19.10) the distributions have a Gaussian shape, with the peak position and area increasing with CM energy (e^+e^-) and Q^2 (DIS).

The predicted energy dependence of the peak in the ξ distribution (see Eq. (19.10)) is explained by soft gluon coherence (angular ordering), *i.e.*, the destructive interference of the color wavefunction of low energy gluon radiation, which correctly predicts the suppression of hadron production at small x. Of course, a decrease at very small x is expected on purely kinematical grounds, but this would occur at particle energies proportional to their masses, *i.e.*, at $x\propto m/\sqrt{s}$ and hence

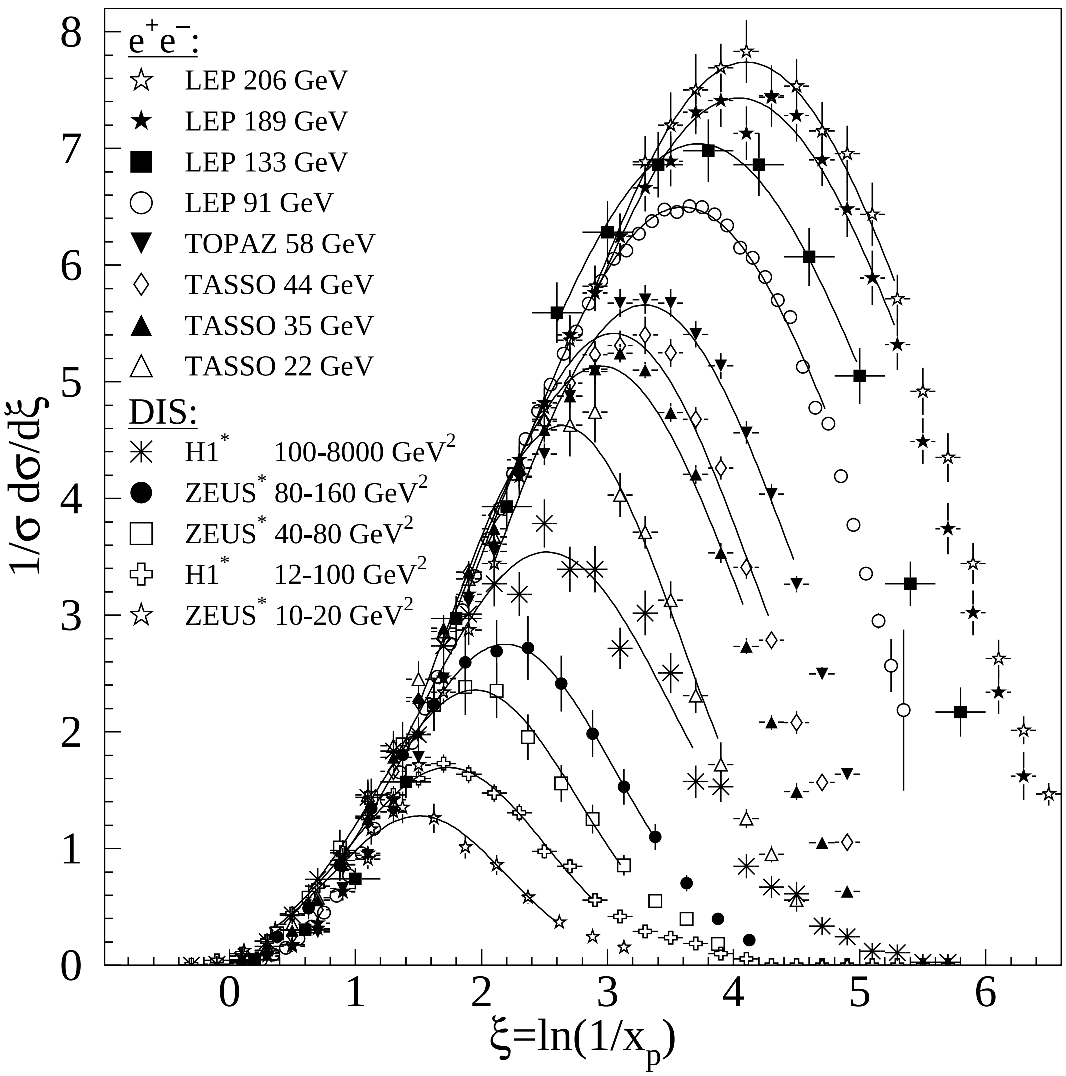

Figure 19.4: Distribution of the normalized fragmentation cross sections in $\xi = \ln(1/x_p)$ at several CM energies (e^+e^-) [18, 19, 24–27, 57, 58, 75–78] and for intervals of Q^2 (DIS). At each energy only one representative measurement is displayed. For clarity some measurements at intermediate CM energies (e^+e^-) or Q^2 ranges (DIS) are not shown. The DIS measurements ($*$) have been scaled by a factor of 2 for direct comparability with the e^+e^- results. Fits of simple Gaussian functions are overlaid for illustration.

$\xi \sim \frac{1}{2} \ln s$. Thus, if the suppression were purely kinematic, the peak position ξ_p would vary twice as rapidly with the energy, which is ruled out by the data in Fig. 19.5. The e^+e^- and DIS data agree well with each other, demonstrating the universality of hadronization and the MLLA prediction. Measurements of the higher moments of the ξ distribution in e^+e^- [27, 78–80] and DIS [58] have also been performed and show consistency with each other.

The average charged-particle multiplicity is another observable sensitive to fragmentation functions for small particle momenta. Perturbative predictions using both NLO [89] and MLLA [90, 91]

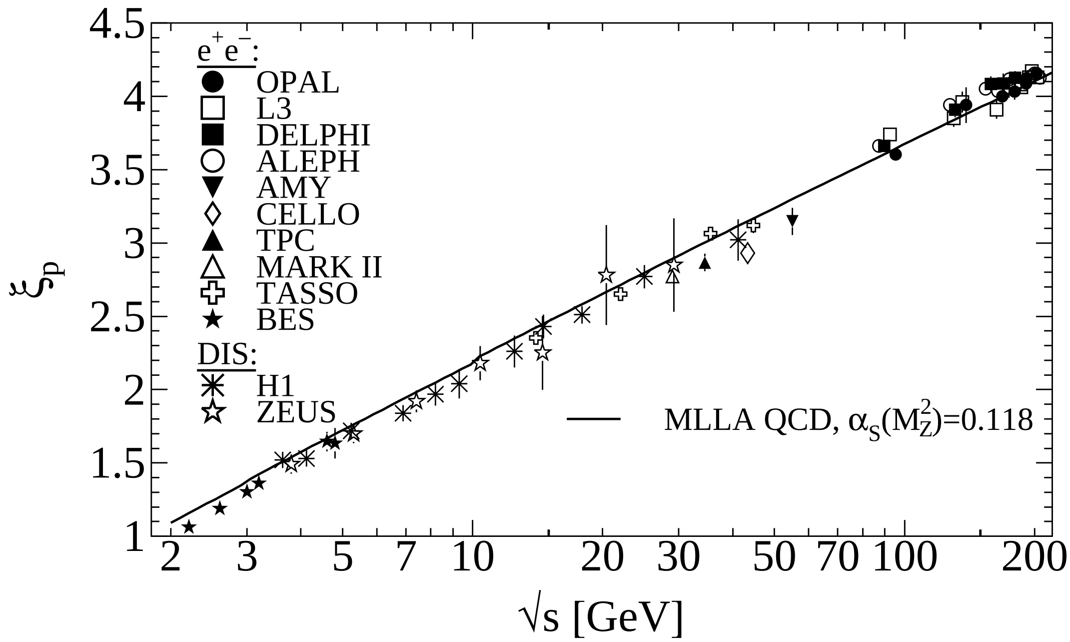

Figure 19.5: Evolution of the peak position, ξ_p, of the ξ distribution with the CM energy $\sqrt{s}$. The MLLA QCD prediction using $\alpha_s(s = M_Z^2) = 0.118$ is superimposed to the data of Refs. [18, 20, 23, 27, 56, 57, 76, 77, 80–88].

