Tests of Conservation Laws

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In keeping with the current interest in tests of conservation laws, we collect together a Table of experimental limits on all weak and electromagnetic decays, mass differences, and moments, and on a few reactions, whose observation would violate conservation laws. The Table is given only in the full Review of Particle Physics (RPP), not in the Particle Physics Booklet, and organizes the data in two main sections: "Discrete Space-Time Symmetries", *i.e.*, C, P, T, CP and CPT; and "Number Conservation Laws", *i.e.*, lepton, baryon, flavor and charge conservation. The references for these data can be found in the Particle Listings. The following text discusses the best limits among those included in the Table and gives a brief overview of the current status. For some topics, a more extensive discussion of the framework for theoretical interpretation is provided, particularly where the analogous discussion does not appear elsewhere in the RPP. References to more extensive review articles are also included where appropriate. Unless otherwise specified, all limits quoted in this review are given at a C.L. of 90%.

DISCRETE SPACE-TIME SYMMETRIES

Charge conjugation (C), parity (P) and time reversal (T) are empirically exact symmetries of the electromagnetic (QED) and strong (QCD) interactions, but they are violated by the weak forces. Owing to the left-handed nature of the $SU(2)_L \otimes U(1)_Y$ electroweak theory, C and P are maximally violated in the fermionic couplings of the $W^{\pm}$ and (up to $\sin^2\theta_W$ corrections) the Z. However, their product CP is still an exact symmetry when only one or two fermion families are considered. With three generations of fermions, CP is violated through the single complex phase present in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. An analogous CP-violating (CPV) phase appears in the lepton sector when non-vanishing neutrino masses are taken into account (plus two additional phases if neutrinos are Majorana particles). The product of the three discrete symmetries, CPT, is an exact symmetry of any local and Lorentz-invariant quantum field theory with a positive-definite hermitian Hamiltonian that preserves micro-causality [1, 2]. Therefore, the breaking of CP implies a corresponding violation of T.

Violations of charge-conjugation symmetry have never been observed in electromagnetic and strong phenomena. The most stringent limits are extracted from C-violating transitions of neutral (self-conjugate) particles such as $\mathrm{Br}(\pi^0 \to 3\gamma) < 3.1 \times 10^{-8}$ [3] and $\mathrm{Br}(J/\psi \to 2\gamma) < 2.7 \times 10^{-7}$ [4]. P (and CP) conservation has been also precisely tested through forbidden decays such as $\mathrm{Br}(\eta \to 4\pi^0) < 6.9 \times 10^{-7}$ [5], but the best limits on P and T are set by the non-observation of electric dipole moments (see section 2). Obviously, the interplay of the weak interaction puts a lower bound in sensitivity for this type of tests, beyond which violations of the corresponding conservation laws should be detected.

1 Violations of CP and T

The first evidence of CP non-invariance in particle physics was the observation in 1964 of $K_L^0 \to \pi^+\pi^-$ decays [6]. For many years afterwards, the non-zero ratio

$$|\eta_{+-}| \equiv |\mathcal{M}(K_L^0 \to \pi^+\pi^-)/\mathcal{M}(K_S^0 \to \pi^+\pi^-)| = (2.232 \pm 0.011) \times 10^{-3} \tag{1}$$

could be explained as a K^0–$\overline{K}^0$ mixing effect, $\eta_{+-} = \epsilon$ (superweak CP violation), which would imply an identical ratio $\eta_{00} \equiv \mathcal{M}(K_L^0 \to \pi^0\pi^0)/\mathcal{M}(K_S^0 \to \pi^0\pi^0)$ in the neutral decay mode and successfully predicts the observed CPV semileptonic asymmetry ($A_L(e) \approx 2\,\mathrm{Re}\,\epsilon$)

$$A_L(e) \equiv \frac{\Gamma(K_L^0 \to \pi^- e^+ \nu_e) - \Gamma(K_L^0 \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K_L^0 \to \pi^- e^+ \nu_e) + \Gamma(K_L^0 \to \pi^+ e^- \bar{\nu}_e)} = (3.34 \pm 0.07) \times 10^{-3}\,. \tag{2}$$

A tiny difference between η_{+-} and η_{00} was reported for the first time in 1988 by the CERN NA31 collaboration [7], and later established at the 7.2σ level with the full data samples from the NA31 [8], E731 [9], NA48 [10] and KTeV [11] experiments:

$$\mathrm{Re}(\epsilon'/\epsilon) \;=\; \frac{1}{3}\,\left(1 - |\eta_{00}/\eta_{+-}|\right) \;=\; (1.66 \pm 0.23) \times 10^{-3}\,. \tag{3}$$

This important measurement confirmed that CP violation is associated with a $\Delta S = 1$ transition, as predicted by the CKM mechanism. The Standard Model (SM) prediction, $\mathrm{Re}(\epsilon'/\epsilon) = (1.4 \pm 0.5) \times 10^{-3}$ [12,13], is in good agreement with the measured ratio, although the theoretical uncertainty is unfortunately large.

Much larger CP asymmetries have been later measured in B meson decays, many of them involving the interference between B^0–$\overline{B}^0$ mixing and the decay amplitude. They provide many successful tests of the CKM unitarity structure, validating the SM mechanism of CP violation (see the review on CP violation in the quark sector). Prominent signals of direct CP violation have been also clearly established in several $B^\pm$, B_d^0 and B_s^0 decays, and, very recently, in charm decays [14]:

$$\Delta A_{CP}^{\mathrm{dir}} \equiv A_{CP}^{\mathrm{dir}}(D^0 \to K^+ K^-) - A_{CP}^{\mathrm{dir}}(D^0 \to \pi^+ \pi^-) = (-15.7 \pm 2.9) \times 10^{-4}\,. \tag{4}$$

These direct CP asymmetries necessarily involve the presence of a strong phase-shift difference between (at least) two interfering amplitudes, which makes very challenging to perform reliable SM predictions for heavy-flavored mesons.

Global fits to neutrino oscillation data provide some hints of a non-zero mixing phase [15,16]. Although the statistical significance is not yet compelling, they suggest that CP-violation effects in neutrino oscillations could be large (see the review on neutrino masses, mixings and oscillations). The future DUNE and Hyper-Kamiokande experiments are expected to confirm the presence of CP violation in the lepton sector or constrain the phase in the leptonic mixing matrix to be smaller than $O(10^\circ)$.

While CP violation implies a breaking of time-reversal symmetry, direct tests of T violation are much more difficult. The CPLEAR experiment observed longtime ago a non-zero difference between the oscillation probabilities of $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$ [17]. Initial neutral kaons with defined strangeness were produced from proton-antiproton annihilations at rest, $p\bar{p} \to K^- \pi^+ K^0, K^+ \pi^- \overline{K}^0$, and tagged by the accompanying charged kaon, while the strangeness of the final neutral kaon was identified through its semileptonic decay: $K^0 \to e^+ \pi^- \nu_e$, $\overline{K}^0 \to e^- \pi^+ \bar{\nu}_e$. The average asymmetry over the time interval from 1 to 20 K_S^0 lifetimes was found to be different from zero at 4σ [17]:

$$\frac{R[\overline{K}^0\,(t=0) \to e^+ \pi^- \nu_e\,(t)] - R[K^0\,(t=0) \to e^- \pi^+ \bar{\nu}_e\,(t)]}{R[\overline{K}^0\,(t=0) \to e^+ \pi^- \nu_e\,(t)] + R[K^0\,(t=0) \to e^- \pi^+ \bar{\nu}_e\,(t)]} \;=\; (6.6 \pm 1.3 \pm 1.0) \times 10^{-3}\,. \tag{5}$$

Since this asymmetry violates also CP, its interpretation as direct evidence of T violation requires a detailed analysis of the underlying K^0–$\overline{K}^0$ mixing process [18–20].

