

# Light Quarks ( $u, d, s$ )

OMITTED FROM SUMMARY TABLE

## $u$ -QUARK MASS

The  $u$ -,  $d$ -, and  $s$ -quark masses are estimates of so-called “current-quark masses,” in a mass-independent subtraction scheme such as  $\overline{MS}$ . The ratios  $m_u/m_d$  and  $m_s/m_d$  are extracted from pion and kaon masses using chiral symmetry. The estimates of  $d$  and  $u$  masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the  $u$  quark could be essentially massless. The  $s$ -quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the  $\overline{MS}$  masses at a renormalization scale of  $\mu = 2$  GeV. Results quoted in the literature at  $\mu = 1$  GeV have been rescaled by dividing by 1.35. The values of “Our Evaluation” were determined in part via Figures 1 and 2.

$\overline{MS}$ MASS (MeV)	DOCUMENT ID	TECN
<b>2.16 <math>^{+0.49}_{-0.26}</math> OUR EVALUATION</b>	See the ideogram below.	
2.130 $\pm$ 0.041	<sup>1</sup> BAZAVOV	18 LATT
2.27 $\pm$ 0.06 $\pm$ 0.06	<sup>2</sup> FODOR	16 LATT
2.36 $\pm$ 0.24	<sup>3</sup> CARRASCO	14 LATT
2.57 $\pm$ 0.26 $\pm$ 0.07	<sup>4</sup> AOKI	12 LATT
2.24 $\pm$ 0.10 $\pm$ 0.34	<sup>5</sup> BLUM	10 LATT
2.01 $\pm$ 0.14	<sup>6</sup> MCNEILE	10 LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●		
2.15 $\pm$ 0.03 $\pm$ 0.10	<sup>7</sup> DURR	11 LATT
1.9 $\pm$ 0.2	<sup>8</sup> BAZAVOV	10 LATT
2.01 $\pm$ 0.14	<sup>6</sup> DAVIES	10 LATT
2.9 $\pm$ 0.2	<sup>9</sup> DOMINGUEZ	09 THEO
2.9 $\pm$ 0.8	<sup>10</sup> DEANDREA	08 THEO
3.02 $\pm$ 0.33	<sup>11</sup> BLUM	07 LATT
2.7 $\pm$ 0.4	<sup>12</sup> JAMIN	06 THEO
1.9 $\pm$ 0.2	<sup>13</sup> MASON	06 LATT
2.8 $\pm$ 0.2	<sup>14</sup> NARISON	06 THEO
1.7 $\pm$ 0.3	<sup>15</sup> AUBIN	04A LATT

<sup>1</sup> BAZAVOV 18 determine the quark masses using a lattice computation with staggered fermions and four active quark flavors.

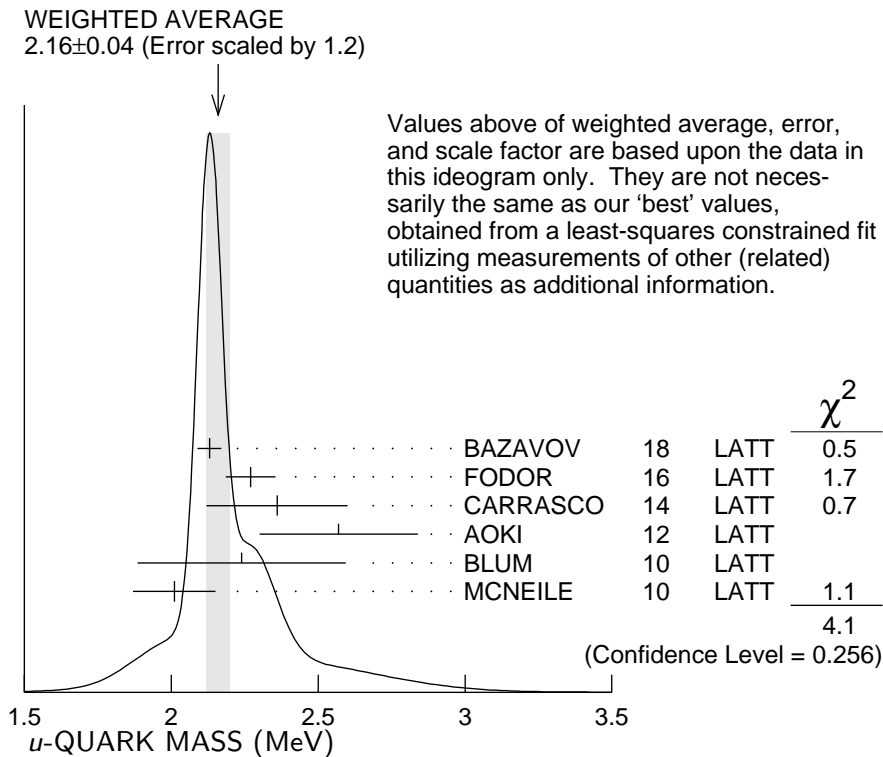
<sup>2</sup> FODOR 16 is a lattice simulation with  $N_f = 2 + 1$  dynamical flavors and includes partially quenched QED effects.

<sup>3</sup> CARRASCO 14 is a lattice QCD computation of light quark masses using  $2 + 1 + 1$  dynamical quarks, with  $m_u = m_d \neq m_s \neq m_c$ . The  $u$  and  $d$  quark masses are obtained separately by using the  $K$  meson mass splittings and lattice results for the electromagnetic contributions.

<sup>4</sup> AOKI 12 is a lattice computation using  $1 + 1 + 1$  dynamical quark flavors.

<sup>5</sup> BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use  $2+1$  dynamical quark flavors.