have been obtained by solving Eq. (19.5) yielding

$$\left\langle n_G(Q^2)\right\rangle \propto \alpha_s^b(Q^2)\cdot\exp\left[\frac{c}{4\pi b_0\sqrt{\alpha_s(Q^2)}}\cdot\left(1+6a_2\frac{\alpha_s(Q^2)}{\pi}\right)\right], \tag{19.11}$$

where $b = \frac{1}{4} + \frac{10}{27}\frac{n_f}{4\pi b_0}$, $c = \sqrt{96\pi}$, with $b_0 = (33 - 2n_f)/(12\pi)$, cf. Section 9 of this *Review*, for n_f contributing quark flavors. Higher-order corrections to Eq. (19.11) are known up to next-to-next-to-next-to-leading order (N^{3}LO), for details and references see [92]. The term proportional to $a_2 \approx -0.502+0.0421\,n_f-0.00036\,n_f^2$ in Eq. (19.11) is the contribution due to NNLO corrections [93]. The quantity $\langle n_G(Q^2)\rangle$ refers to the average number of gluons, while for $\langle n_q(Q^2)\rangle$ for quarks a correction factor $1/r$ is required due to the different color factors in quark and gluon couplings, so that $\langle n_q(Q^2)\rangle = \langle n_G(Q^2)\rangle/r$. The correction factor depends only weakly on Q^2; higher-order corrections up to N^{3}LO on the asymptotic value $r = C_A/C_F = 9/4$ [94] are quoted in [92].

Employing the hypothesis of 'Local Parton-Hadron Duality' (LPHD) [90], *i.e.*, that the color charge of partons is balanced locally in phase space and, hence, their hadronization occurs locally such that (Mellin transformed) parton and hadron inclusive distributions directly correspond, Eq. (19.11) can be applied to describe average charged particle multiplicities obtained in e^+e^- annihilation. The equation can also be applied to $e^\pm p$ scattering if the current fragmentation region of the Breit frame is considered for measuring the average charged-particle multiplicity. Fig. 19.6 shows corresponding data and fits of Eq. (19.11) where apart from an LPHD normalization factor a constant offset has been allowed for, so that $\langle n_{ch}(Q)\rangle = K_{\rm LHPD}\cdot\langle n_G(Q)\rangle/r + n_0$.

In hadron-hadron collisions beam remnants, e.g. from single-diffractive (SD) scattering where one colliding proton is negligibly deflected while hadrons related with the other colliding proton are well-separated in rapidity from the former proton, contribute to the measurement of the hadron multiplicity from a hard parton-parton scattering, making interpretation of the data more model

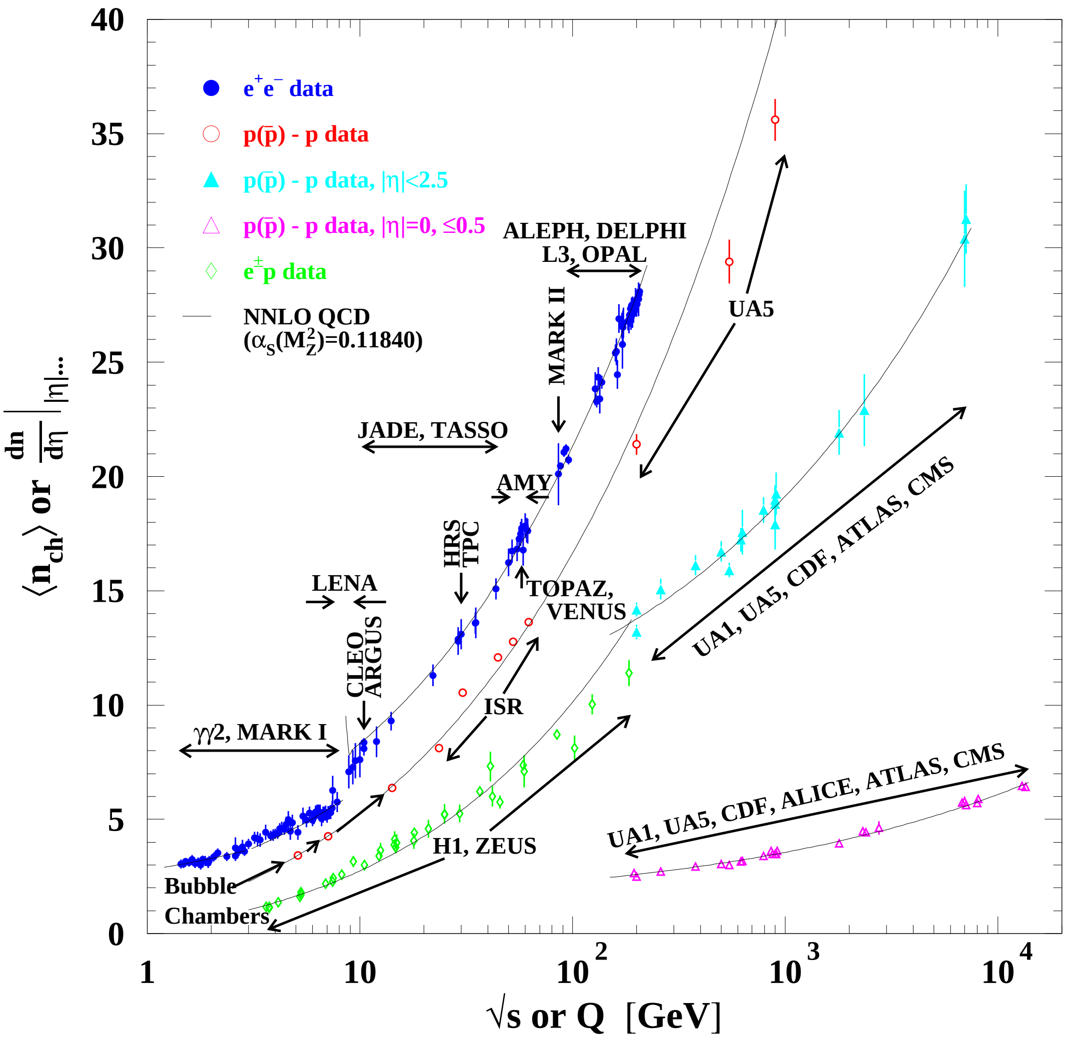

Figure 19.6: Average charged-particle multiplicity $\langle n_{\mathrm{ch}} \rangle$ as a function of $\sqrt{s}$ or Q for e^+e^- and $p\overline{p}$ annihilations, and pp and ep collisions. The indicated errors are statistical and systematic uncertainties added in quadrature, except when no systematic uncertainties are given. All NNLO QCD curves are from Eq. (19.11) with fitted normalization, K_{LHPD}, and offset, n_0, using a fixed $\alpha_{\mathrm{s}}(M_Z^2) = 0.1184$ [95] and for e^+e^- annihilation data $n_f = 3$, 4, or 5 depending on $\sqrt{s}$, else $n_f = 3$. $\mathbf{e^+e^-}$: Contributions from K_S^0 and $\mathit{\Lambda}$ decays included. Data compiled from Refs. [16, 18, 24, 24, 30, 77, 83, 96–106]; $\mathbf{e^\pm p}$: Multiplicities have been measured in the current fragmentation region of the Breit frame. Data compiled from Refs. [53, 57, 58, 60, 107]; $\mathbf{p^{(}\overline{p}^{)}}$: Measured values above 20 GeV refer to non-single diffractive (NSD) processes. Central pseudorapidity multiplicities $(\mathrm{d}n/\mathrm{d}\eta)|_{|\eta|\ldots}$ refer to either $|\eta| < 2.5$ (CMS: $|\eta| < 2.4$) or $|\eta| = 0$ (UA5, CMS, ALICE: $|\eta| < 0.5$). Data compiled from Refs. [108–123].

dependent. Experimental results are usually given for inelastic processes or for non-single diffractive processes (NSD). Due to the large beam particle momenta at Tevatron and LHC, not all final state particles can be detected within the limited detector acceptance. Therefore, experiments at Tevatron and LHC quote particle multiplicities for limited ranges of pseudo-rapidity $\eta = -\ln\tan(\vartheta/2)$ or at central rapidity, i.e. $\eta = 0$, as shown in Fig. 19.6.