More recently, the exchange of initial and final states has been made possible in B decays, taking advantage of the entanglement of the two daughter mesons produced in the decay $\Upsilon(4S) \to B\bar{B}$ which allows for both flavor ($B^0 \to \ell^+ X$, $\overline{B}^0 \to \ell^- X$) and CP ($B_+ \to J/\psi K_L^0$, $B_- \to J/\psi K_S^0$) tagging. Selecting events where one B candidate is reconstructed in a CP eigenstate and the flavor

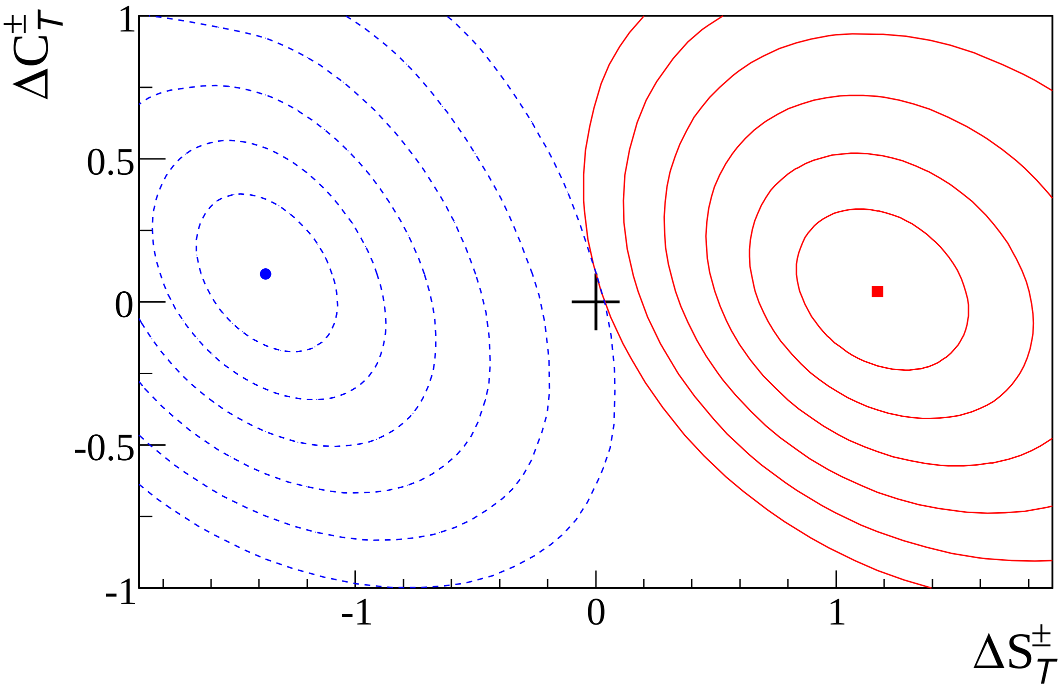

Figure 1: Measured values of ΔS_T^+, ΔC_T^+ (blue point, dashed lines) and ΔS_T^-, ΔC_T^- (red square, solid lines) [24]. The two-dimensional contours correspond to $1 - \mathrm{CL} = 0.317$, 4.55×10^{-2}, 2.70×10^{-3}, 6.33×10^{-5}, 5.73×10^{-7}, and 1.97×10^{-9}. The $+$ sign indicates the T-invariant point.

of the other B is identified, one can compare the rates of the $\overline{B}^0 \to B_\pm$ and $B^0 \to B_\pm$ transitions with their T-reversed $B_\pm \to \overline{B}^0$ and $B_\pm \to B^0$ processes, as a function of the time difference Δt between the two B decays [21–23]. Neglecting the small width difference between the two B_d^0 mass eigenstates, each of these eight transitions has a time-dependent decay rate of the form $\mathrm{e}^{-\Gamma_d \Delta t}\,\{1 + S^{\pm}_{\alpha,\beta}\,\sin{(\Delta m_d \Delta t)} + C^{\pm}_{\alpha,\beta}\,\cos{(\Delta m_d \Delta t)}\}$, where Γ_d is the average decay width, Δm_d the B_d^0 mass difference, the subindices $\alpha = \ell^+, \ell^-$ and $\beta = K_S^0, K_L^0$ stand for the reconstructed final states of the two B mesons and the superindex $+$ or $-$ indicates whether the decay to the flavor final state α occurs before or after the decay to the CP final state β. Figure 1 shows confidence-level contours for the T-asymmetry parameters $\Delta S_T^{\pm} \equiv S^{\mp}_{\ell^-,K_L^0} - S^{\pm}_{\ell^+,K_S^0}$ and $\Delta C_T^{\pm} \equiv C^{\mp}_{\ell^-,K_L^0} - C^{\pm}_{\ell^+,K_S^0}$, reported by the BABAR experiment [24], which clearly demonstrate a violation of T in $\Delta S_T^{\pm}$, with a significance of 14σ.

2 Electric dipole moments

Among the most powerful tests of CP invariance is the search for a permanent electric dipole moment (EDM) of an elementary fermion or non-degenerate quantum system. The EDM of an elementary spin-1/2 fermion f is defined by the effective, non-renormalizable interaction

$$\mathcal{L}_{\mathrm{EDM}} = -\frac{i}{2} d_f\, \bar{f} \sigma_{\mu\nu} \gamma_5 f\, F^{\mu\nu} \tag{6}$$

where $F^{\mu\nu}$ is the QED field strength tensor. The values for d_f are conventionally expressed in units of $e\,$cm. The interaction (6) is separately odd under T and P. In the non-relativistic limit, Eq. (6) reduces to

$$\mathcal{L}_{\mathrm{EDM}} \to d_f\, \chi_f^\dagger \vec{\sigma} \chi \cdot \vec{E} \tag{7}$$

where χ is a two-component Pauli spinor and $\vec{E}$ is the electric field. Note the interaction (7) is manifestly T-odd and carries no direct information on CP. The observation of a non-zero EDM of a non-relativistic (and non-degenerate) quantum system, such as the mercury atom (see below) would imply CP violation under the assumption of CPT invariance.

To date, no experimental observation of an EDM of an elementary particle or non-degenerate bound quantum system has been observed. The most stringent limits have been obtained for the EDMs of the electron, mercury atom, and neutron. A selection of the representative, most stringent limits is given in Table 1. The limits on the electron EDM are inferred from experiments involving polar molecules, paramagnetic systems with an unpaired electron spin. In contrast, the neutron and ^{199}Hg atom are diamagnetic. A variety of experimental efforts aimed at improved sensitivities are underway. For reviews of the experimental and theoretical situation, see, *e.g.* [25–28].

Table 1: Most stringent limits on electric dipole moments.

EDM	Limit ($e\,$cm)		Source
Electron	1.1×10^{-29}	(90% C.L.)	ThO [29]
	1.3×10^{-28}	(90% C.L.)	HfF^+ [30]
Muon	1.9×10^{-19}	(95% C.L.)	[31]
Neutron	3.6×10^{-26}	(95% C.L.)	[32]
^{199}Hg Atom	7.4×10^{-30}	(95% C.L.)	[33]
^{129}Xe Atom	1.5×10^{-27}	(95% C.L.)	[34]

EDMs in the Standard Model

The SM provides two sources of d_f: the CPV phase in the CKM matrix and the P- and T-odd 'θ term' in the QCD Lagrangian. The former is characterized by the Jarlskog invariant [35]

$$\mathcal{J} = \mathrm{Im}(V_{us}V_{cs}^* V_{cb} V_{ub}^*) \sim A^2\lambda^6\eta < 10^{-4}\,, \tag{8}$$

while the latter is given by

$$\mathcal{L}_{\bar\theta} = -\frac{g_3^2}{16\pi^2}\,\bar\theta\;\mathrm{Tr}\left(G^{\mu\nu}\tilde{G}_{\mu\nu}\right), \tag{9}$$

where $G_{\mu\nu}$ ($\tilde{G}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}\,G^{\alpha\beta}/2$) is the QCD field strength tensor (dual).

The CKM-induced EDMs of quarks and charged leptons arise at three- and four-loop orders, respectively [36–39]. The resulting numerical impact for the experimental observables (see below) falls well below present and prospective experimental sensitivities. The most important impact of $\mathcal{J}$ for the EDMs of the neutron and diamagnetic atoms arise via induced hadronic interactions. The resulting theoretical expectations for the electron, neutron and ^{199}Hg EDMs are

$$|d_e|_{\mathrm{CKM}} \approx 10^{-44}\;e\,\mathrm{cm}\;\;[39], \tag{10a}$$

$$|d_n|_{\mathrm{CKM}} \approx (1-6)\times 10^{-32}\;e\,\mathrm{cm}\;\;[40], \tag{10b}$$

$$|d_A(^{199}\mathrm{Hg})|_{\mathrm{CKM}} \lesssim 4\times 10^{-34}\;e\,\mathrm{cm}\;\;[25]. \tag{10c}$$

For d_n and $d_A(^{199}\mathrm{Hg})$, the dominant CKM contributions arise from four-quark operators (generated after integrating out the electroweak gauge bosons) rather than from the EDMs of the individual quarks. The corresponding sensitivities to the QCD $\bar\theta$ parameter are given by

$$|d_n|_{\bar\theta} \approx (0.9-1.2)\times 10^{-16}\;\bar\theta\;e\,\mathrm{cm}\;\;[40], \tag{11a}$$

$$|d_A(^{199}\mathrm{Hg})|_{\bar\theta} \approx (0.07-8)\times 10^{-20}\;\bar\theta\;e\,\mathrm{cm}\;\;[25,26], \tag{11b}$$

where the ranges quoted include the impacts of hadronic, nuclear, and atomic theory uncertainties. The neutron EDM puts then a stringent limit on 'strong' CP violation: $\bar{\theta} \lesssim 3 \times 10^{-10}$. The corresponding limit from $d_A(^{199}\mathrm{Hg})$ is weaker due to the large theoretical uncertainty.