- 6 DAVIES 10 and MCNEILE 10 determine  $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$  using a lattice computation with  $N_f = 2 + 1$  dynamical fermions of the pseudoscalar meson masses. Mass  $m_u$  is obtained from this using the value of  $m_c$  from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios,  $m_s/\overline{m}$  and  $m_u/m_d$ .
- 7 DURR 11 determine quark mass from a lattice computation of the meson spectrum using  $N_f = 2 + 1$  dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual  $m_u$ ,  $m_d$  values are obtained using the lattice determination of the average mass  $m_{ud}$  and of the ratio  $m_s/m_{ud}$  and the value of  $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$  as determined from  $\eta \rightarrow 3\pi$  decays.
- 8 BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- 9 DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order  $\alpha_s^4$ .
- 10 DEANDREA 08 determine  $m_u - m_d$  from  $\eta \rightarrow 3\pi^0$ , and combine with the PDG 06 lattice average value of  $m_u + m_d = 7.6 \pm 1.6$  to determine  $m_u$  and  $m_d$ .
- 11 BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- 12 JAMIN 06 determine  $m_u(2 \text{ GeV})$  by combining the value of  $m_s$  obtained from the spectral function for the scalar  $K\pi$  form factor with other determinations of the quark mass ratios.
- 13 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate  $u$  and  $d$  quarks. Perturbative corrections were included at NNLO order. The quark masses  $m_u$  and  $m_d$  were determined from their  $(m_u + m_d)/2$  measurement and AUBIN 04A  $m_u/m_d$  value.
- 14 NARISON 06 uses sum rules for  $e^+e^- \rightarrow \text{hadrons}$  to order  $\alpha_s^3$  to determine  $m_s$  combined with other determinations of the quark mass ratios.
- 15 AUBIN 04A employ a partially quenched lattice calculation of the pseudoscalar meson masses.



***d*-QUARK MASS**

See the comment for the *u* quark above.

We have normalized the  $\overline{MS}$  masses at a renormalization scale of  $\mu = 2$  GeV. Results quoted in the literature at  $\mu = 1$  GeV have been rescaled by dividing by 1.35. The values of “Our Evaluation” were determined in part via Figures 1 and 2.

$\overline{MS}$ MASS (MeV)	DOCUMENT ID	TECN
<b>4.67 <math>^{+0.48}_{-0.17}</math> OUR EVALUATION</b>	See the ideogram below.	
4.675 $\pm$ 0.056	<sup>1</sup> BAZAVOV	18 LATT
4.67 $\pm$ 0.06 $\pm$ 0.06	<sup>2</sup> FODOR	16 LATT
5.03 $\pm$ 0.26	<sup>3</sup> CARRASCO	14 LATT
3.68 $\pm$ 0.29 $\pm$ 0.10	<sup>4</sup> AOKI	12 LATT
4.65 $\pm$ 0.15 $\pm$ 0.32	<sup>5</sup> BLUM	10 LATT
4.77 $\pm$ 0.15	<sup>6</sup> MCNEILE	10 LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●		
4.79 $\pm$ 0.07 $\pm$ 0.12	<sup>7</sup> DURR	11 LATT
4.6 $\pm$ 0.3	<sup>8</sup> BAZAVOV	10 LATT
4.79 $\pm$ 0.16	<sup>6</sup> DAVIES	10 LATT
5.3 $\pm$ 0.4	<sup>9</sup> DOMINGUEZ	09 THEO
4.7 $\pm$ 0.8	<sup>10</sup> DEANDREA	08 THEO
5.49 $\pm$ 0.39	<sup>11</sup> BLUM	07 LATT
4.8 $\pm$ 0.5	<sup>12</sup> JAMIN	06 THEO
4.4 $\pm$ 0.3	<sup>13</sup> MASON	06 LATT
5.1 $\pm$ 0.4	<sup>14</sup> NARISON	06 THEO
3.9 $\pm$ 0.5	<sup>15</sup> AUBIN	04A LATT

<sup>1</sup> BAZAVOV 18 determine the quark masses using a lattice computation with staggered fermions and four active quark flavors.

<sup>2</sup> FODOR 16 is a lattice simulation with  $N_f = 2 + 1$  dynamical flavors and includes partially quenched QED effects.

<sup>3</sup> CARRASCO 14 is a lattice QCD computation of light quark masses using  $2 + 1 + 1$  dynamical quarks, with  $m_u = m_d \neq m_s \neq m_c$ . The *u* and *d* quark masses are obtained separately by using the *K* meson mass splittings and lattice results for the electromagnetic contributions.

<sup>4</sup> AOKI 12 is a lattice computation using  $1 + 1 + 1$  dynamical quark flavors.

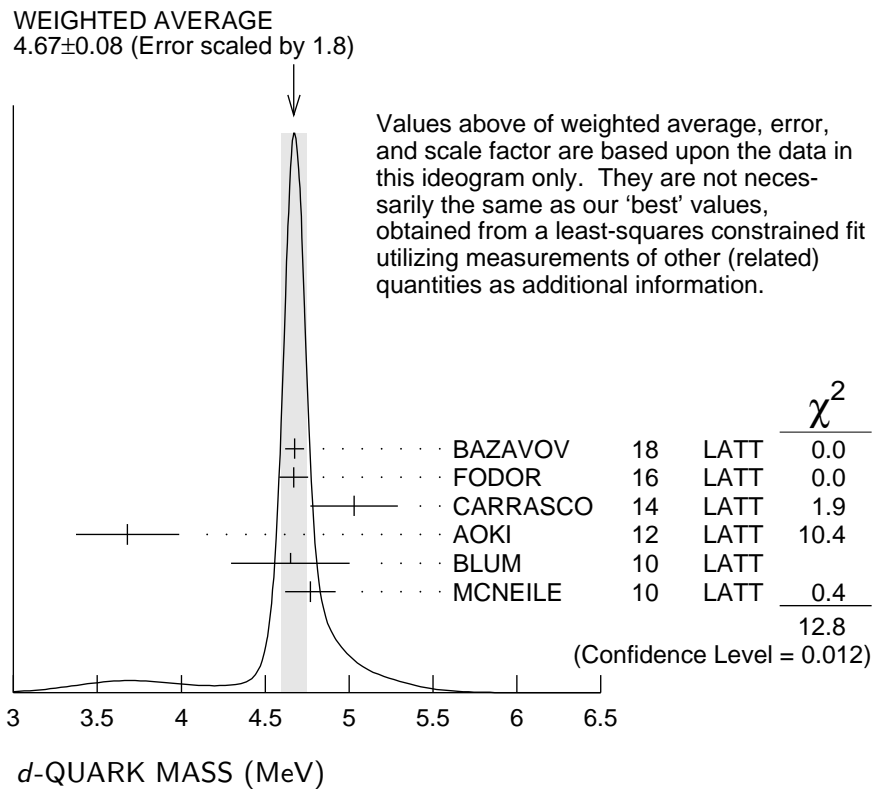
<sup>5</sup> BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use  $2+1$  dynamical quark flavors.