A universality of the average particle multiplicities in e^+e^- and $p^{(}\overline{p}^{)}$ processes has been reported in Ref. [124] when considering an effective collision energy $Q_{\rm eff} = \sqrt{s}/k$ in $p\overset{(}{\overline{p}}{}^{)}$ reduced by a factor of $k \approx 3$, plus a constant offset of $n_0 \approx 2$. A more detailed review is available in Ref. [125]. According to the investigations presented in Ref. [126] the universality of the energy dependence of average particle multiplicities also applies to hadron-hadron and nucleus-nucleus collisions for both full and central rapidity multiplicities. Evidence for this universality is given by the good agreement for the energy dependence of Eq. (19.11) when fit to the $p\overset{(}{\overline{p}}{}^{)}$ data as shown in Fig. 19.6.

19.4 Fragmentation models

Although the scaling violations can be calculated perturbatively, the actual form of the parton fragmentation functions is non-perturbative. Perturbative evolution gives rise to a shower of quarks and gluons (partons). Multi-parton final states from leading and higher order matrix element calculations are linked to these parton showers using factorization prescriptions, also called matching schemes, see Ref. [127] for an overview.

Phenomenological schemes are then used to model the carry-over of parton momenta and flavor to the hadrons. Implemented in Monte Carlo event generators (see Section 41 of this *Review*), these schemes have been tuned using e^+e^- data and provide good description of hadron collisions as well, thus providing evidence of the universality of fragmentation. However, e^+e^- mainly fix the quark jet fragmentation while it provides less constraints for modelling the gluon jet fragmentation.

19.5 Phenomenology of quark and gluon fragmentation functions

The fragmentation functions are solutions to the evolution equations Eq. (19.5), but need to be specified at some initial scale μ_0^2 (usually around 1 GeV^2 for light quarks and gluons, and at $m_{\rm Q}^2$ for heavy quarks). A typical parameterization for a given light hadron is [128–136]

$$D_i^h(x,\mu_0^2) = N_i\, x^{\alpha_i}(1-x)^{\beta_i}\left(1+\gamma_i(1-x)^{\delta_i}\right) \ , \tag{19.12}$$

where as indicated the normalization N_i, and the parameters α_i, β_i, γ_i and δ_i depend on the type i of the fragmenting parton. Heavy flavor fragmentation into heavy mesons is discussed in Sec. 19.8 below. The parameters of Eq. (19.12) are obtained by performing global fits to data on various hadron types for different combinations of partons and hadrons in e^+e^-, lepton-hadron and hadron-hadron collisions. We note that the choice of parameterization of the fragmentation functions at the initial scale necessarily introduces a bias since it imposes a certain form of the functions. This bias is largely avoided in neural network approaches which offer a wide flexibility of the initial functions and have recently been applied to fragmentation functions as well [137]. Sets of fragmentation functions are now available for pions, kaons, protons, neutrons, η mesons, Λ baryons, and charged hadrons [129–140]. They are all at NLO level, except for Refs. [137, 139] which have been performed at NNLO level. The latter sets are restricted to the analysis of e^+e^- annihilation data. Recently, data from hadron-hadron collisions have been added in the framework of the neural network approach at NLO accuracy for charged hadrons [141]. It is noteworthy that the NNLO effects lead to an improvement in the theoretical description of the data in e^+e^- annihilation.

Data from e^+e^- annihilation present the cleanest experimental source for the measurement of fragmentation functions, but cannot be used to disentangle quark from antiquark fragmentation. Since the bulk of the e^+e^- annihilation data is obtained at the mass of the Z-boson, where the electroweak couplings are roughly the same for the different partons, it provides the most precise determination of the flavor-singlet combination of quark and antiquark fragmentation functions. Flavor-tagged results [142], distinguishing between the light quark, charm and bottom contributions are of particular value for flavor decomposition, even though those measurements cannot be unambiguously interpreted in perturbative QCD.

The most relevant source for quark-antiquark (and also flavor) separation is provided by SIDIS data. Semi-inclusive measurements are usually performed at much lower scales than for e^+e^-

annihilation. The inclusion of SIDIS data in global fits allows for a wider coverage in the evolution of the fragmentation functions, resulting at the same time in a stringent test of the universality of the distributions. Charged-hadron production data in hadronic collisions also have sensitivity to (anti-)quark fragmentation functions.

The gluon fragmentation function $D_g^h(x)$ can be extracted, in principle, from the longitudinal fragmentation structure function F_L in Eq. (19.2), as the coefficient functions $C_{L,i}$ for quarks and gluons are comparable at order α_s. However at NLO, *i.e.*, including the $\mathcal{O}(\alpha_\mathrm{s}^2)$ coefficient functions $C_{L,i}^{(2)}$ [13], quark fragmentation is dominant in F_L over a large part of the kinematic range, reducing the sensitivity to D_g^h. This distribution could be determined also by analyzing the scale evolution of the fragmentation functions. This possibility is limited by the lack of sufficiently precise data at energy scales away from the Z-resonance and the dominance of the quark contributions at medium and large values of x. In e^+e^- annihilation, D_g^h can also be deduced from the study of three-jet events in which the gluon jet is identified, for example, by tagging the other two jets with heavy quark decays. To leading order, the measured distributions of $x = E_\mathrm{had}/E_\mathrm{jet}$ for particles in gluon jets can be identified directly with the gluon fragmentation function $D_g^h(x)$.

Data for $p\overset{(-)}{p} \to h + X$ provide much more direct constraint on D_g^h. At variance with e^+e^- annihilation and SIDIS, here gluon fragmentation contributes already at the lowest order in the coupling constant. At large $x \gtrsim 0.5$, where information from e^+e^- is sparse, data from hadronic colliders significantly improve extractions of D_g^h [128, 129, 135, 138]. Recent LHC data has been included in the NLO analyses [135, 136] of pion-fragmentation functions; see Sec.(17.7) for more details. Note that these analyses are currently the only ones that 'globally' incorporate available data from all sources, $e^+e^- \to h + X$, $ep \to e'h + X$ and $pp \to h + X$.

We note that recently a 'hybrid' type of high-p_T jet/hadron observable has also been considered both theoretically [143–149] and experimentally [150–157]. It is defined by an identified specific hadron found inside a fully reconstructed jet. This gives rise to a *same-side* hadron-jet momentum correlation that may be addressed using perturbative methods. One of several relevant kinematical variables (see [148] for an overview) is $z_h \equiv (\vec{p}_T^{\,h} \cdot \vec{p}_T^{\,\mathrm{jet}})/(p_T^{\mathrm{jet}})^2$, where $\vec{p}_T^{\,h}$ and $\vec{p}_T^{\,\mathrm{jet}}$ are the transverse momenta of the hadron and the jet, respectively. The observable provides an alternative window on fragmentation functions in a more exclusive setting, enabling novel tests of the universality of fragmentation functions. Varying z_h and/or the hadron species, one can map out the fragmentation functions 'locally' as functions of x. This is in contrast to the single-inclusive observable $pp \to h+X$, which inevitably samples over a broad range of x. Although hadron-in-jet data are not yet routinely included in analyses of fragmentation functions, a 'proof-of-principle' analysis does exist [158] that shows the potential of the observable in providing constraint on fragmentation functions.

A comparison of recent NLO fits of fragmentation functions for $\pi^+ + \pi^-$ obtained by DSS14 [135], AKK08 [129] and NNPDF1.0 [137] is shown in Fig. 19.7. Differences among the functions for these sets are large, especially for the gluon fragmentation function over the full range of x and for the quark functions at large momentum fractions. The differences are even larger for other species of hadrons like kaons and protons [128,129,133,138]. Recent analyses [133,135–137,159,160] estimate the uncertainties involved in the extraction of fragmentation functions.