EDMs Beyond the Standard Model

It is possible that the next generation of EDM searches will yield a non-zero result, arising from the $\bar{\theta}$-term interaction and/or physics beyond the SM (BSM). Most of the considered BSM scenarios involve new particles with masses well above the electroweak scale. At energies much lower than the BSM mass scale Λ, the dynamics can be described through an effective field theory (SMEFT) involving an infinite set of non-renormalizable operators $\mathcal{O}_k^{(d)}$, with dimensions $d > 4$, that are invariant under the SM gauge group:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{k,d} \alpha_k^{(d)} \left(\frac{1}{\Lambda}\right)^{d-4} \mathcal{O}_k^{(d)} . \tag{12}$$

The operators contain only SM fields, while all short-distance information on the BSM physics is encoded in their Wilson coefficients $\alpha_k^{(d)}$. The $d = 4$ term corresponds to the SM Lagrangian.

For the systems of Table 1 and for many BSM scenarios of recent interest, it suffices to consider the leading contributions from $d = 6$ operators. Considering only the first-generation SM fermions, there exist 12 independent CPV pertinent operators. For a complete listing, see *e.g.*, Refs. [26, 41]. For a given elementary fermion f, two of these operators reduce to the EDM interaction in Eq. (6). Of the remaining, the most relevant include the chromo-electric dipole moments (cEDMs) of the quarks; a CP-odd three gluon operator; three semileptonic, four-fermion operators; two four-quark operators; and a CPV interaction involving two Higgs fields and a right-handed quark current. For the dipole operators, it is useful to define a rescaled Wilson coefficient $\alpha_{fV_j}^{(6)} \equiv g_j\, C_{fV_j}$, where V_j ($j = 1, 2, 3$) denote the gauge bosons for the three SM gauge groups with corresponding couplings g_j; for all other $d = 6$ operators we correspondingly identify $\alpha_k^{(6)} \equiv C_k$. In this case, one has for the EDM (d_f) and cEDM ($\tilde{d}_q$)

$$d_f = -(1.13 \times 10^{-16}\ e\ \mathrm{cm}) \left(\frac{v}{\Lambda}\right)^2 \mathrm{Im}\ C_{f\gamma}\, , \tag{13a}$$

$$\tilde{d}_q = -(1.13 \times 10^{-16}\ \mathrm{cm}) \left(\frac{v}{\Lambda}\right)^2 \mathrm{Im}\ C_{qG}\, , \tag{13b}$$

with $\mathrm{Im}\ C_{f\gamma} = \mathrm{Im}\ C_{fB} + 3I_3^f\, \mathrm{Im}\ C_{fW}$. As the expressions (13a,13b) illustrate, the magnitude of the BSM contributions scales with two inverse powers of the scale Λ. A similar conclusion holds for the contributions from the other $d = 6$ operators to the EDMs of Table 1.

It is important to emphasize that if the BSM mediators are light, with masses below the weak scale, the effective field theory description of Eq. (12) does not apply. For recent studies along these lines, see, *e.g.* [42, 43].

EDM Interpretation: From Short Distances to the Atomic Scale

The EDM limits in Table 1 are obtained using composite quantum systems, wherein the relevant dynamics involve physics at the hadronic, nuclear, atomic and molecular scales. The manifestation of a given CPV source (CKM, $\bar{\theta}$ term, BSM) involves an interplay of these dynamics. In all cases, one must first evolve the Wilson coefficients from the weak scale to the hadronic scale, then match onto the relevant low-energy degrees of freedom (electrons, nucleons, pions, *etc.*). At this level, the most straightforward interpretation involves the paramagnetic systems, for which two sources dominate: the electron EDM and the electron spin-dependent semileptonic interaction $\bar{e}\gamma_5 e\, \bar{q}q$. The

Table 2: Pertinent dimension-six EDM and cEDM sources (first generation fermions only).

System	$d = 6$ Source	Wilson Coefficient
Paramagnetic	Electron EDM	Im $C_{e\gamma}$
	Electron-quark	$C_{eq}^{(\pm)}$
Diamagnetic	Quark EDM	Im $C_{q\gamma}$
	Quark cEDM	Im C_{qG}
	Three gluon	$C_{\tilde{G}}$
	Four quark	Im $C_{quqd}^{(1,8)}$
	Quark-Higgs	Im $C_{\varphi ud}$
	Electron-quark tensor*	Im $C_{\ell equ}^{(3)}$

*Applicable only to atoms.

latter gives rise to an spin-independent Hamiltonian, for an atom with Z electrons/protons and N neutrons,

$$\hat{H}_S = \frac{iG_F}{\sqrt{2}}\,\delta(\vec{r})\,\left[(Z+N)\,C_S^{(0)} + (Z-N)\,C_S^{(1)}\right]\gamma_0\gamma_5\,, \tag{14}$$

where $C_S^{(0)}$ ($C_S^{(1)}$) is proportional to $C_{eq}^{(+)}$ ($C_{eq}^{(-)}$). The computation of $C_S^{(0,1)}$ is relatively free from theoretical uncertainty since the operator $\bar{q}q$ essentially counts the number of quarks of flavor q in the nucleus. Experimental results for paramagnetic systems, thus, often quote bounds on

$$C_S \equiv C_S^{(0)} + \left(\frac{Z-N}{Z+N}\right) C_S^{(1)} \tag{15}$$

as well as on d_e, assuming only one of these two sources is non-vanishing. Combining results from ThO and HfF^+ (see Figure 2) allows one to obtain the global, 90% C.L. bounds

$$|d_e| < 1.8\times 10^{-28}\ e\ \mathrm{cm}\,, \qquad\qquad |C_S| < 9.8\times 10^{-9}\,. \tag{16}$$

Note that the limits on d_e given in Table 1 have been obtained assuming $C_S = 0$.

For the diamagnetic systems, the situation is considerably more involved. For the neutron, a variety of approaches – including lattice QCD, chiral perturbation theory, QCD sum rules, and the quark model – have been employed to compute the relevant hadronic matrix elements of the CPV sources (see, *e.g.*, [26, 27, 45, 46]). For diamagnetic atoms, the non-leptonic sources of Table 2 give rise to the EDM of the nucleus as well as other P- and T-odd nuclear moments, as allowed by the nuclear spin. However, according to a theorem by Schiff [47], the nuclear EDM generates no contribution to the neutral-atom EDM due to screening by atomic electrons. The leading contribution from these sources, instead, arises via the nuclear Schiff moment, $\vec{S}$, an r^3-weighted moment of the T- and P-odd component of the nuclear charge density. The resulting effective atomic Hamiltonian is

$$\hat{H}_{\mathrm{Schiff}} = -4\pi\,\vec{\nabla}\rho_e(0)\cdot\vec{S}\,, \tag{17}$$

where $\vec{\nabla}\rho_e(0)$ is the gradient of the electron density at the nucleus. To date, computations of the nuclear Schiff moment have assumed that the leading contribution arises from a pion-exchange induced nuclear force, with the P- and T-odd πN interaction given by

$$\mathcal{L}_{\pi N}^{T,P} = \bar{N}\left[\bar{g}_\pi^{(0)}\vec{\tau}\cdot\vec{\pi} + \bar{g}_\pi^{(1)}\pi^0 + \bar{g}_\pi^{(2)}(3\tau_3\pi^0 - \vec{\tau}\cdot\vec{\pi})\right]N\,. \tag{18}$$