<sup>6</sup> DAVIES 10 and MCNEILE 10 determine  $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$  using a lattice computation with  $N_f = 2 + 1$  dynamical fermions of the pseudoscalar meson masses. Mass  $m_d$  is obtained from this using the value of  $m_c$  from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios,  $m_s/\overline{m}$  and  $m_u/m_d$ .

<sup>7</sup> DURR 11 determine quark mass from a lattice computation of the meson spectrum using  $N_f = 2 + 1$  dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual  $m_u$ ,  $m_d$  values are obtained using the lattice determination of the average mass  $m_{ud}$  and of the ratio  $m_s/m_{ud}$  and the value of  $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$  as determined from  $\eta \rightarrow 3\pi$  decays.

<sup>8</sup> BAZAVOV 10 is a lattice computation using  $2+1$  dynamical quark flavors.

- <sup>9</sup> DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order  $\alpha_s^4$ .
- <sup>10</sup> DEANDREA 08 determine  $m_u - m_d$  from  $\eta \rightarrow 3\pi^0$ , and combine with the PDG 06 lattice average value of  $m_u + m_d = 7.6 \pm 1.6$  to determine  $m_u$  and  $m_d$ .
- <sup>11</sup> BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- <sup>12</sup> JAMIN 06 determine  $m_d(2 \text{ GeV})$  by combining the value of  $m_s$  obtained from the spectral function for the scalar  $K\pi$  form factor with other determinations of the quark mass ratios.
- <sup>13</sup> MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate  $u$  and  $d$  quarks. Perturbative corrections were included at NNLO order. The quark masses  $m_u$  and  $m_d$  were determined from their  $(m_u + m_d)/2$  measurement and AUBIN 04A  $m_u/m_d$  value.
- <sup>14</sup> NARISON 06 uses sum rules for  $e^+e^- \rightarrow \text{hadrons}$  to order  $\alpha_s^3$  to determine  $m_s$  combined with other determinations of the quark mass ratios.
- <sup>15</sup> AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses, and one-loop perturbative renormalization constant.



$$\bar{m} = (m_u + m_d)/2$$

See the comments for the  $u$  quark above.

We have normalized the  $\overline{MS}$  masses at a renormalization scale of  $\mu = 2$  GeV. Results quoted in the literature at  $\mu = 1$  GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

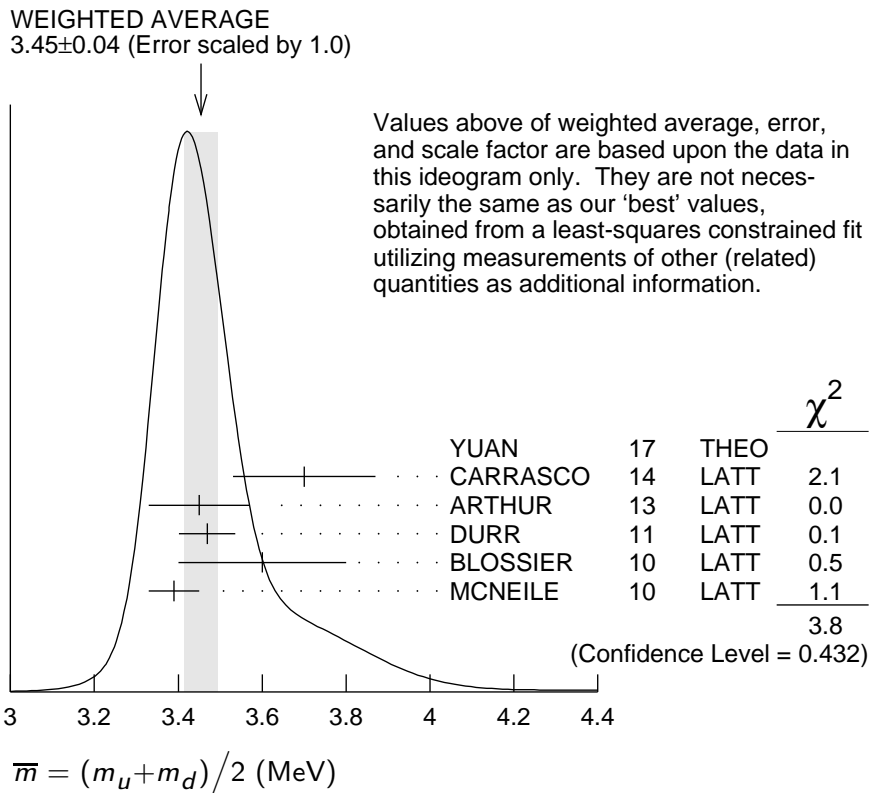
$\overline{MS}$ MASS (MeV)	DOCUMENT ID	TECN
<b>3.45 <math>^{+0.55}_{-0.15}</math> OUR EVALUATION</b>	See the ideogram below.	
4.7 $^{+0.8}_{-0.7}$	1 YUAN	17 THEO
3.70 $\pm 0.17$	2 CARRASCO	14 LATT
3.45 $\pm 0.12$	3 ARTHUR	13 LATT
3.469 $\pm 0.047 \pm 0.048$	4 DURR	11 LATT
3.6 $\pm 0.2$	5 BLOSSIER	10 LATT
3.39 $\pm 0.06$	6 MCNEILE	10 LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●		
3.59 $\pm 0.21$	7 AOKI	11A LATT
3.40 $\pm 0.07$	6 DAVIES	10 LATT
4.1 $\pm 0.2$	8 DOMINGUEZ	09 THEO
3.72 $\pm 0.41$	9 ALLTON	08 LATT
3.85 $\pm 0.12 \pm 0.4$	10 BLOSSIER	08 LATT
$\geq$ 4.85 $\pm 0.20$	11 DOMINGUEZ...08B	THEO
3.55 $^{+0.65}_{-0.28}$	12 ISHIKAWA	08 LATT
4.026 $\pm 0.048$	13 NAKAMURA	08 LATT
4.25 $\pm 0.35$	14 BLUM	07 LATT
4.08 $\pm 0.25 \pm 0.42$	15 GOCKELER	06 LATT
4.7 $\pm 0.2 \pm 0.3$	16 GOCKELER	06A LATT
3.2 $\pm 0.3$	17 MASON	06 LATT
3.95 $\pm 0.3$	18 NARISON	06 THEO
2.8 $\pm 0.3$	19 AUBIN	04 LATT
4.29 $\pm 0.14 \pm 0.65$	20 AOKI	03 LATT
3.223 $\pm 0.3$	21 AOKI	03B LATT
4.4 $\pm 0.1 \pm 0.4$	22 BECIREVIC	03 LATT
4.1 $\pm 0.3 \pm 1.0$	23 CHIU	03 LATT