Photonic fragmentation functions play a relevant role in the theoretical understanding of inclusive photon production in (leptonic and hadronic) high energy processes. In the sprit of the analogy between parton fragmentation functions and parton distribution functions, also photonic fragmentation functions are analogous to the photon structure function F_2^γ and to the proton's photonic parton distributions (see review on structure functions in Section 18 of this *Review*). Since photons have a pointlike coupling to quarks [161], the corresponding fragmentation functions obey inhomogeneous evolution equations and are generally decomposed into a perturbative and a

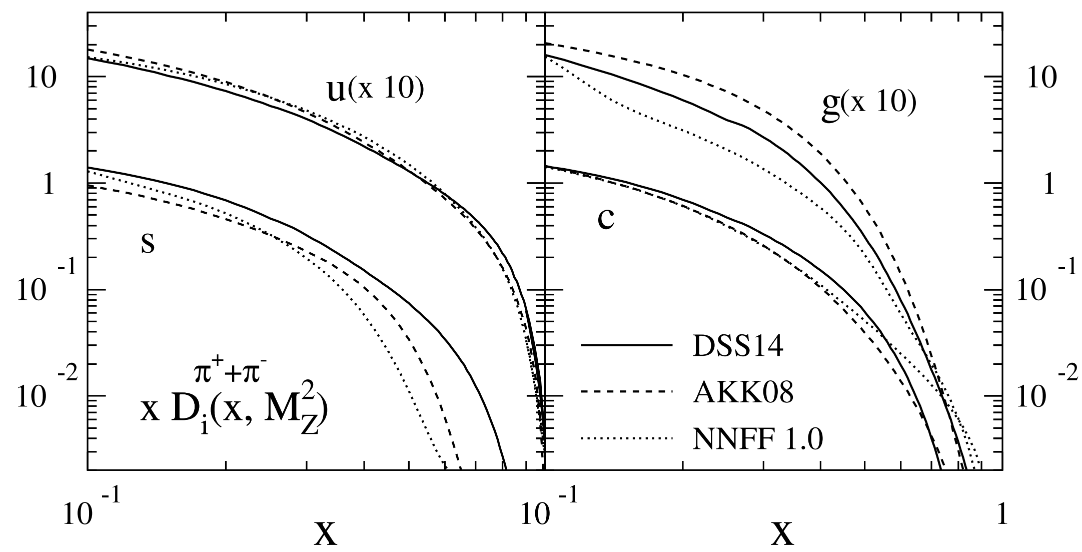

Figure 19.7: Comparison of up, strange, charm and gluon NLO fragmentation functions for $\pi^+ + \pi^-$ at the mass of the Z. The different lines correspond to the results of the analyses performed in Refs. [129, 135, 137].

non-perturbative component [132, 162, 163]. The hadronic part, sometimes approximated by the Vector Meson Dominance Model, can in principle be obtained by performing a global analysis to the available prompt photon production data [7,20,23,27–29,85,164–166], although in practice this has not been done. We note that also the cross section for photons produced in fully reconstructed jets has been proposed [167] as a new tool for obtaining access to photon fragmentation functions, in analogy to the hadron-in-jet cross section discussed above.

19.6 Identified particles in e^+e^- and semi-inclusive DIS

There is a great wealth of measurements of e^+e^- fragmentation into identified particles. A collection of references for data on fragmentation into identified particles is provided in Table 51.1 of this *Review*. As a representative example, Figure 19.8 shows differential charged-hadron spectra as functions of the scaled hadron momentum at several CM energies.

Quantitative results of studies of scaling violations in e^+e^- fragmentation have been reported in [7, 29, 168, 169]. Scaling violations may be used to extract a value of α_s; the values obtained are consistent with the world average (see review on QCD in Section 9 of this *Review*).

Many studies have been made of production of identified particles in lepton-hadron scattering, although fewer particle species have been measured than in e^+e^- collisions. References [170–177] and [178–184] are representative of the data from fixed target and *ep* collider experiments, respectively. QCD calculations performed at NLO provide an overall good description of the HERA data [51,52,58,184–186], both for SIDIS [187] and for the hadron transverse momentum distribution [46, 188] in the kinematic regions in which the calculations are predictive. A first step towards an NNLO calculation for SIDIS has been presented in [189].

Fig. 19.9(a) compares lower-energy fixed-target and HERA data on strangeness production, showing that the HERA spectra have substantially increased multiplicities, albeit with statistical precision that is insufficient to study scaling violations. The fixed-target data show that the Λ rate substantially exceeds the $\overline{\Lambda}$ rate in the remnant region, owing to the conserved baryon number from the baryon target. Fig. 19.9(b) shows $1/N \cdot dn/dz$ for neutral and charged pion production, where z is defined as the ratio of the pion energy to that of the exchanged boson, both measured in

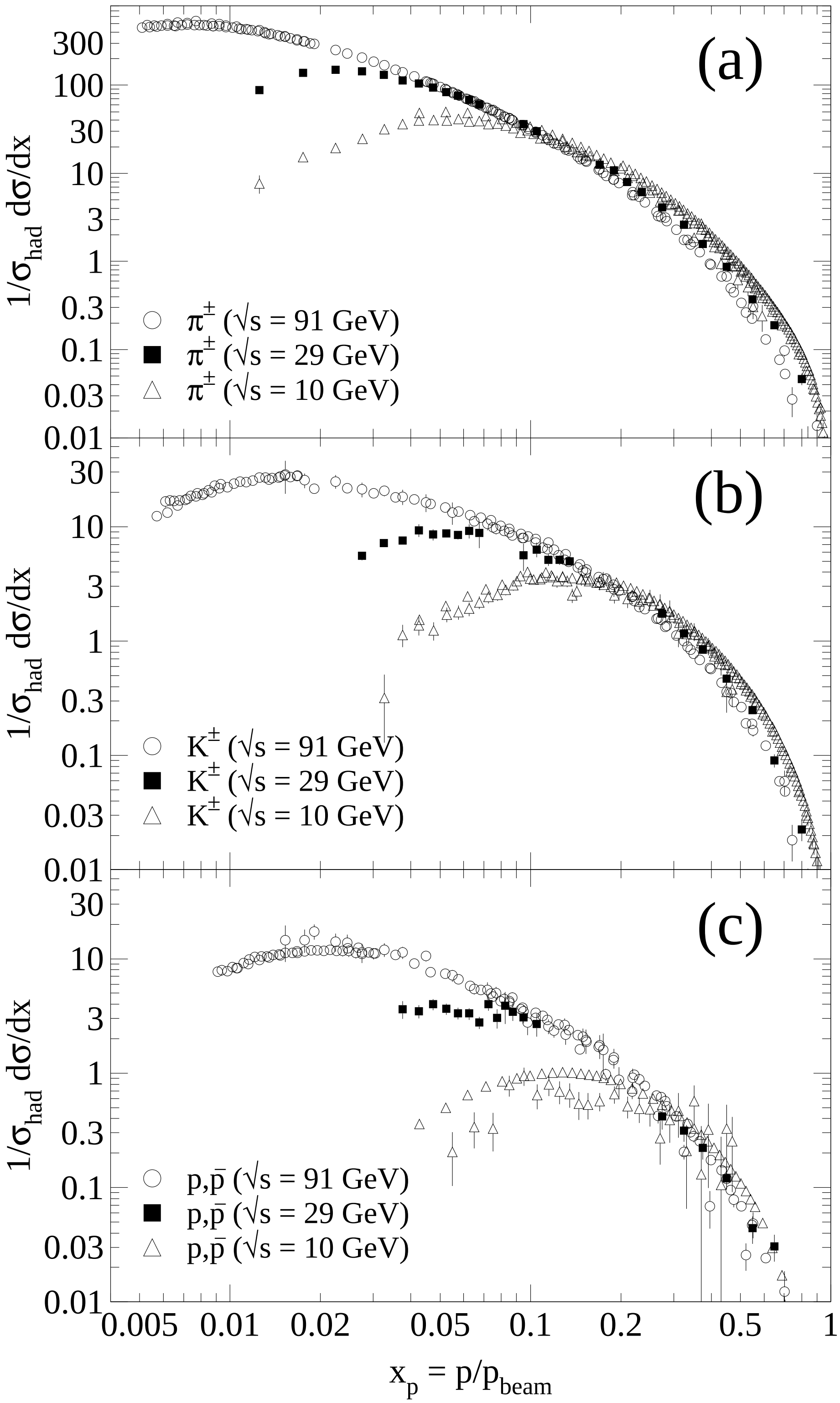

Figure 19.8: Scaled momentum spectra of (a) $\pi^\pm$, (b) $K^\pm$, and (c) $p, \overline{p}$ at $\sqrt{s} = 10$, 29, and 91 GeV [32, 34, 85, 165, 190].

the laboratory frame. Results are shown from the HERMES and the EMC experiments, where the HERMES data have been evolved to $\langle Q^2 \rangle = 25$ GeV^2 at NLO QCD, in order to be comparable with the EMC data. Each of the experiments uses various kinematic cuts to ensure that the measured particles lie in the region that is expected to be associated with the struck quark. In the DIS kinematic regime accessed at these experiments, and over the range in z shown in Fig. 19.9, the z and x_F variables have similar values [48]. The precision data on identified particles can be used in the study of the quark flavor content of the proton [159, 191, 192].