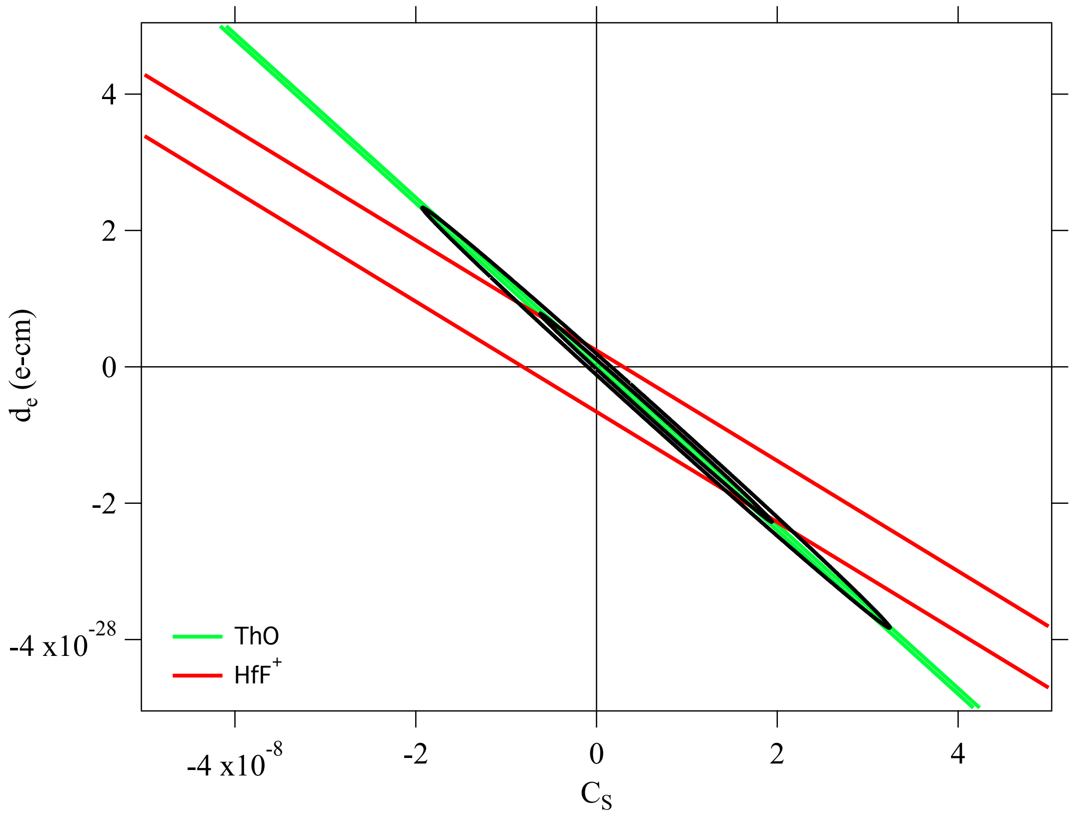

Figure 2: Constraints on d_e and C_S from EDM searches using polar molecules (updated by [44] from Ref. [25]).

Chiral effective field theory power counting implies that in general the magnitude of $\bar{g}_\pi^{(2)}$ is suppressed with respect to the isoscalar and isovector couplings. The CPV sources then generate a diamagnetic atom EDM d_A via the sequence

$$\text{CPV source} \longrightarrow \bar{g}_\pi^{(i)} \longrightarrow \vec{S} \longrightarrow d_A \,. \tag{19}$$

The steps in this sequence involve dynamics at the hadronic, nuclear, and atomic scales, respectively. In addition, d_A may receive contributions from the nuclear spin-dependent interaction generated by the semileptonic tensor interaction listed in Table 2, with the corresponding atomic Hamiltonian

$$\hat{H}_T = \frac{2iG_F}{\sqrt{2}}\,\delta(\vec{r})\,\sum_N \left[C_T^{(0)} + C_T^{(1)}\tau_3\right]\vec{\sigma}_N \cdot \vec{\gamma}\,, \tag{20}$$

where σ_N is the nucleon spin Pauli matrix and $C_T^{(0,1)} \propto \text{Im}\, C_{\ell equ}^{(3)}$.

Given the large number of CPV sources and existing diagmagnetic EDM limits, it is not possible to obtain a set of global constraints on the former. One may, however, do so for the low-energy effective parameters $\bar{g}_\pi^{(0,1)}$, $C_T^{(0,1)}$ and $\bar{d}_n^{\text{sr}}$, where the latter denotes a 'short-range' contribution to the neutron EDM [25, 48]. In this context, the dominant source of theoretical uncertainty involves computations of the nuclear Schiff moment. From the bounds on the low-energy parameters, one may then derive constraints on the CPV sources by utilizing computations of the hadronic matrix elements. Reducing the degree of theoretical hadronic and nuclear physics uncertainty is an area of active effort.

3 Tests of CPT

CPT symmetry implies the equality of the masses and widths of a particle and its antiparticle. The most constraining limits are extracted from the neutral kaons [49, 50]:

$$2\,\frac{|m_{K^0}-m_{\overline{K}^0}|}{(m_{K^0}+m_{\overline{K}^0})}\;<\;6\times10^{-19}\,,\qquad 2\,\frac{|\Gamma_{K^0}-\Gamma_{\overline{K}^0}|}{(\Gamma_{K^0}+\Gamma_{\overline{K}^0})}\;=\;(8\pm8)\times10^{-18}\,.\tag{21}$$

The limit on the K^0–$\overline{K}^0$ mass difference assumes that there is no other source of CPT violation. An upper bound on CPT breaking in $K^0_L\to2\pi$ has been also set through the measured phase difference of the CPV ratios η_{00} and η_{+-}, $\phi_{00}-\phi_{+-}=(0.34\pm0.32)^\circ$, thanks to the small value of $(1-|\eta_{00}/\eta_{+-}|)$ (see the review on CP violation in K^0_L decays).

The measured masses and electric charges of the electron, the proton and their antiparticles provide also strong limits on CPT violation [51–53]:

$$2\,\frac{|m_{e^+}-m_{e^-}|}{m_{e^+}+m_{e^-}}\;<\;8\times10^{-9}\,,\qquad \frac{|q_{e^+}+q_{e^-}|}{e}\;<\;4\times10^{-8}\,,\qquad \left|\frac{q_{\bar p}/m_{\bar p}}{q_p/m_p}\right|-1\;=\;(0.1\pm6.9)\times10^{-11}\,.\tag{22}$$

Worth mentioning are also the tight constraints derived from the lepton and antilepton magnetic moments [54, 55],

$$2\,\frac{g_{e^+}-g_{e^-}}{g_{e^+}+g_{e^-}}\;=\;(-0.5\pm2.1)\times10^{-12}\,,\qquad 2\,\frac{g_{\mu^+}-g_{\mu^-}}{g_{\mu^+}+g_{\mu^-}}\;=\;(-0.11\pm0.12)\times10^{-8}\,,\tag{23}$$

those of the proton and antiproton [56],

$$2\left|\frac{g_{\bar p}-g_p}{g_{\bar p}+g_p}\right|\;<\;6\times10^{-12}\qquad(95\%\ \text{C.L.})\,,\tag{24}$$

and the recent measurement of the 1S-2S atomic transition in antihydrogen which agrees with the corresponding frequency spectral line in hydrogen at a relative precision of 2×10^{-12} [57].

A violation of CPT in an interacting local quantum field theory would imply that Lorentz symmetry is also violated [58]. Signatures of Lorentz-invariance violation have been searched for with atomic clocks, penning traps, matter and antimatter spectroscopy, colliders and astroparticle experiments, with so far negative results [59]. A compilation of experimental bounds is given in Ref. [60], parametrized through the coefficients of the so-called Standard Model Extension (SME) Lagrangian which contains all possible Lorentz- and CPT-violating operators preserving gauge invariance, renormalizability, locality and observer causality [61].

QUANTUM-NUMBER CONSERVATION LAWS

Conservation laws of several quantum numbers have been empirically established with a very high degree of confidence. They are usually associated with some global phase symmetry. However, while some of them are deeply rooted in basic principles such as gauge invariance (charge conservation; local symmetry implies global symmetry) or Lorentz symmetry (fermion number conservation), others appear to be accidental symmetries of the SM Lagrangian and could be broken by new physics interactions.

In fact, if one only assumes the SM gauge symmetries and particle content, the most general dynamics at energies below the BSM mass scale is described by the SMEFT Lagrangian in Eq. (12). All $d=4$ operators (*i.e.*, the SM) happen to preserve the B and L quantum numbers, but this is nolonger true for the gauge-invariant structures of higher dimensionality. There is only one operator with $d=5$ (up to hermitian conjugation and flavor assignments), and it violates lepton number by two units [62], giving rise to Majorana neutrino masses after the electroweak spontaneous symmetry breaking. With $d = 6$, there are five operators that violate B and L [63, 64]. Thus, violations of these quantum numbers can be generically expected, unless there is an explicit symmetry protecting them.