<sup>1</sup> YUAN 17 determine  $\bar{m}$  using QCD sum rules in the isospin  $I=0$  scalar channel. At the end of the "Numerical Results" section of YUAN 17 the authors discuss the significance of their larger value of the light quark mass compared to previous determinations.

<sup>2</sup> CARRASCO 14 is a lattice QCD computation of light quark masses using  $2 + 1 + 1$  dynamical quarks, with  $m_u = m_d \neq m_s \neq m_c$ . The  $u$  and  $d$  quark masses are obtained separately by using the  $K$  meson mass splittings and lattice results for the electromagnetic contributions.

<sup>3</sup> ARTHUR 13 is a lattice computation using  $2+1$  dynamical domain wall fermions. Masses at  $\mu = 3$  GeV have been converted to  $\mu = 2$  GeV using conversion factors given in their paper.

<sup>4</sup> DURR 11 determine quark mass from a lattice computation of the meson spectrum using  $N_f = 2 + 1$  dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.



- <sup>5</sup> BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using  $N_f=2$  dynamical twisted-mass Wilson fermions.
- <sup>6</sup> DAVIES 10 and MCNEILE 10 determine  $\bar{m}_c(\mu)/\bar{m}_s(\mu) = 11.85 \pm 0.16$  using a lattice computation with  $N_f = 2 + 1$  dynamical fermions of the pseudoscalar meson masses. Mass  $\bar{m}$  is obtained from this using the value of  $m_c$  from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratio,  $m_s/\bar{m}$ .
- <sup>7</sup> AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using  $N_f = 2 + 1$  dynamical flavors of domain wall fermions.
- <sup>8</sup> DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order  $\alpha_s^4$ .
- <sup>9</sup> ALLTON 08 use a lattice computation of the  $\pi$ ,  $K$ , and  $\Omega$  masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- <sup>10</sup> BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- <sup>11</sup> DOMINGUEZ-CLARIMON 08B obtain an inequality from sum rules for the scalar two-point correlator.
- <sup>12</sup> ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of  $\mathcal{O}(a)$  improved Wilson quarks, and one-loop perturbative renormalization.
- <sup>13</sup> NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- <sup>14</sup> BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- <sup>15</sup> GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with  $N_f = 2$  dynamical light quark flavors, and non-perturbative renormalization, to obtain  $\bar{m}(2 \text{ GeV}) = 4.08 \pm 0.25 \pm 0.19 \pm 0.23 \text{ MeV}$ , where the first error

- is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.
- <sup>16</sup> GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with  $N_f = 2$  dynamical light quark flavors, and non-perturbative renormalization.
- <sup>17</sup> MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate  $u$  and  $d$  quarks. Perturbative corrections were included at NNLO order.
- <sup>18</sup> NARISON 06 uses sum rules for  $e^+ e^- \rightarrow$  hadrons to order  $\alpha_s^3$  to determine  $m_s$  combined with other determinations of the quark mass ratios.
- <sup>19</sup> AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- <sup>20</sup> AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory.
- <sup>21</sup> The errors given in AOKI 03B were  $^{+0.046}_{-0.069}$ . We changed them to  $\pm 0.3$  for calculating the overall best values. AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the  $\mathcal{O}(a)$  improved Wilson action.
- <sup>22</sup> BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses  $\mathcal{O}(a)$  improved Wilson action and nonperturbative renormalization.
- <sup>23</sup> CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.