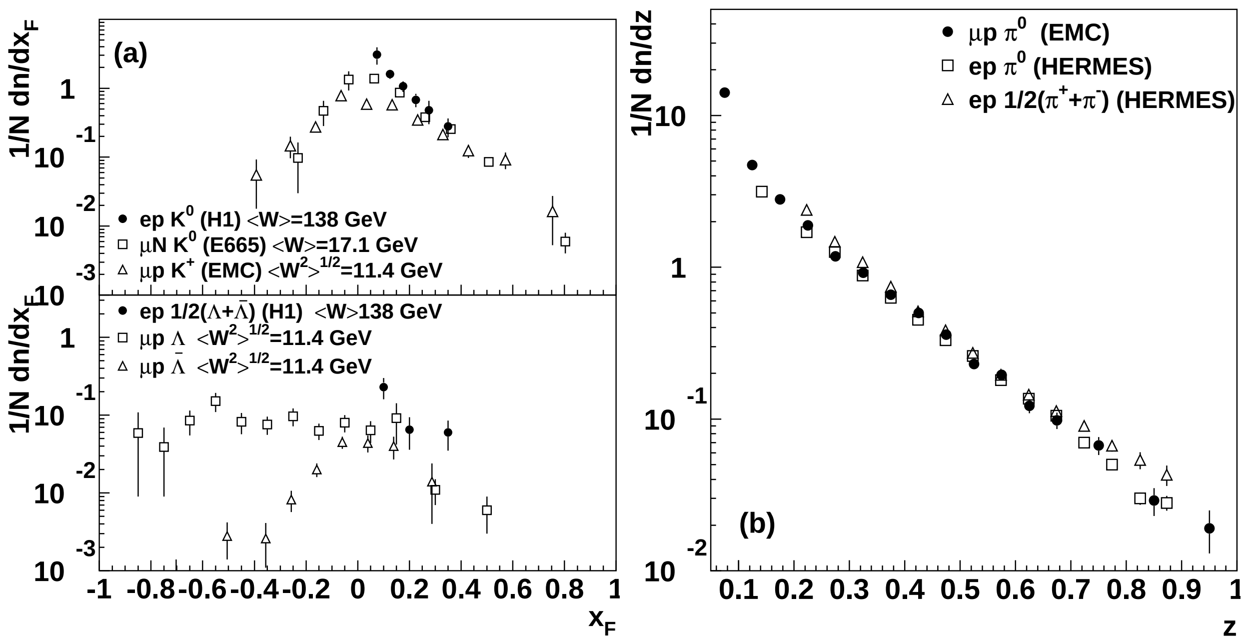

Figure 19.9: (a) $1/N \cdot dn/dx_F$ for identified strange particles in DIS at various values of W [170, 173, 178]. (b) $1/N \cdot dn/dz$ for measurements of pions in fixed-target DIS experiments [171, 174, 177].

Data on identified particle production can aid the investigation of the universality of jet fragmentation in e^+e^- and DIS. The strangeness suppression factor γ_s, as derived principally from tuning the Lund string model [193] within JETSET [194], is typically found to be around 0.3 in e^+e^- experiments [75], although values closer to 0.2 [195] have also been obtained. A number of measurements of so-called V^0-particles (K^0, Λ^0) and the relative rates of V^0's and inclusively produced charged particles have been performed at HERA [178, 179, 196] and fixed target experiments [170]. These typically favour a stronger suppression ($\gamma_s \approx 0.2$) than usually obtained from e^+e^- data, although values close to 0.3 have also been obtained [197, 198].

However, when comparing the description of QCD-based models for lepton-hadron interactions and e^+e^- collisions, it is important to note that the overall description by event generators of inclusively produced hadronic final states is more accurate in e^+e^- collisions than in lepton-hadron interactions [199]. Predictions of particle rates in lepton-hadron scattering are affected by uncertainties in the modelling of the parton composition of the proton and photon, the extended target remnant, and initial and final-state QCD radiation. Furthermore, the tuning of event generators for e^+e^- collisions is typically based on a larger set of parameters and uses more observables [75] than are used when optimizing models for lepton-hadron data [200].

19.7 Fragmentation in hadron-hadron collisions

An extensive set on high-transverse momentum (p_T) single-inclusive hadron data has been collected in $h_1h_2 \to hX$ scattering processes, both at high energy colliders and fixed-target experiments [166, 201–231]. Fig. 19.10 shows the invariant cross sections $E\mathrm{d}^3\sigma/\mathrm{d}p^3$ for a compilation of neutral-pion and charged-hadron production data for energies in the range $\sqrt{s} \approx 23$ - 7000 GeV.

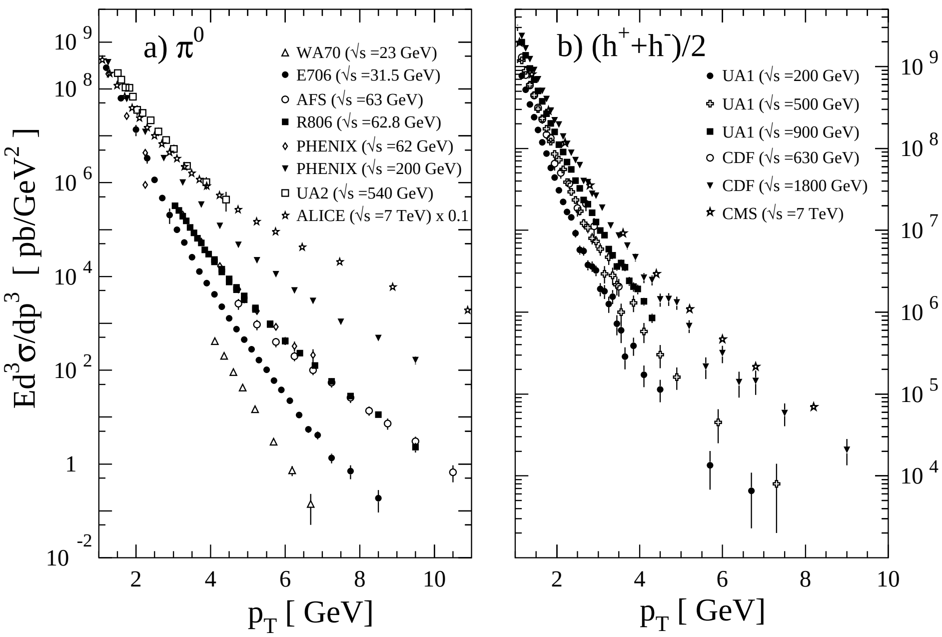

Figure 19.10: Selection of inclusive (a) π^0 and (b) charged-hadron production data from pp [118, 207, 221, 225–228] and $p\bar{p}$ [114, 201, 205] collisions.

The differential cross section for high-transverse momentum hadron production has been computed to NLO accuracy in perturbative QCD [232]. The NLO corrections are typically large and can even double the prediction for the cross section at fixed-target energies. Nevertheless, the NLO calculations significantly under-predict the cross-section for several fixed-target energy data sets [223, 233, 234]. Different strategies have been developed to ameliorate the theoretical description at fixed-target energies. A possible phenomenological approach involves the introduction of a non-perturbative intrinsic partonic transverse momentum [223, 228, 235, 236]. Furthermore, the resummation of the dominant higher order corrections at threshold produces an enhancement of the theoretical calculation that significantly improves the description of the data [237, 238].