4 Electric charge

The conservation of electric charges is associated with the QED gauge symmetry. The most precise tests are the non-observation of the decays $e \to \nu_e \gamma$ (lifetime larger than 6.6×10^{28} yr [65]) and $n \to p \nu_e \bar{\nu}_e$ (Br $< 8 \times 10^{-27}$, 68% C.L. [66]). The neutrality of matter can be also interpreted as a test of electric charge conservation. Worth mentioning are the experimental limits on the electric charge of the neutron, $q_n/e = (-0.2 \pm 0.8) \times 10^{-21}$, and on the sum of the proton and electron charges, $|q_p + q_e|/e < 1 \times 10^{-21}$ (68% CL) [67].

The isotropy of the cosmic microwave background has been used to set stringent limits on a possible charge asymmetry of the Universe [68]. Assuming that charge asymmetries produced by different particles are not anticorrelated, this implies upper bounds on the photon ($|q_\gamma|/e < 1 \times 10^{-35}$) and neutrino ($|q_\nu|/e < 4 \times 10^{-35}$) electric charges. A much stronger upper bound on the photon charge ($|q_\gamma|/e < 1 \times 10^{-46}$) has been derived from the non-observation of Aharonov-Bohm phase differences in interferometric experiments with photons that have traversed cosmological distances, under the assumption that both positive and negative charged photons exist [69].

5 Lepton family numbers

In the SM with massless left-handed neutrinos there is a separate conservation number for each lepton family. However, neutrino oscillations show that neutrinos have tiny masses and there are sizable mixings among the different lepton flavors. Compelling evidence from solar, atmospheric, accelerator and reactor neutrino experiments has established a quite solid pattern of neutrino mass differences and mixing angles [15, 16] (see the review on neutrino masses, mixings and oscillations). Nevertheless, flavor mixing among the different charged leptons has never been observed.

If neutrino masses and mixings among the three active neutrinos were the only sources of lepton-flavor violation (LFV), neutrinoless transitions from one charged lepton flavor to another would be heavily suppressed by powers of m_{ν_i} (GIM mechanism), leading to un-observably small rates; for instance [70–75],

$$\mathrm{Br}(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_i U^*_{\mu i} U_{ei} \, \frac{m^2_{\nu_i} - m^2_{\nu_1}}{M^2_W} \right|^2 < 10^{-54} \,, \tag{25}$$

where U_{ia} are the relevant elements of the PMNS mixing matrix. This contribution is clearly too small to be observed in any realistic experiment, so any experimentally accessible effect would arise from BSM physics with sources of LFV not related to m_{ν_i}. The search for charged LFV (CLFV) remains an area of active interest, which has the potential to probe physics at scales much higher than the TeV.

Among the most sensitive probes are searches for the CLFV decays of the muon, $\mu \to e\gamma$ and $\mu \to 3e$, as well as the conversion process $\mu^- + A(N, Z) \to e^- + A(N, Z)$, where $A(N, Z)$ denotes a nucleus with N neutrons and Z protons. Searches for rare τ decays such as $\tau \to \ell\gamma$ ($\ell = e, \mu$) also provide interesting probes of CLFV. A variety of BSM scenarios predict that rates for these CLFV processes could be sufficiently large to be observed in the present or planned searches. To date, no observation has been reported, and the resulting null results place strong constraints on BSM scenarios. For extensive reviews of the experimental and theoretical status and prospects, see Refs. [76–78].

A detailed set of upper bounds on CLFV branching rations is given in the listings for the muon and tau leptons. Here we emphasize those with the strongest limits:

$$\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13} \quad [79], \qquad \mathrm{Br}(\mu \to 3e) < 1.0 \times 10^{-12} \quad [80] \tag{26}$$

and

$$B_{\mu\to e} \equiv \frac{\Gamma(\mu^- + A(N,Z) \to e^- + A(N,Z)}{\Gamma(\mu^- + A(N,Z) \to \nu + A(N+1, Z-1)} \tag{27}$$

with the best limit so far, $B_{\mu\to e} < 7\times 10^{-13}$ [81], obtained with gold. Several proposed experiments aim to improve these limits by several orders of magnitude with different atoms.

One may interpret both $\mu \to e\gamma$ and $\mu \to e$ conversion in terms of the amplitudes to emit a real or virtual photon:

$$\mathcal{M}_{\mu\to e\gamma^{(*)}} = eG_\mu \varepsilon^{\alpha *}\bar{e}(p-q)\Big[\Big(q^2\gamma_\alpha - \not{q} q_\alpha\Big)\Big(\tilde{A}_1^R P_R + \tilde{A}_1^L P_L\Big) \tag{28a}$$

$$+ i m_\mu \sigma_{\alpha\beta} q^\beta \Big(\tilde{A}_2^R P_R + \tilde{A}_2^L P_L\Big)\Big]\mu(p)\,, \tag{28b}$$

where it is conventional to normalize the amplitude to the Fermi constant. One then has

$$\mathrm{Br}(\mu \to e\gamma) = 48\pi^3\alpha \left(|\tilde{A}_2^R|^2 + |\tilde{A}_2^L|^2\right)\,. \tag{29}$$

For the conversion process, the virtual photon is absorbed by the quarks in the nucleus, yielding an effective four-fermion operator. In general, the exchange of other particles could lead to similar or alternate Lorentz structures, and it is not possible to distinguish between the exchange of a virtual photon or other particle. It is conventional to write the most general four-fermion amplitude, valid for energies below the electroweak scale as (adapted from Ref. [82])

$$\mathcal{M}_{\mu\to e} = G_\mu \sum_{n,a,q} a_{a,q}^{(n)}\, \bar{e}\Gamma^n P_a \mu\, \bar{q}\Gamma_n q\,, \tag{30}$$

where P_a $(a = L, R)$ denote the left and right-handed projectors and Γ^n denotes 1, γ_5, γ^μ, $\gamma^\mu\gamma_5$, and $\sigma_{\mu\nu}$. If any of the coefficients $a_{a,q}^{(n)}$ are generated by physics at a scale $\Lambda > v$, then their effects would be encoded in the SMEFT Lagrangian (12). For scenarios in which the leading CLFV operators occur at $d = 6$, the $a_{a,q}^{(n)}$ will scale as $(v/\Lambda)^2$. The corresponding decay and conversion rates will then scale as $(v/\Lambda)^4$. Note that the scalar and time component of the vector interactions are coherent over the nucleus, essentially counting the number of quarks. Consequently, these interactions typically yield the greatest sensitivities to high BSM mass scales.

It is sometimes convenient to compare the relative sensitivities of the decay and conversion processes using the following simplified effective Lagrangian [77]:

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{CLFV}} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\,\bar{\mu}_R\sigma_{\mu\nu}e_L\,F^{\mu\nu} + \frac{\kappa}{(\kappa+1)\Lambda^2}\,\bar{\mu}\gamma_\mu e \sum_q \bar{q}\gamma^\mu q + \mathrm{h.c.}\,. \tag{31a}$$

Note that one may replace the second term in Eq. (31a) by any one of the other four-fermion interactions given in Eq. (30). An analogous expression applies to the process $\mu \to 3e$ when replacing the sum over quarks by the corresponding electron bilinear. A comparison of the present and prospective sensitivities for various muon CLFV searches in this framework is shown in Figure 3.