### $m_u/m_d$ MASS RATIO

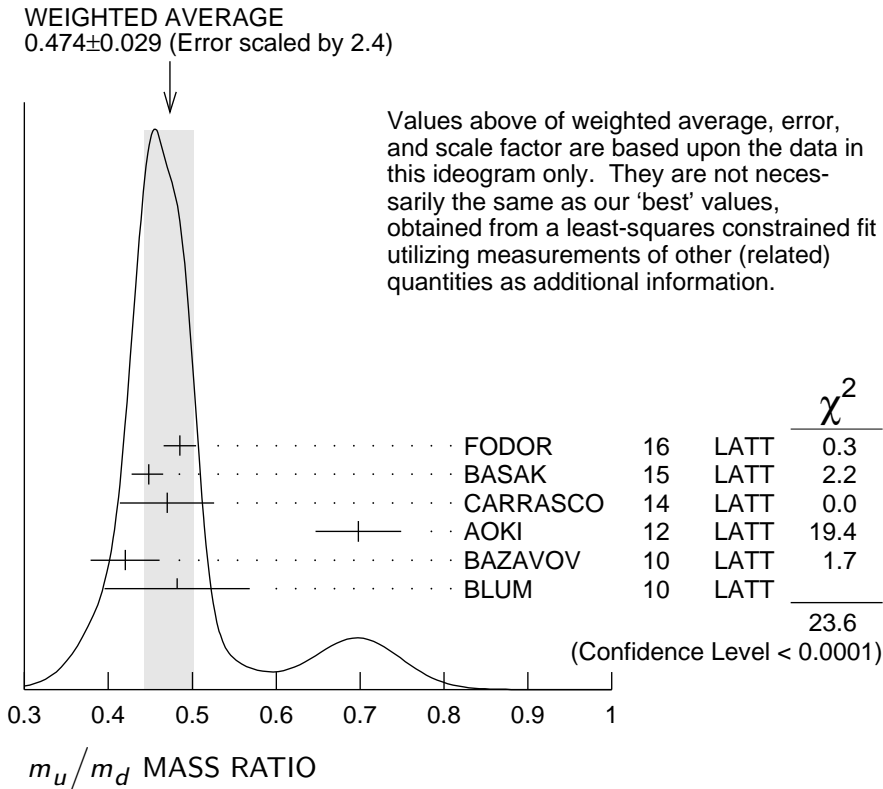
VALUE	DOCUMENT ID	TECN	COMMENT
<b>0.47 <math>^{+0.06}_{-0.07}</math> OUR EVALUATION</b>	See the ideogram below.		
0.485 $\pm 0.011$ $\pm 0.016$	<sup>1</sup> FODOR	16	LATT
0.4482 $^{+0.0173}_{-0.0206}$	<sup>2</sup> BASAK	15	LATT
0.470 $\pm 0.056$	<sup>3</sup> CARRASCO	14	LATT
0.698 $\pm 0.051$	<sup>4</sup> AOKI	12	LATT
0.42 $\pm 0.01$ $\pm 0.04$	<sup>5</sup> BAZAVOV	10	LATT
0.4818 $\pm 0.0096$ $\pm 0.0860$	<sup>6</sup> BLUM	10	LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
0.550 $\pm 0.031$	<sup>7</sup> BLUM	07	LATT
0.43 $\pm 0.08$	<sup>8</sup> AUBIN	04A	LATT
0.410 $\pm 0.036$	<sup>9</sup> NELSON	03	LATT
0.553 $\pm 0.043$	<sup>10</sup> LEUTWYLER	96	THEO Compilation

<sup>1</sup> FODOR 16 is a lattice simulation with  $N_f = 2 + 1$  dynamical flavors and includes partially quenched QED effects.

<sup>2</sup> BASAK 15 is a lattice computation using 2+1 dynamical quark flavors.

<sup>3</sup> CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with  $m_u = m_d \neq m_s \neq m_c$ . The  $u$  and  $d$  quark masses are obtained separately by using the  $K$  meson mass splittings and lattice results for the electromagnetic contributions.

- <sup>4</sup> AOKI 12 is a lattice computation using 1 + 1 + 1 dynamical quark flavors.
- <sup>5</sup> BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- <sup>6</sup> BLUM 10 is a lattice computation using 2+1 dynamical quark flavors.
- <sup>7</sup> BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- <sup>8</sup> AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses.
- <sup>9</sup> NELSON 03 computes coefficients in the order  $p^4$  chiral Lagrangian using a lattice calculation with three dynamical flavors. The ratio  $m_u/m_d$  is obtained by combining this with the chiral perturbation theory computation of the meson masses to order  $p^4$ .
- <sup>10</sup> LEUTWYLER 96 uses a combined fit to  $\eta \rightarrow 3\pi$  and  $\psi' \rightarrow J/\psi (\pi, \eta)$  decay rates, and the electromagnetic mass differences of the  $\pi$  and  $K$ .



### s-QUARK MASS

See the comment for the  $u$  quark above.

We have normalized the  $\overline{MS}$  masses at a renormalization scale of  $\mu = 2$  GeV. Results quoted in the literature at  $\mu = 1$  GeV have been rescaled by dividing by 1.35.

$\overline{MS}$ MASS (MeV)	DOCUMENT ID	TECN
<b>93</b> $+11$ $-5$ <b>OUR EVALUATION</b>	See the ideogram below.	
92.47 ± 0.69	1 BAZAVOV 18	LATT
87.6 ± 6.0	2 ANANTHANA..16	THEO
93.6 ± 0.8	3 CHAKRABOR..15	LATT
99.6 ± 4.3	4 CARRASCO 14	LATT