Data collected at high energy colliders are either included in global fit analyses or used as a test for the universality of fragmentation functions. A certain tension has been observed between data sets from RHIC and the LHC [239]. The tension can be largely resolved [135] by excluding data with transverse momentum smaller than ~ 5 GeV from the analysis, where fixed-order pQCD calculations are not expected to provide an accurate description of the process. Still, after removing these smaller p_T values where the data sets appear to be mutually exclusive in the global fit, the RHIC data show a preference towards harder gluon fragmentation at large x than the LHC data.

Transverse momentum distributions can usually be fit by power laws [255]. An approach to describe the low p_T particle spectra is the Tsallis distribution [256–258], which is based on a non-extensive generalization of the Boltzmann-Gibbs statistics. The functional form [259]

$$\frac{\mathrm{d}^2N}{\mathrm{d}p_T\mathrm{d}y} \;=\; p_T\frac{\mathrm{d}N}{\mathrm{d}y}\frac{(n-1)(n-2)}{nT(nT+m_0(n-2))}\left[1+\frac{m_T-m_0}{nT}\right]^{-n} \tag{19.13}$$

is frequently used to fit the transverse momentum spectra, where $\mathrm{d}N/\mathrm{d}y$ is the particle's multiplicity, T and n are fit parameters of the Tsallis distribution, m_0 is the either the mass of the most

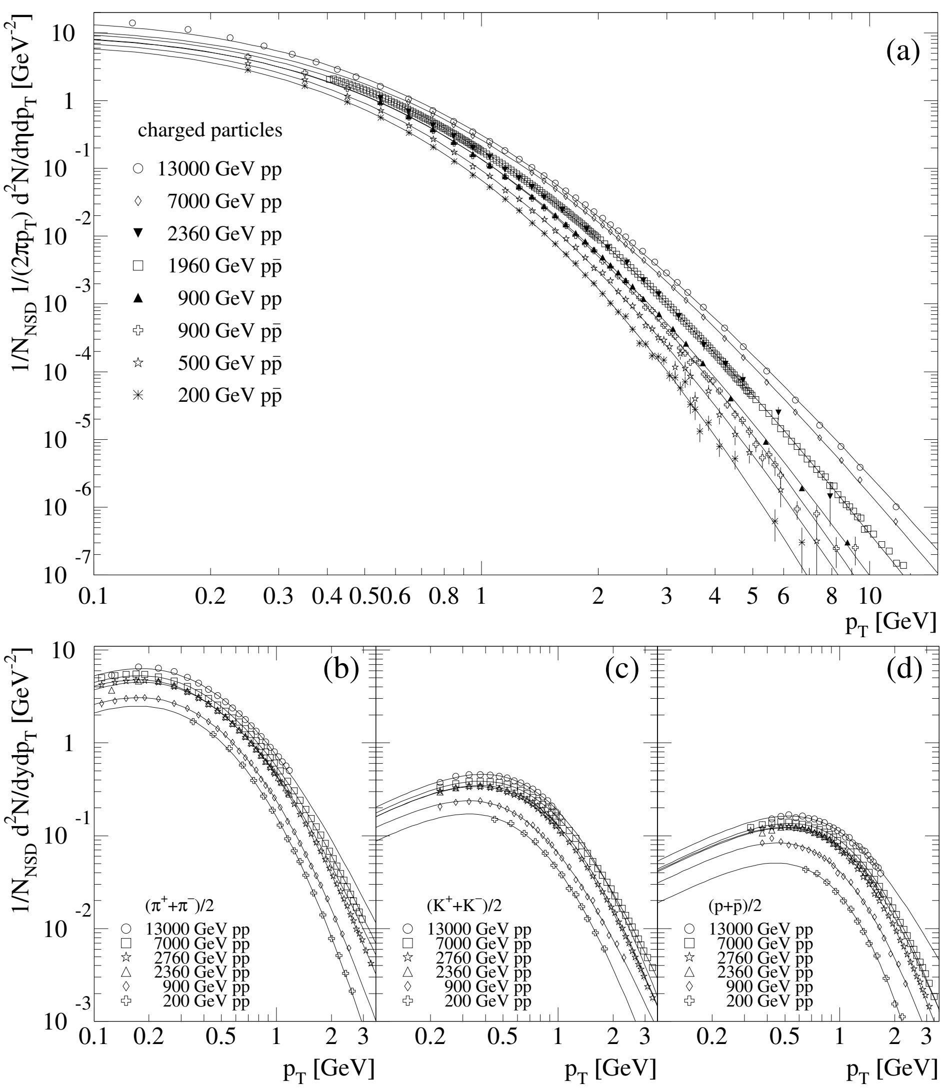

Figure 19.11: (a) Selection of inclusive charged-particle transverse momentum spectra [114, 117, 119,121,201,240,241], normalized to the non-single diffractive cross section (NSD). (b)-(d) Selection of identified charged-particle transverse momentum spectra [242–247] normalized to the NSD cross section. All spectra are scaled to the NSD cross-section using measurements of total, inelastic, elastic, single, or non-single diffractive cross sections from [248–252, 252–254]. The overall normalization uncertainty of about 3% is not shown. Superimposed are fits of the Tsallis distribution in Eq. (19.13).

abundant particle, i.e. the pion for inclusive spectra, or the mass of an identified particle, and $m_T = \sqrt{p_T^2 + m_0^2}$. The parameter n is related to the non-extensive parameter $q = n/(n-1)$ of the original Tsallis formula [260], and T is connected to the temperature in the Boltzmann-Gibbs statistics. The Tsallis distribution has been very successfully fit to measured transverse momentum distributions of both inclusive charged particles and identified particle spectra for hadron-hadron collisions, see for example [261–263], for collisions of heavy nuclei, see for example [264], and also for e^+e^- collisions, see for example [265]. The energy dependence of the fitted Tsallis parameters has also been investigated in detail, see [259, 266]. Fig. 19.11 shows examples of hadron production data in pp and $p\bar{p}$ collisions compared to Tsallis distributions.

Hadron production provides a critical observable for probing the high energy-density matter produced in heavy-ion collisions. Measurements at colliders show a suppression of inclusive hadron yields at high transverse momentum for AA collisions compared to pp scattering, indicating the formation of a dense medium opaque to quark and gluons, see e.g. [267].

19.8 Heavy quark fragmentation

It was recognized very early [268] that a heavy flavored meson should retain a large fraction of the momentum of the primordial heavy quark, and therefore its fragmentation function should be much harder than that of a light hadron. In the limit of a very heavy quark, one expects the fragmentation function for a heavy quark to go into any heavy hadron to be peaked near $x = 1$.

When the heavy quark is produced at a momentum much larger than its mass, one expects important perturbative effects, enhanced by powers of the logarithm of the transverse momentum over the heavy quark mass, to intervene and modify the shape of the fragmentation function. In leading logarithmic order (*i.e.*, including all powers of $\alpha_s \log(m_Q/p_T)$), the total (*i.e.*, summed over all hadron types) perturbative fragmentation function is simply obtained by solving the leading evolution equation for fragmentation functions, Eq. (19.5), with the initial condition due to the finite mass of the heavy quark given by $D_Q(x,\mu^2)|_{\mu^2=m_Q^2} = \delta(1-x)$ and $D_i(x,\mu^2)|_{\mu^2=m_Q^2} = 0$ for $i \neq Q$ (here $D_i(x,\mu^2)$, stands for the probability to produce a heavy quark Q from parton i with a fraction x of the parton momentum).

Several extensions of the leading logarithmic result have appeared in the literature. Next-to-leading-log (NLL) order results for the perturbative heavy quark fragmentation function have been obtained in [269]. The resummation of the dominant logarithmic contributions at large x was performed in [270] to next-to-leading-log accuracy. Fixed-order calculations of the fragmentation function at order α_s^2 in e^+e^- annihilation have appeared in [271] while the initial condition for the perturbative heavy quark fragmentation function has been extended to NNLO in [272].