Stringent limits have been also set on the LFV decay modes of the τ lepton [84]. As shown in Figure 4, the large τ data samples collected at the B factories have made possible to reach a 10^{-8} sensitivity for many of its leptonic ($\tau \to \ell\gamma$, $\tau \to \ell'\ell^+\ell^-$) and semileptonic ($\tau \to \ell P^0$, $\tau \to \ell V^0$, $\tau \to \ell P^0 P^0$, $\tau \to \ell P^+ P'^-$) neutrinoless LFV decays, and BELLE-II is expected to push these

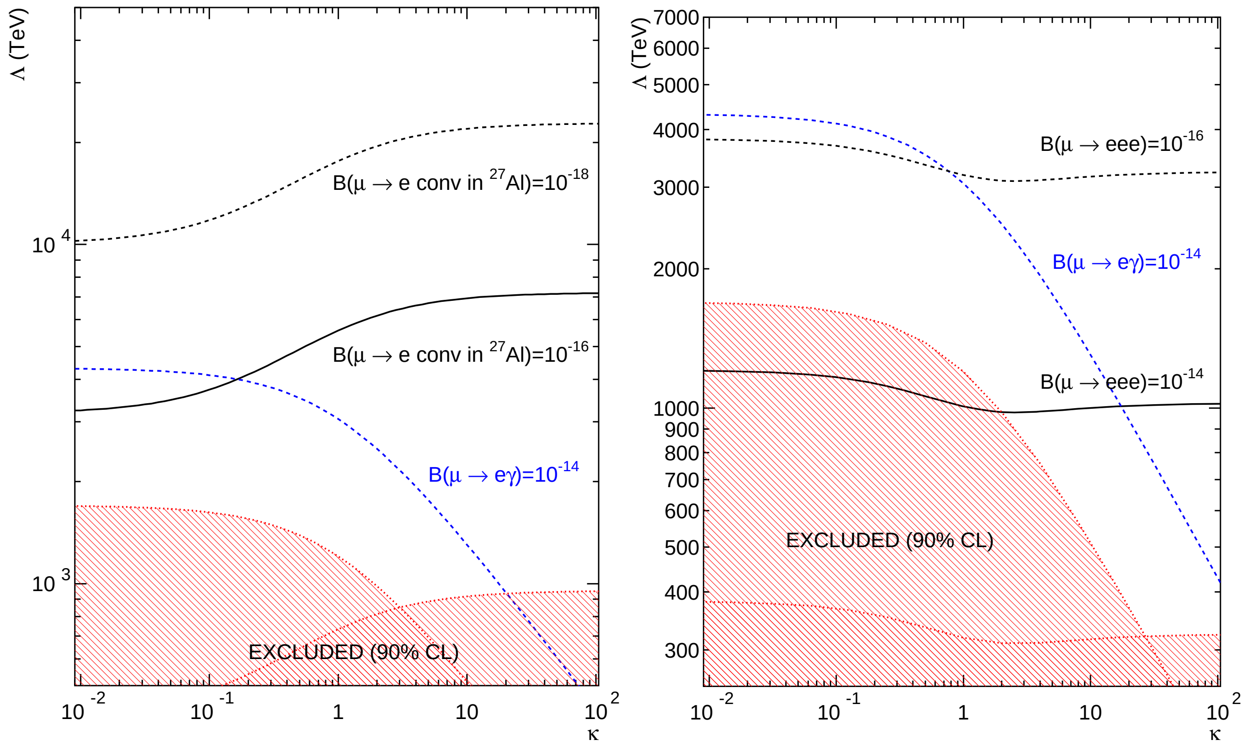

Figure 3: Model-independent CLFV sensitivities based on Eq (31a). Left panel shows the comparison of present constraints with prospective future sensitivities for $\mu \to e\gamma$ and $\mu \to e$ conversion. Right panel gives analogous comparison for $\mu \to e\gamma$ and $\mu \to 3e$. Updated by [83] from Ref. [77].

limits beyond the 10^{-9} level [85]. Being a third generation lepton, the τ could be more sensitive to heavier new-physics scales, which makes his LFV decays particularly interesting. Compared to the muon, the τ decay amplitudes could be enhanced by a chirality ratio $(m_\tau/m_\mu)^2 \sim 280$ and/or by lepton-mixing factors such as $|U_{\tau 3}/U_{e3}|^2 \sim 20$, but the exact relation is model dependent. In any case, the τ LFV decays provide a rich data set that is very complementary to the μ bounds. If LFV is finally observed, the correlations between μ and τ data, and among different LFV τ decays will allow to probe the underlying mechanism of lepton flavor breaking.

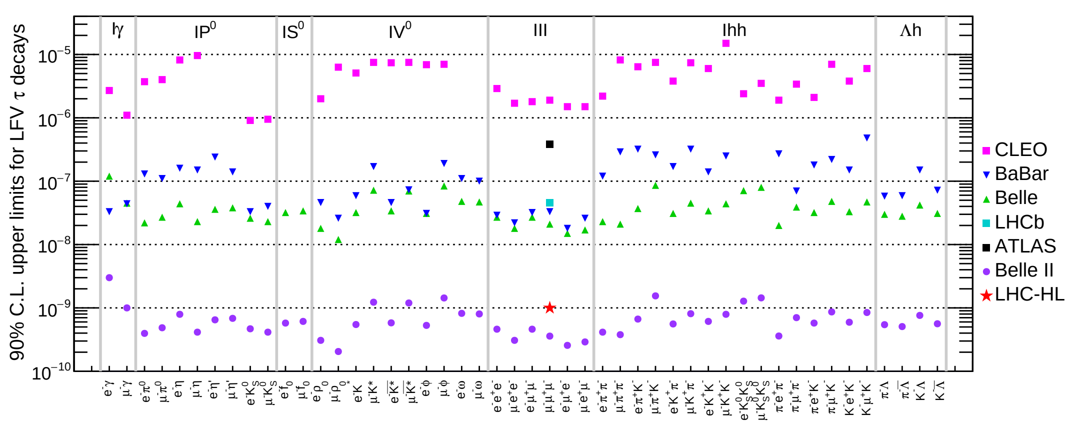

Figure 4: Current experimental limits on neutrinoless LFV τ decays [86]. Also shown are the future projections at Belle-II [85] and at the HL-LHC [87].

Interesting limits on LFV are also obtained in meson decays. The best bounds come from kaon experiments, e.g., $\mathrm{Br}(K_L^0 \to e^\pm \mu^\mp) < 4.7 \times 10^{-12}$ [88], $\mathrm{Br}(K^+ \to \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11}$ [89]. Quite strong limits have also been set in decays of B and D mesons, the best upper bounds being $\mathrm{Br}(B^0 \to e^\pm \mu^\mp) < 1.0 \times 10^{-9}$ [90] and $\mathrm{Br}(D^0 \to e^\pm \mu^\mp) < 1.3 \times 10^{-8}$ [91].

The LFV decays of the Z boson were probed at LEP at the 10^{-5} to 10^{-6} level. The LHC ATLAS collaboration has put recently a stronger bound on the $Z \to e^\pm \mu^\mp$ decay mode [92]. Currently, the best (95% C.L.) limits are [92–94]:

$$\mathrm{Br}(Z \to e^\pm \mu^\mp) < 7.5 \times 10^{-7}\,, \qquad \mathrm{Br}(Z \to e^\pm \tau^\mp) < 9.8 \times 10^{-6}\,, \qquad \mathrm{Br}(Z \to \mu^\pm \tau^\mp) < 1.2 \times 10^{-5}\,. \tag{32}$$

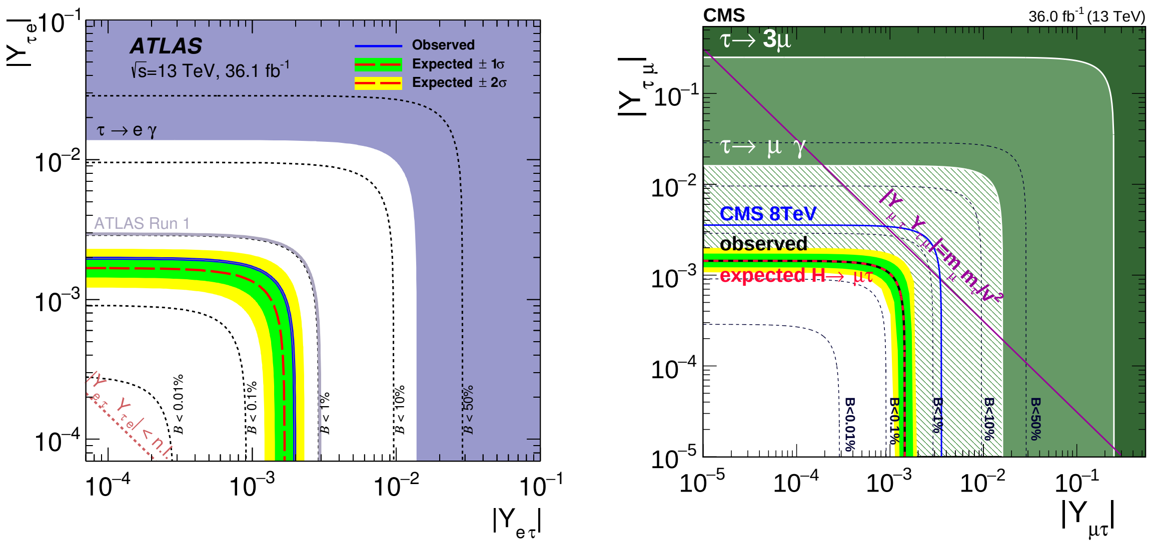

Figure 5: Current limits on the Higgs LFV τ Yukawas from direct $H^0 \to \ell^\pm \tau^\mp$ decays ($\ell = e, \mu$), and indirect constraints from τ decays [95, 96].