94.4 ± 2.3	5	ARTHUR	13	LATT
94 ± 9	6	BODENSTEIN	13	THEO
102 ± 3 ± 1	7	FRITZSCH	12	LATT
95.5 ± 1.1 ± 1.5	8	DURR	11	LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
96.2 ± 2.7	9	AOKI	11A	LATT
95 ± 6	10	BLOSSIER	10	LATT
97.6 ± 2.9 ± 5.5	11	BLUM	10	LATT
92.4 ± 1.5	12	DAVIES	10	LATT
92.2 ± 1.3	12	MCNEILE	10	LATT
107.3 ± 11.7	13	ALLTON	08	LATT
105 ± 3 ± 9	14	BLOSSIER	08	LATT
102 ± 8	15	DOMINGUEZ	08A	THEO
90.1 <sup>+17.2</sup> <sub>-6.1</sub>	16	ISHIKAWA	08	LATT
105.6 ± 1.2	17	NAKAMURA	08	LATT
119.5 ± 9.3	18	BLUM	07	LATT
105 ± 6 ± 7	19	CHETYRKIN	06	THEO
111 ± 6 ± 10	20	GOCKELER	06	LATT
119 ± 5 ± 8	21	GOCKELER	06A	LATT
92 ± 9	22	JAMIN	06	THEO
87 ± 6	23	MASON	06	LATT
104 ± 15	24	NARISON	06	THEO
≥ 71 ± 4, ≤ 151 ± 14	25	NARISON	06	THEO
96 <sup>+5</sup> <sub>-3</sub> <sup>+16</sup> <sub>-18</sub>	26	BAIKOV	05	THEO
81 ± 22	27	GAMIZ	05	THEO
125 ± 28	28	GORBUNOV	05	THEO
93 ± 32	29	NARISON	05	THEO
76 ± 8	30	AUBIN	04	LATT
116 ± 6 ± 0.65	31	AOKI	03	LATT
84.5 <sup>+12</sup> <sub>-1.7</sub>	32	AOKI	03B	LATT
106 ± 2 ± 8	33	BECIREVIC	03	LATT
92 ± 9 ± 16	34	CHIU	03	LATT
117 ± 17	35	GAMIZ	03	THEO
103 ± 17	36	GAMIZ	03	THEO

<sup>1</sup>BAZAVOV 18 determine the quark masses using a lattice computation with staggered fermions and four active quark flavors.

<sup>2</sup>ANANTHANARAYAN 16 determine  $\overline{m}_s(2 \text{ GeV}) = 106.70 \pm 9.36 \text{ MeV}$  and  $74.47 \pm 7.77 \text{ MeV}$  from fits to ALEPH and OPAL  $\tau$  decay data, respectively. We have used the weighted average of the two.

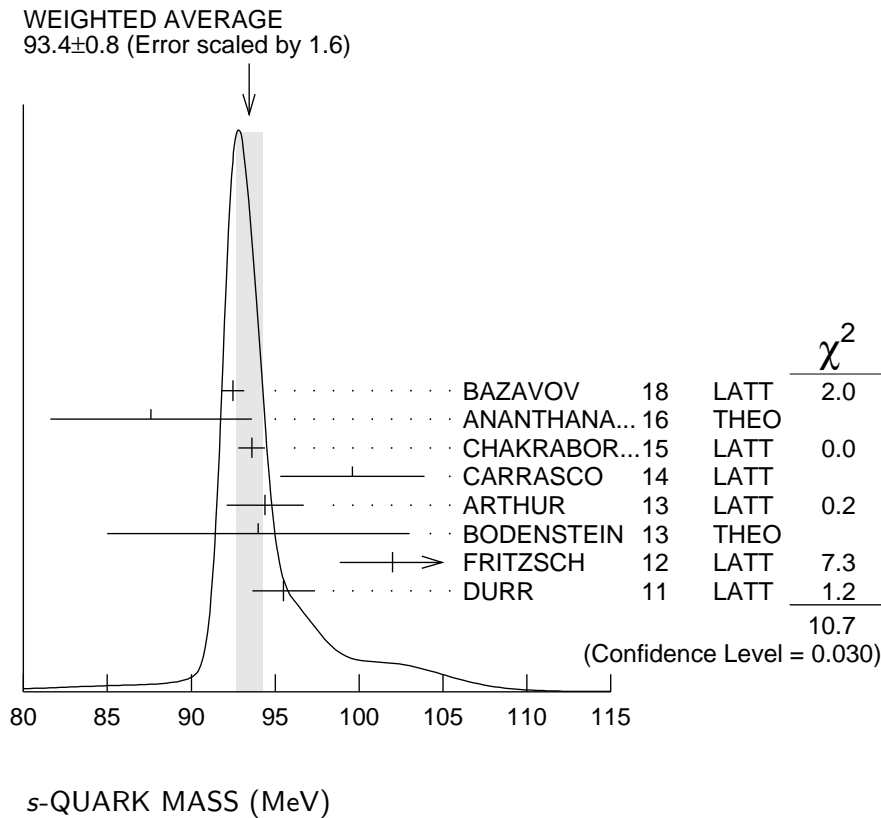
<sup>3</sup>CHAKRABORTY 15 is a lattice QCD computation that determines  $m_c$  and  $m_c/m_s$  using pseudoscalar mesons masses tuned on gluon field configurations with 2+1+1 dynamical flavors of HISQ quarks with  $u/d$  masses down to the physical value.

<sup>4</sup>CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with  $m_u = m_d \neq m_s \neq m_c$ . The  $u$  and  $d$  quark masses are obtained separately by using the  $K$  meson mass splittings and lattice results for the electromagnetic contributions.

<sup>5</sup>ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions. Masses at  $\mu = 3 \text{ GeV}$  have been converted to  $\mu = 2 \text{ GeV}$  using conversion factors given in their paper.