Inclusion of non-perturbative effects in the calculation of the heavy-quark fragmentation function is done by convoluting the perturbative result with a phenomenological non-perturbative form. This form follows from the simple kinematical consideration that the formation of a hadron by attaching light quarks/anti-quarks to the heavy quark will slightly decelerate the heavy quark. Thus its shape will show a peak that becomes increasingly centered next to $x = 1$ the higher the quark mass. Among the most popular parameterizations we have the following:

$$\text{Peterson } et\ al.\ [273]: \quad D_{\rm np}(x) \propto \frac{1}{x}\left(1-\frac{1}{x}-\frac{\epsilon}{1-x}\right)^{-2}, \tag{19.14}$$

$$\text{Kartvelishvili } et\ al.\ [274]: \quad D_{\rm np}(x) \propto x^{\alpha}(1-x), \tag{19.15}$$

$$\text{Collins \& Spiller } [275]: \quad D_{\rm np}(x) \propto \left(\frac{1-x}{x}+\frac{(2-x)\epsilon_C}{1-x}\right)\times (1+x^2)\times \left(1-\frac{1}{x}-\frac{\epsilon_C}{1-x}\right)^{-2} \tag{19.16}$$

$$\text{Colangelo \& Nason } [276]: \quad D_{\rm np}(x) \propto (1-x)^{\alpha}x^{\beta} \tag{19.17}$$

$$\text{Bowler } [277]: \quad D_{\rm np}(x) \propto x^{-(1+bm_{h,\perp}^2)} \times (1-x)^a \exp\left(-\frac{bm_{h,\perp}^2}{x}\right) \tag{19.18}$$

$$\text{Braaten } et\ al.\ [278]: \quad \text{(see Eqs. (31), (32) in [278])} \tag{19.19}$$

where ϵ, ϵ_C, a, $bm^2_{h,\perp}$, α, and β are non-perturbative parameters that depend on the heavy hadron considered. The parameters entering the non-perturbative forms are fitted together with some model of hard radiation, which can be either a shower Monte Carlo, a leading-log or NLL calculation (which may or may not include Sudakov resummation), or a fixed order calculation. In [271], for example, the Peterson *et al.* [273] ϵ parameter for charm and bottom production is fitted from the measured distributions of Refs. [279, 280] for charm, and of [281] for bottom. If the leading-logarithmic approximation (LLA) is used for the perturbative part, one finds $\epsilon_c \approx 0.05$ and $\epsilon_b \approx 0.006$; if a second order calculation is used one finds $\epsilon_c \approx 0.035$ and $\epsilon_b \approx 0.0033$; if a NLL improved fixed order $\mathcal{O}(\alpha_s^2)$ calculation is used instead of NLO $\mathcal{O}(\alpha_s)$ one finds $\epsilon_c \approx 0.022$ and $\epsilon_b \approx 0.0023$. The larger values found in the LL approximation are consistent with what is obtained in the context of parton shower models [282], as expected. The ϵ parameter for charm and bottom scales roughly with the inverse square of the heavy flavor mass. This behavior can be justified by several arguments [268, 283, 284]. It can be used to relate the non-perturbative parts of the fragmentation functions of charm and bottom quarks [271, 276, 285].

A more conventional approach [286] involves the introduction of a unique set of heavy quark fragmentation functions of non-perturbative nature that obey the usual massless evolution equations in Eq. (19.5). Finite mass terms of the form $(m_Q/p_T)^n$ are kept in the corresponding short distance coefficient function for each scattering process. Within this approach, the initial condition for the perturbative fragmentation function provides the term needed to define the correct subtraction scheme to match the massless limit for the coefficient function (see e.g. [287]). Such an implementation is in line with the variable flavor number scheme introduced for parton distributions functions, as described in Section 18 of this *Review*.

High statistics data for charmed-meson production near the Υ resonance (excluding decay products of B mesons) have been published [288, 289]. They include results for D and D^*, D_s (see also [290, 291]) and Λ_c. Shown in Fig. 19.12(a) are the CLEO and BELLE inclusive cross-sections times branching ratio $\mathcal{B}$, $s\mathcal{B}d\sigma/dx_p$, for the production of D^0 and D^{*+}. The variable x_p approximates the light-cone momentum fraction x, but is not identical to it. The two measurements are consistent with each other.

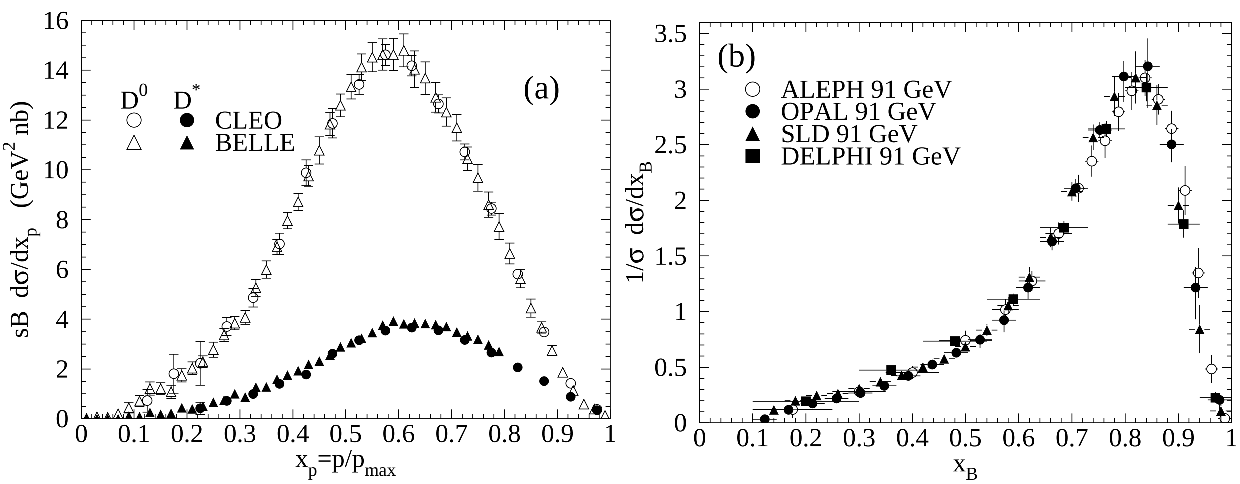

Figure 19.12: (a) Efficiency-corrected inclusive cross-section measurements for the production of D^0 and D^{*+} in e^+e^- measurements at $\sqrt{s} \approx 10.6$ GeV, excluding B decay products [288] [289]. (b) Measured e^+e^- fragmentation function of b quarks into B hadrons at $\sqrt{s} \approx 91$ GeV [292].

The branching ratio $\mathcal{B}$ represents $D^0 \to K^-\pi^+$ for the D^0 results and for the D^{*+} the product of the branching fractions for $D^{*+} \to D^0\pi^+$ and $D^0 \to K^-\pi^+$. Given the high precision of CLEO's and BELLE's data, a superposition of different parametric forms for the non-perturbative contribution is needed to obtain a good fit [45]. Older studies are reported in Refs. [280, 293, 294]. Charmed meson spectra on the Z peak have been published by OPAL and ALEPH [295, 296].

Charm quark production has also been extensively studied at HERA by the H1 and ZEUS collaborations. Measurements have been made of $D^{*\pm}$, $D^\pm$, and $D_s^\pm$ mesons and the Λ_c baryon. See, for example, Refs. [297, 298].

Experimental studies of the fragmentation function for b quarks, shown in Fig. 19.12(b), have been performed at LEP and SLD [281, 292, 299]. Commonly used methods identify the B meson through its semileptonic decay or based upon tracks emerging from the B secondary vertex. Heavy flavor contributions from gluon splitting are usually explicitly removed before fitting for the fragmentation functions. The studies in [292] fit the B spectrum using a Monte Carlo shower model supplemented with non-perturbative fragmentation functions yielding consistent results.

The experiments measure primarily the spectrum of B mesons. This defines a fragmentation function that includes the effect of the decay of higher mass excitations, like the B^* and B^{**}. In the literature (cf. details in Ref. [300]), there is sometimes ambiguity in what is defined to be the bottom fragmentation function. Instead of using what is directly measured (*i.e.*, the B meson spectrum), in some cases corrections are applied to account for B^* or B^{**} production.