LHC is now starting to test LFV in Higgs decays, within the available statistics. From the current (95% C.L.) experimental upper bounds [95–97],

$$\mathrm{Br}(H^0 \to e^\pm \mu^\mp) < 6.1 \times 10^{-5}\,, \qquad \mathrm{Br}(H^0 \to e^\pm \tau^\mp) < 0.47\%\,, \qquad \mathrm{Br}(H^0 \to \mu^\pm \tau^\mp) < 0.25\%\,, \tag{33}$$

one can derive direct limits on the LFV Yukawa couplings of the Higgs boson,

$$\mathcal{L}_Y = -H^0 \sum_{i \neq j} \left(Y_{\ell_i \ell_j}\; \bar{\ell}^i_L \ell^j_R + \mathrm{h.c.} \right) . \tag{34}$$

From $H^0 \to e^\pm \mu^\mp$, one obtains $\sqrt{Y_{\mu e}^2 + Y_{e\mu}^2} < 2.2 \times 10^{-4}$, which is not yet competitive with the indirect limit set by $\mu \to e\gamma$ through a (one-loop) virtual Higgs exchange:

$$\sqrt{Y_{\mu e}^2 + Y_{e\mu}^2} < 3.6 \times 10^{-6}\,. \tag{35}$$

However, the LHC data provides at present the strongest bounds on the LFV τ Yukawas [95, 96]:

$$\sqrt{Y_{e\tau}^2 + Y_{\tau e}^2} < 2.0 \times 10^{-3}\,, \qquad \sqrt{Y_{\mu\tau}^2 + Y_{\tau\mu}^2} < 1.4 \times 10^{-3}\,. \tag{36}$$

Figure 5 compares the Higgs exclusion limits on the τ Yukawas with the current indirect constraints from LFV τ decays.

6 Baryon and Lepton Number

The transitions discussed in the previous section preserve the total lepton number $L = L_e + L_\mu + L_\tau$. In the SM, conservation of $B - L$ is an accidental symmetry of the Lagrangian. At the classical level, $B + L$ is also conserved, though it is violated at the loop level by the anomaly. The latter is a topological effect that is highly suppressed at zero temperature and, moreover, does not contribute to the processes discussed in the review. Going beyond renormalizable interactions, there exists a tower of operators in the SMEFT Lagrangian (12), containing only SM fields, that break one or both of these symmetries. We briefly review these possibilities in turn.

Lepton Number

The lowest-dimension operator containing only SM fields that breaks baryon or lepton number is the $d = 5$, lepton-number-violating (LNV) 'Weinberg' neutrino-mass operator [62]:

$$\mathcal{L}^{\rm LNV} = \frac{y}{\Lambda}\, \bar{L}^C H H^T L\,. \tag{37}$$

When the neutral component of the Higgs field obtains its vacuum expectation value, this $\Delta L = 2$ interaction yields a Majorana mass for the light, active neutrinos. The most comprehensive approach for probing this effect is the search for neutrinoless double-beta decay ($0\nu\beta\beta$) of atomic nuclei, $(Z, A) \to (Z + 2, A) + e^- + e^-$ [98,99] (see the review on neutrinoless double-β decay). The detection of a non-zero $0\nu\beta\beta$ signal could represent a spectacular evidence of Majorana neutrinos. The current best limit, $\tau_{1/2} > 1.07 \times 10^{26}$ yr, was obtained by the KamLAND-Zen experiment with ^{136}Xe [100].

Theoretically, the interaction (37) can arise from BSM interactions in the well-known see-saw mechanism for neutrino mass (for a review, see [101]). In this context, the conventional choice for the scale Λ is of order the GUT scale, yielding light neutrino masses of order eV and below when the couplings y are of order the charged elementary fermion Yukawa couplings. BSM theories may also give rise to LNV observables in other contexts. In these scenarios, if the LNV scale is of order 1 TeV, one may observe signatures of LNV not only in $0\nu\beta\beta$ but also in collider searches for final states containing same sign dileptons. Searches for same sign dileptons plus a di-jet pair at the LHC have placed constraints on TeV-scale LNV [102, 103] that in some cases complement those obtained from $0\nu\beta\beta$.

Stringent constraints on violations of L have been also set in $\mu^- \to e^+$ conversion in muonic atoms, the best limit being $\sigma(\mu^-\text{Ti} \to e^+\text{Ca})/\sigma(\mu^-\text{Ti} \to \text{all}) < 3.6 \times 10^{-11}$ [104], and at the flavor factories through L-violating decays of the τ lepton and K, D and B mesons. Some representative examples are $\text{Br}(\tau^- \to e^+\pi^-\pi^-) < 2.0 \times 10^{-8}$ [105], $\text{Br}(K^+ \to \pi^-\mu^+\mu^+) < 4.2 \times 10^{-11}$ [106], $\text{Br}(D^+ \to \pi^-\mu^+\mu^+) < 2.2 \times 10^{-8}$ [107] and $\text{Br}(B^+ \to K^-e^+e^+) < 3.0 \times 10^{-8}$ [108]. All these $|\Delta L| = 2$ processes could be mediated by a massive Majorana neutrino. They provide useful bounds on the effective Majorana neutrino mass matrix $m_{\ell\ell'} \sim \sum_i U_{\ell i} U_{\ell' i} m_{\nu_i}$ [109], although not as strong as the $0\nu\beta\beta$ constraint on m_{ee}.

Baryon Number

Grand Unified Theories (GUTs) combine leptons and quarks in the same symmetry multiplets and, therefore, predict the violation of the baryon and lepton quantum numbers. Many experiments have searched for B-violating transitions, but no positive signal has been identified so far. Proton decay would be the most relevant violation of B, as it would imply the unstability of matter. The current lower bound on the proton lifetime is 2.1×10^{29} yr [110]. Stronger limits have been set for particular decay modes, such as $\tau(p \to e^+\pi^0) > 1.6 \times 10^{34}$ yr [111]. For a discussion of proton decay in the context of GUTs, see the review on Grand Unified Theories.

Another spectacular signal would be neutron-antineutron oscillations. Searches have been performed for quasi-free n–$\bar{n}$ oscillations and for $n\bar{n}$ annihilation products in a nucleus. The latter would arise when the $\bar{n}$ produced through oscillations annihilates with another neutron in the nuclear medium. The corresponding best limits, expressed in terms of the free and bound oscillation times, $\tau_{n\bar{n}}$ and τ_m, respectively, are:

$$\tau_{n\bar{n}} > 0.86 \times 10^8 \text{ s} \quad [112], \tag{38a}$$

$$\tau_m > 1.9 \times 10^{32} \text{ yr} \quad [113]. \tag{38b}$$

From the latter, one may infer a bound $\tau_{n\bar{n}} > 2.7 \times 10^8$ s, as discussed below. See Ref. [114] for a recent review.

The theoretical interpretation of these bounds starts with an assumed, effective Hamiltonian for the free (anti-)neutron, $\mathcal{H}_{\text{eff}}$ that contains a B-violating part, yielding matrix elements

$$\langle n|\mathcal{H}_{\text{eff}}|n\rangle = \langle \bar{n}|\mathcal{H}_{\text{eff}}|\bar{n}\rangle = m - i\frac{\lambda}{2}\,, \tag{39a}$$

$$\langle n|\mathcal{H}_{\text{eff}}|\bar{n}\rangle = \langle \bar{n}|\mathcal{H}_{\text{eff}}|n\rangle \equiv \delta m\,, \tag{39b}$$

where CPT is assumed to be conserved, the neutron lifetime $\tau_n = 1/\lambda$ and $\tau_{n\bar{n}} = 1/|\delta m|$. The rate for a neutron to oscillate into an antineutron after a time t is given by

$$\mathcal{P}_{n\bar{n}}(t) = \sin^2\left(\frac{t}{\tau_{n\bar{n}}}\right) e^{-\lambda t}\,. \tag{40}$$

For $t << \tau_n << \tau_{n\bar{n}}$, one has

$$\mathcal{P}_{n\bar{n}}(t) \to (t/\tau_{n\bar{n}})^2\,. \tag{41}$$

In realistic experiments, there exist effects, such as background magnetic fields, that split the energies of the neutron and antineutron. One must ensure that the observation time is sufficiently short so that these effects do not overwhelm the small B-violating term δm and that Eq. (40) applies.