- <sup>6</sup> BODENSTEIN 13 determines  $m_s$  from QCD finite energy sum rules, and the perturbative computation of the pseudoscalar correlator to five-loop order.
- <sup>7</sup> FRITZSCH 12 determine  $m_s$  using a lattice computation with  $N_f = 2$  dynamical flavors.
- <sup>8</sup> DURR 11 determine quark mass from a lattice computation of the meson spectrum using  $N_f = 2 + 1$  dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.
- <sup>9</sup> AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using  $N_f = 2 + 1$  dynamical flavors of domain wall fermions.
- <sup>10</sup> BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using  $N_f=2$  dynamical twisted-mass Wilson fermions.
- <sup>11</sup> BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- <sup>12</sup> DAVIES 10 and MCNEILE 10 determine  $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$  using a lattice computation with  $N_f = 2 + 1$  dynamical fermions of the pseudoscalar meson masses. Mass  $m_s$  is obtained from this using the value of  $m_c$  from ALLISON 08 or MCNEILE 10.
- <sup>13</sup> ALLTON 08 use a lattice computation of the  $\pi$ ,  $K$ , and  $\Omega$  masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- <sup>14</sup> BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- <sup>15</sup> DOMINGUEZ 08A make determination from QCD finite energy sum rules for the pseudoscalar two-point function computed to order  $\alpha_s^4$ .
- <sup>16</sup> ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of  $\mathcal{O}(a)$  improved Wilson quarks, and one-loop perturbative renormalization.
- <sup>17</sup> NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- <sup>18</sup> BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- <sup>19</sup> CHETYRKIN 06 use QCD sum rules in the pseudoscalar channel to order  $\alpha_s^4$ .
- <sup>20</sup> GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with  $N_f = 2$  dynamical light quark flavors, and non-perturbative renormalization, to obtain  $\overline{m}_s(2 \text{ GeV}) = 111 \pm 6 \pm 4 \pm 6 \text{ MeV}$ , where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.
- <sup>21</sup> GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with  $N_f = 2$  dynamical light quark flavors, and non-perturbative renormalization.
- <sup>22</sup> JAMIN 06 determine  $\overline{m}_s(2 \text{ GeV})$  from the spectral function for the scalar  $K\pi$  form factor.
- <sup>23</sup> MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate  $u$  and  $d$  quarks. Perturbative corrections were included at NNLO order.
- <sup>24</sup> NARISON 06 uses sum rules for  $e^+ e^- \rightarrow$  hadrons to order  $\alpha_s^3$ .
- <sup>25</sup> NARISON 06 obtains the quoted range from positivity of the spectral functions.
- <sup>26</sup> BAIKOV 05 determines  $\overline{m}_s(M_\tau) = 100^{+5+17}_{-3-19}$  from sum rules using the strange spectral function in  $\tau$  decay. The computations were done to order  $\alpha_s^3$ , with an estimate of the  $\alpha_s^4$  terms. We have converted the result to  $\mu = 2 \text{ GeV}$ .
- <sup>27</sup> GAMIZ 05 determines  $\overline{m}_s(2 \text{ GeV})$  from sum rules using the strange spectral function in  $\tau$  decay. The computations were done to order  $\alpha_s^2$ , with an estimate of the  $\alpha_s^3$  terms.
- <sup>28</sup> GORBUNOV 05 use hadronic tau decays to N3LO, including power corrections.
- <sup>29</sup> NARISON 05 determines  $\overline{m}_s(2 \text{ GeV})$  from sum rules using the strange spectral function in  $\tau$  decay. The computations were done to order  $\alpha_s^3$ .

- 30 AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- 31 AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory. Determines  $m_s=113.8 \pm 2.3^{+5.8}_{-2.9}$  using  $K$  mass as input and  $m_s=142.3 \pm 5.8^{+22}_0$  using  $\phi$  mass as input. We have performed a weighted average of these values.
- 32 AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the  $\mathcal{O}(a)$  improved Wilson action.
- 33 BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses  $\mathcal{O}(a)$  improved Wilson action and nonperturbative renormalization. They also quote  $\overline{m}/m_s=24.3 \pm 0.2 \pm 0.6$ .
- 34 CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.
- 35 GAMIZ 03 determines  $m_s$  from SU(3) breaking in the  $\tau$  hadronic width. The value of  $V_{us}$  is chosen to satisfy CKM unitarity.
- 36 GAMIZ 03 determines  $m_s$  from SU(3) breaking in the  $\tau$  hadronic width. The value of  $V_{us}$  is taken from the PDG.



## OTHER LIGHT QUARK MASS RATIOS

### $m_s/m_d$ MASS RATIO

VALUE	DOCUMENT ID	TECN	COMMENT
<b>17-22 OUR EVALUATION</b>			
20.0	1 GAO	97	THEO
18.9±0.8	2 LEUTWYLER	96	THEO Compilation

21	<sup>3</sup> DONOGHUE 92	THEO
18	<sup>4</sup> GERARD 90	THEO
18 to 23	<sup>5</sup> LEUTWYLER 90B	THEO

<sup>1</sup> GAO 97 uses electromagnetic mass splittings of light mesons.

<sup>2</sup> LEUTWYLER 96 uses a combined fit to  $\eta \rightarrow 3\pi$  and  $\psi' \rightarrow J/\psi(\pi,\eta)$  decay rates, and the electromagnetic mass differences of the  $\pi$  and  $K$ .

<sup>3</sup> DONOGHUE 92 result is from a combined analysis of meson masses,  $\eta \rightarrow 3\pi$  using second-order chiral perturbation theory including nonanalytic terms, and  $(\psi(2S) \rightarrow J/\psi(1S)\pi)/(\psi(2S) \rightarrow J/\psi(1S)\eta)$ .

<sup>4</sup> GERARD 90 uses large  $N$  and  $\eta$ - $\eta'$  mixing.

<sup>5</sup> LEUTWYLER 90B determines quark mass ratios using second-order chiral perturbation theory for the meson and baryon masses, including nonanalytic corrections. Also uses Weinberg sum rules to determine  $L_7$ .

## $m_s/\bar{m}$ MASS RATIO

$$\bar{m} \equiv (m_u + m_d)/2$$

VALUE	DOCUMENT ID	TECN
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**27.3  $^{+0.7}_{-1.3}$  OUR EVALUATION** See the ideogram below.

$27.35 \pm 0.05^{+0.10}_{-0.07}$	<sup>1</sup> BAZAVOV 14A	LATT
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$26.66 \pm 0.32$	<sup>2</sup> CARRASCO 14	LATT
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$27.36 \pm 0.54$	<sup>3</sup> ARTHUR 13	LATT
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$27.53 \pm 0.20 \pm 0.08$	<sup>4</sup> DURR 11	LATT
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• • • We do not use the following data for averages, fits, limits, etc. • • •

$26.8 \pm 1.4$	<sup>5</sup> AOKI 11A	LATT
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$27.3 \pm 0.9$	<sup>6</sup> BLOSSIER 10	LATT
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$28.8 \pm 1.65$	<sup>7</sup> ALLTON 08	LATT
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$27.3 \pm 0.3 \pm 1.2$	<sup>8</sup> BLOSSIER 08	LATT
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$23.5 \pm 1.5$	<sup>9</sup> OLLER 07A	THEO
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$27.4 \pm 0.4$	<sup>10</sup> AUBIN 04	LATT
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<sup>1</sup> BAZAVOV 14A is a lattice computation using 4 dynamical flavors of HISQ fermions.