Heavy-flavor production in e^+e^- collisions is the primary source of information for the role of fragmentation effects in heavy-flavor production in hadron-hadron and lepton-hadron collisions. The QCD calculations tend to underestimate the data in certain regions of phase space. Some experimental results from LHC summarized in [301] show such deviations e.g. at high transverse jet momentum and also at low di-jet separation angles, see [302] for details, and were already theoretically investigated in [303].

Both bottomed- and charmed-meson spectra have been measured at the Tevatron with unprecedented accuracy [304]. The measured spectra are in good agreement with QCD calculations (including non-perturbative fragmentation effects inferred from e^+e^- data [305]).

The HERA collaborations have produced a number of measurements of beauty production; see, for example, Refs. [297, 306–309]. As for the Tevatron data, the HERA results are described well by QCD-based calculations using fragmentation models optimised with e^+e^- data.

Besides degrading the fragmentation function by gluon radiation, QCD evolution can also generate soft heavy quarks, increasing in the small x region as $\sqrt{s}$ increases. Several theoretical studies are available on the issue of how often $b\bar{b}$ or $c\bar{c}$ pairs are produced indirectly via a gluon splitting mechanism [310–312]. Experimental results from studies on charm and bottom production via gluon splitting, given in [296, 313–317], yield weighted averages of $\overline{n}_{g\to c\bar{c}} = 3.05 \pm 0.45\%$ and $\overline{n}_{g\to b\bar{b}} = 0.277 \pm 0.072\%$, respectively. The production of bottom-antibottom quark pairs via gluon splitting has also been investigated at hadron colliders, see for example [318–320].

19.9 Spin-dependent and transverse-momentum dependent fragmentation functions

The fragmentation functions we have considered so far apply to the spin-averaged case in which the polarization of the produced hadron is not observed, or the hadron has spin-0. We have also only considered 'collinear' fragmentation functions $D_i^h(x, \mu^2)$ which carry only one kinematical variable, the momentum fraction x. New insights into fragmentation and hadronization become available when also the dependence of fragmentation functions on the spin of the produced hadron and/or its relative transverse momentum with respect to the fragmenting parton are considered. In the latter case, one refers to the fragmentation functions as 'transverse-momentum dependent (TMD)' fragmentation functions.

Staying first with collinear fragmentation functions, two types of spin-dependent fragmentation functions to spin-1/2 hadrons can be considered. The helicity-dependent fragmentation function measures the transfer of longitudinal spin from the fragmenting parton to the hadron [12,321–324]. It is given by

$$\Delta D_i^h(x,\mu^2) \equiv D_{i^+}^{h^+}(x,\mu^2) - D_{i^+}^{h^-}(x,\mu^2)\,, \tag{19.20}$$

where the superscripts $\pm$ refer to the helicities of the parton and hadron. Λ hyperons are ideally suited for measurements of the ΔD_i^h, thanks to their self-analyzing weak decay $\Lambda \to \pi p$. Measurements of the longitudinal spin transfer to Lambda hyperons have been presented in e^+e^- (on the Z resonance), ℓp, and pp scattering in Refs. [325–331]. One may readily extend Eq. (19.20) to the case of transverse polarization of hadrons and quarks [332], where the corresponding fragmentation functions are known as 'transversity' fragmentation functions. There are also measurements constraining these fragmentation functions [326, 333, 334].

If the transverse-momentum (k_T) dependence of fragmentation functions is considered, there are eight types of leading-twist functions, defined by the correlations among the hadronic and partonic spin vectors and transverse-momentum vectors they represent. (For review, see [5]). We note that the eight fragmentation functions given in the table below exist separately for each quark and antiquark flavor, and a similar set may be introduced for gluons. Upon integration over the transverse momentum k_T the collinear unpolarized, helicity, and transversity fragmentation functions are reproduced.

	quark polarization		
hadron pol.	unpolarized	long. polarized	transv. polarized
unpol.	D	–	$\frac{\left[\vec{k}_T \times \vec{s}_T\right]}{m_h}\, H^\perp$
long. pol.	–	$\lambda\, \Lambda\, G_L$	$\frac{\vec{k}_T \cdot \vec{s}_T}{m_h}\, \Lambda\, H_L^\perp$
transv. pol.	$\frac{\left[\vec{k}_T \times \vec{S}_T\right]}{m_h}\, D_T^\perp$	$\frac{\vec{k}_T \cdot \vec{S}_T}{m_h}\, \lambda\, G_T^\perp$	$(\vec{s}_T \cdot \vec{S}_T)\, H_T + \frac{\vec{k}_T \cdot \vec{S}_T}{m_h}\frac{\vec{k}_T \cdot \vec{s}_T}{m_h}\, H_T^\perp$

Table 19.1: Classification of spin- and transverse-momentum dependent quark fragmentation functions. For simplicity we have left out the ubiquitous label for flavor i of the fragmenting quark and for hadron species h. Each of the functions carries the argument $(x, x^2k_T^2)$ (plus dependence on a factorization scale), where $xk_T = p_T^h$ is the hadron's transverse momentum. λ and Λ are the quark's and hadron's helicities, respectively, and $\vec{s}_T$ and $\vec{S}_T$ are their transverse spin vectors. We have defined $[\vec{a} \times \vec{b}\,] \equiv a^1b^2 - a^2b^1$. Finally, m_h is the mass of the produced hadron.

The various fragmentation functions may be obtained from spin asymmetries and angular distributions in hadron production processes. There is a large body of precision data by now on transverse-momentum distributions in e^+e^- annihilation [335] and SIDIS [175, 336] that provide constraints on the unpolarized TMD fragmentation functions D_i^h, which have been analyzed theoretically, partly also including TMD evolution effects and high orders of perturbation theory [337–342].

Besides the unpolarized functions D most of the attention in experiment and theory has been on the function $H^\perp$ which describes the production of unpolarized (or spin-0) hadrons by transversely polarized quarks. This function is known as the 'Collins function' [343]. Its importance also derives from the fact that it may be used to probe the quark transversity PDF of the nucleon [344] which gives the probability of finding a transversely polarized quark with its spin aligned or anti-aligned with the spin of a transversely polarized nucleon. The transversity function is chiral-odd, and therefore not accessible through measurements of inclusive lepton-hadron scattering. The Collins effect in semi-inclusive DIS, on the other hand, provides an avenue for accessing transversity. The Collins fragmentation function is chiral-odd and T-odd, leading to a characteristic single-spin asymmetry in the azimuthal angular distribution of the produced hadron in the hadron scattering plane. A number of SIDIS [345–356] and e^+e^- experiments [357–361] have performed measurements of the Collins effect, for charged pions and kaons. These have been analyzed theoretically [362,363], leading to an extraction of the nucleon's transversity distributions [363]. The Collins effect has also been studied in pp scattering, where one considers azimuthal transverse single-spin asymmetries for distributions of hadrons inside jets [157, 364, 365].

In the context of extractions of transversity PDFs also fragmentation functions for same-side pairs of hadrons with small invariant mass, *dihadrons*, have been introduced and studied [366–374]. Compared to the Collins effect, dihadron fragmentation functions have the advantage that they may be defined purely in collinear factorization. The relevant spin-dependent dihadron fragmentation function exploits a correlation between the transverse polarization of the fragmenting quark and the relative momentum of the two hadrons. In SIDIS with a transversely polarized hadron beam, the dihadron cross section then contains a specific modulation in the azimuthal orientation of the plane containing the momenta of the two hadrons. The coefficient of this modulation is a product of the spin-dependent dihadron fragmentation function and the target's transversity PDF. The dihadron fragmentation functions may be separately extracted from measurements in e^+e^- annihiliation, and the Belle experiment has presented data [375] that have been analyzed theoretically [376, 377]. In lepton scattering, HERMES [378] and COMPASS [379, 380] have reported data sensitive to the spin-dependent dihadron fragmentation functions, and recently the STAR experiment at RHIC has presented data in the azimuthal distribution of $\pi^+\pi^-$ pairs produced in pp scattering with one transversely polarized proton [381]. The results have been successfully used for the extraction of transversity PDFs [377, 382–384].

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