In nuclei, the interactions of neutrons and antineutrons with the surrounding medium are sufficiently distinct that one must take the corresponding matter potentials into account. In particular, the matrix elements in Eq. (39a) become

$$\langle n|\mathcal{H}_{\text{eff}}|n\rangle = m + V_n\,, \qquad \langle \bar{n}|\mathcal{H}_{\text{eff}}|\bar{n}\rangle = m + V_{\bar{n}}\,, \tag{42}$$

with V_n being essentially real ($V_n \equiv V_{nR}$) and $V_{\bar{n}} = V_{\bar{n}R} - iV_{\bar{n}I}$. The imaginary part $V_{\bar{n}I}$ characterizes the annihilation of the antineutron with bound nucleons into secondary hadrons. The rate for a bound neutron to disappear is given by

$$\Gamma_m = \frac{2(\delta m)^2|V_{\bar{n}I}|}{(V_{nR} - V_{\bar{n}R})^2 + V_{\bar{n}I}^2} \equiv \left(R\,\tau_{n\bar{n}}^2\right)^{-1}\,. \tag{43}$$

For the nuclei of experimental interest, nuclear theory computations yield $R \sim 10^{23} s^{-1}$. Null results of bound n-$\bar{n}$ oscillation searches thus allow one to infer a bound on $\tau_{n\bar{n}}$ via Eq. (43).

From an elementary particle standpoint, n-$\bar{n}$ oscillations involve the conversion of three quarks into three antiquarks (and vice-versa). The lowest-dimension operators mediating such process arise at dimension nine in the SMEFT:

$$\mathcal{L}_{n\text{-}\bar{n}} = \frac{1}{\Lambda^5}\sum_j \alpha_j^{(9)}\,\mathcal{O}_j^{\text{BNV}}\,. \tag{44}$$

Consequently, one expects

$$\delta m \sim \alpha_j^{(9)} \frac{\Lambda_{\mathrm{HAD}}^6}{\Lambda^5}\,, \tag{45}$$

where Λ_{HAD} is a hadronic scale set by the n-$\bar{n}$ matrix elements in Eq. (39b). Taking Λ_{HAD} to be of order the QCD scale and using the present bounds on $\tau_{n\bar{n}}$ yields a lower bound on the B-violating mass scale of ~ 100 TeV.

The search for B-violating decays of short-lived particles such as Z bosons, τ leptons and B mesons provides also relevant constraints. The best limits are $\mathrm{Br}(Z \to pe, p\mu) < 1.8 \times 10^{-6}$ (95% C.L.) [115], $\mathrm{Br}(\tau^- \to \Lambda\pi^-) < 7.2 \times 10^{-8}$ [116] and $\mathrm{Br}(B^+ \to \Lambda e^+) < 3.2 \times 10^{-8}$ [117].

7 Quark flavors

While strong and electromagnetic forces preserve the quark flavor, the charged-current weak interactions generate transitions among the different quark species (see the review on the CKM quark-mixing matrix). Since the SM flavor-changing mechanism is associated with the $W^\pm$ fermionic vertices, the tree-level transitions satisfy a $\Delta F = \Delta Q$ rule where ΔQ denotes the change in charge of the relevant hadrons. Remember that the flavor quantum number F is defined to be $+1$ for positively charged quarks ($F = U, C, T$) and -1 for quarks with negative charges ($F = D, S, B$). The strongest tests on this conservation law have been obtained in kaon decays such as $\mathrm{Br}(K^+ \to \pi^+\pi^+e^-\bar{\nu}_e) < 1.3 \times 10^{-8}$ [118], and $(\mathrm{Re}\,x, \mathrm{Im}\,x) = (-0.002 \pm 0.006, 0.0012 \pm 0.0021)$ [119,120] where $x \equiv \mathcal{M}(\overline{K}^0 \to \pi^-\ell^+\nu)/\mathcal{M}(K^0 \to \pi^-\ell^+\nu)$.

The $\Delta F = \Delta Q$ rule can be violated through quantum loop contributions giving rise to flavor-changing neutral-current transitions (FCNCs). Owing to the GIM mechanism, processes of this type are very suppressed in the SM, which makes them a superb tool in the search for new physics associated with the flavor dynamics. Within the SM itself, these transitions are also sensitive to the heavy-quark mass scales and have played a crucial role identifying the size of the charm (K^0–$\overline{K}^0$ mixing) and top (B^0–$\overline{B}^0$ mixing) masses before the discovery of those quarks. In addition to the well-established $\Delta F = 2$ mixings in neutral K and B mesons, $\Delta M_{K^0} \equiv M_{K_L^0} - M_{K_S^0} = (0.5293 \pm 0.0009) \times 10^{10}\ \mathrm{s}^{-1}$, $\Delta M_{B^0} \equiv M_{B_H^0} - M_{B_L^0} = (0.5065 \pm 0.0019) \times 10^{12}\ \mathrm{s}^{-1}$ and $\Delta M_{B_s^0} \equiv M_{B_{sH}^0} - M_{B_{sL}^0} = (17.757 \pm 0.0021) \times 10^{12}\ \mathrm{s}^{-1}$, there is now strong evidence for the mixing of the D^0 meson and its antiparticle [121],

$$x_D \equiv (M_{D_H^0} - M_{D_L^0})/\Gamma_{D^0} = (3.9\,^{+\,1.1}_{-\,1.2}) \times 10^{-3}\,, \tag{46}$$

showing that there is a nonzero mass difference between the two neutral charm-meson eigenstates, of the expected size. The SM prediction for x_D is dominated by long-distance physics, because it involves virtual loops with down-type light quarks, and has unfortunately quite large uncertainties [122].

The FCNC kaon decays into lepton-antilepton pairs put stringent constraints on new flavor-changing interactions. The measured $K_L^0 \to \mu^+\mu^-$ rate, $\mathrm{Br}(K_L^0 \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$, is completely dominated by the known 2γ absorptive contribution, leaving very little room for new-physics, and $\mathrm{Br}(K_L^0 \to e^+e^-) = (9^{+6}_{-4}) \times 10^{-12}$ [123] (the tiniest branching ratio ever measured) also agrees with the SM expectation [124]. The experimental K_S^0 upper bounds on the electron, $\mathrm{Br}(K_S^0 \to e^+e^-) < 9 \times 10^{-9}$ [125], and muon, $\mathrm{Br}(K_S^0 \to \mu^+\mu^-) < 8 \times 10^{-10}$ [126], modes are still five and two orders of magnitude, respectively, larger than their SM predictions [124]. Another very clean test of FCNCs will be soon provided by the decay $K^+ \to \pi^+\nu\bar{\nu}$. With a predicted SM branching fraction of $(7.8 \pm 0.8) \times 10^{-11}$ [127], the CERN NA62 experiment is aiming to collect around one hundred events. Even more interesting is the CP-violating neutral mode $K_L^0 \to \pi^0\nu\bar{\nu}$, expected at a rate of $(2.4 \pm 0.4) \times 10^{-11}$ [127] that is still far away from the current upper bound of

3.0×10^{-9} [128]. The KOTO experiment at KEK is expected to substantially increase the sensitivity to this mode.

The strongest bound on FCNC transitions in charm decays is $\mathrm{Br}(D^0 \to \mu^+\mu^-) < 6.2 \times 10^{-9}$ [129], while in B decays the LHC experiments have recently reached the SM sensitivity: $\mathrm{Br}(B_d^0 \to \mu^+\mu^-) < 3.4 \times 10^{-10}$ (95% C.L.) and $\mathrm{Br}(B_s^0 \to \mu^+\mu^-) = (3.0 \pm 0.4) \times 10^{-9}$. At present, there is a lot of interest on the decays $B \to K^{(*)}\ell^+\ell^-$ where sizable discrepancies between the measured data and the SM predictions have been reported [130]. In particular, the LHCb experiment has found the ratios of produced muons versus electrons to be around 2.5σ below the SM predictions, both in $B \to K^*\ell^+\ell^-$ [131] and in $B^+ \to K^+\ell^+\ell^-$ [132] (for dilepton invariant-masses squared in the range $q^2 \leq 6\ \mathrm{GeV}^2$), suggesting a significant violation of lepton universality. The current Belle-II measurements of these ratios [133,134] are consistent with the SM, but they are also compatible with the LHCb results. Future analyses from LHCb and Belle-II are expected to clarify the situation.

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