<sup>2</sup> CARRASCO 14 is a lattice QCD computation of light quark masses using  $2 + 1 + 1$  dynamical quarks, with  $m_u = m_d \neq m_s \neq m_c$ . The  $u$  and  $d$  quark masses are obtained separately by using the  $K$  meson mass splittings and lattice results for the electromagnetic contributions.

<sup>3</sup> ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions.

<sup>4</sup> DURR 11 determine quark mass from a lattice computation of the meson spectrum using  $N_f = 2 + 1$  dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.

<sup>5</sup> AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using  $N_f = 2 + 1$  dynamical flavors of domain wall fermions.

<sup>6</sup> BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using  $N_f=2$  dynamical twisted-mass Wilson fermions.

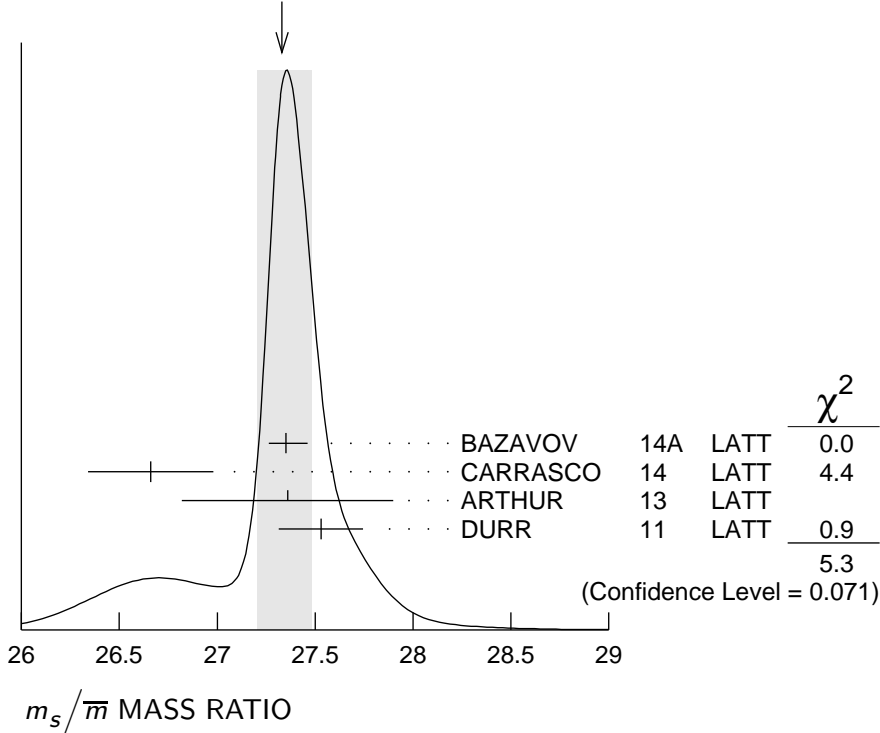
<sup>7</sup> ALLTON 08 use a lattice computation of the  $\pi$ ,  $K$ , and  $\Omega$  masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.

<sup>8</sup> BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.

<sup>9</sup> OLLER 07A use unitarized chiral perturbation theory to order  $p^4$ .

<sup>10</sup> Three flavor dynamical lattice calculation of pseudoscalar meson masses.

WEIGHTED AVERAGE  
 27.33±0.15-0.12 (Error scaled by 1.6)



### Q MASS RATIO

$$Q \equiv \sqrt{(m_s^2 - \bar{m}^2)/(m_d^2 - m_u^2)}; \quad \bar{m} \equiv (m_u + m_d)/2$$

VALUE	DOCUMENT ID	TECN
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• • • We do not use the following data for averages, fits, limits, etc. • • •

22.1±0.7	1 COLANGELO	18	THEO
22.0±0.7	2 COLANGELO	17	THEO
21.6±1.1	3 GUO	17	THEO
23.4±0.4±0.5	4 FODOR	16	LATT
21.4±0.4	5 GUO	15F	THEO
22.8±0.4	6 MARTEMYA...	05	THEO
22.7±0.8	7 ANISOVICH	96	THEO

<sup>1</sup> COLANGELO 18 obtain  $Q$  from a dispersive analysis of  $\eta \rightarrow 3\pi$  decay.

<sup>2</sup> COLANGELO 17 obtain  $Q$  from a dispersive analysis of KLOE collaboration data on  $\eta \rightarrow \pi^+ \pi^- \pi^0$  decays and chiral perturbation theory input.

<sup>3</sup> GUO 17 determine  $Q$  from a dispersive model fit to KLOE and WASA-at-COSY data on  $\eta \rightarrow \pi^+ \pi^- \pi^0$  decay and matching to chiral perturbation theory .

<sup>4</sup> FODOR 16 is a lattice simulation with  $N_f = 2 + 1$  dynamical flavors and includes partially quenched QED effects.

<sup>5</sup> GUO 15F determine  $Q$  from a Khuri-Treiman analysis of  $\eta \rightarrow 3\pi$  decays.

<sup>6</sup> MARTEMYANOV 05 determine  $Q$  from  $\eta \rightarrow 3\pi$  decay.

<sup>7</sup> ANISOVICH 96 find  $Q$  from  $\eta \rightarrow \pi^+ \pi^- \pi^0$  decay using dispersion relations and chiral perturbation theory.

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AOKI	11A	PR D83 074508	Y. Aoki <i>et al.</i> (RBC-UKQCD Collab.)
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