

Light Quarks (u, d, s)

OMITTED FROM SUMMARY TABLE

u -QUARK MASS

The u -, d -, and s -quark masses are estimates of so-called “current-quark masses,” in a mass-independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s -quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of “Our Evaluation” were determined in part via Figures 1 and 2.

$\overline{\text{MS}}$ MASS (MeV)	DOCUMENT ID	TECN
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2.2 $\begin{smallmatrix} +0.6 \\ -0.4 \end{smallmatrix}$ OUR EVALUATION See the ideogram below.

2.27 ± 0.06 ± 0.06	1 FODOR	16	LATT
2.36 ± 0.24	2 CARRASCO	14	LATT
2.57 ± 0.26 ± 0.07	3 AOKI	12	LATT
2.15 ± 0.03 ± 0.10	4 DURR	11	LATT
1.9 ± 0.2	5 BAZAVOV	10	LATT
2.24 ± 0.10 ± 0.34	6 BLUM	10	LATT
2.01 ± 0.14	7 MCNEILE	10	LATT
2.9 ± 0.2	8 DOMINGUEZ	09	THEO

• • • We do not use the following data for averages, fits, limits, etc. • • •

2.01 ± 0.14	7 DAVIES	10	LATT
2.9 ± 0.8	9 DEANDREA	08	THEO
3.02 ± 0.33	10 BLUM	07	LATT
2.7 ± 0.4	11 JAMIN	06	THEO
1.9 ± 0.2	12 MASON	06	LATT
2.8 ± 0.2	13 NARISON	06	THEO
1.7 ± 0.3	14 AUBIN	04A	LATT

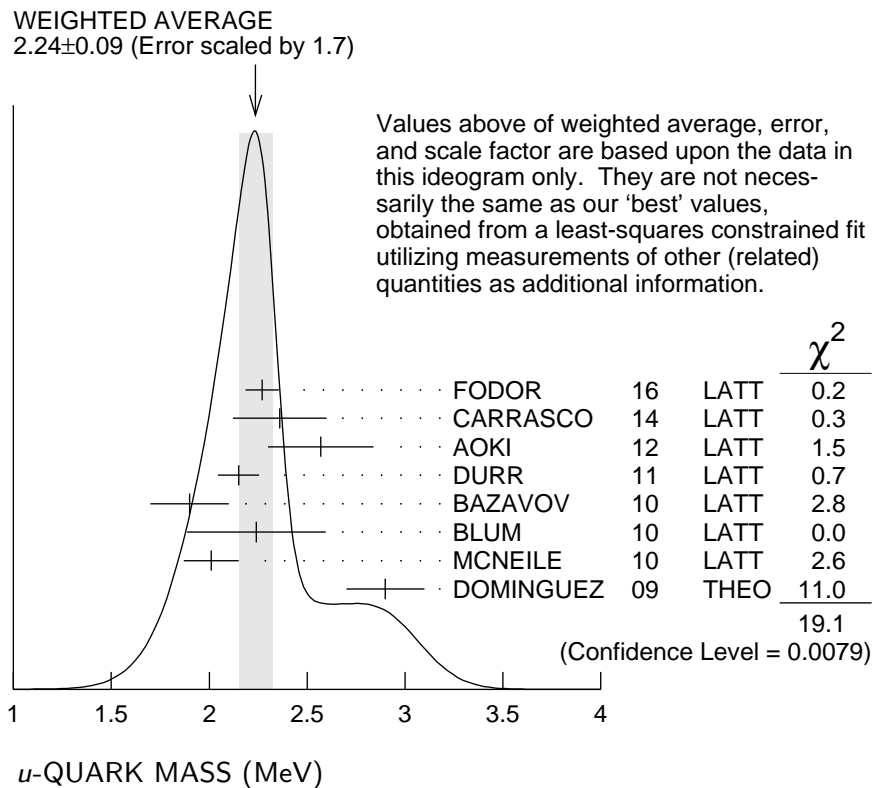
¹FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

²CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

³AOKI 12 is a lattice computation using $1 + 1 + 1$ dynamical quark flavors.

⁴DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual m_u, m_d values are obtained using the lattice determination of the average mass m_{ud} and of the ratio m_s/m_{ud} and the value of $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$ as determined from $\eta \rightarrow 3\pi$ decays.

- ⁵ BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.
- ⁶ BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- ⁷ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_u is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios, m_s/\overline{m} and m_u/m_d .
- ⁸ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .
- ⁹ DEANDREA 08 determine $m_u - m_d$ from $\eta \rightarrow 3\pi^0$, and combine with the PDG 06 lattice average value of $m_u + m_d = 7.6 \pm 1.6$ to determine m_u and m_d .
- ¹⁰ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ¹¹ JAMIN 06 determine $m_u(2 \text{ GeV})$ by combining the value of m_s obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.
- ¹² MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order. The quark masses m_u and m_d were determined from their $(m_u + m_d)/2$ measurement and AUBIN 04A m_u/m_d value.
- ¹³ NARISON 06 uses sum rules for $e^+e^- \rightarrow \text{hadrons}$ to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- ¹⁴ AUBIN 04A employ a partially quenched lattice calculation of the pseudoscalar meson masses.



***d*-QUARK MASS**

See the comment for the *u* quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of “Our Evaluation” were determined in part via Figures 1 and 2.

\overline{MS} MASS (MeV)	DOCUMENT ID	TECN
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4.7^{+0.5}_{-0.4} OUR EVALUATION	See the ideogram below.	
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4.67±0.06±0.06	1 FODOR	16	LATT
5.03±0.26	2 CARRASCO	14	LATT
3.68±0.29±0.10	3 AOKI	12	LATT
4.79±0.07±0.12	4 DURR	11	LATT
4.6 ±0.3	5 BAZAVOV	10	LATT
4.65±0.15±0.32	6 BLUM	10	LATT
4.77±0.15	7 MCNEILE	10	LATT
5.3 ±0.4	8 DOMINGUEZ	09	THEO
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
4.79±0.16	7 DAVIES	10	LATT
4.7 ±0.8	9 DEANDREA	08	THEO
5.49±0.39	10 BLUM	07	LATT
4.8 ±0.5	11 JAMIN	06	THEO
4.4 ±0.3	12 MASON	06	LATT
5.1 ±0.4	13 NARISON	06	THEO
3.9 ±0.5	14 AUBIN	04A	LATT

¹ FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

² CARRASCO 14 is a lattice QCD computation of light quark masses using $2 + 1 + 1$ dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The *u* and *d* quark masses are obtained separately by using the *K* meson mass splittings and lattice results for the electromagnetic contributions.

³ AOKI 12 is a lattice computation using $1 + 1 + 1$ dynamical quark flavors.

⁴ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed. The individual m_u , m_d values are obtained using the lattice determination of the average mass m_{ud} and of the ratio m_s/m_{ud} and the value of $Q = (m_s^2 - m_{ud}^2) / (m_d^2 - m_u^2)$ as determined from $\eta \rightarrow 3\pi$ decays.

⁵ BAZAVOV 10 is a lattice computation using $2+1$ dynamical quark flavors.

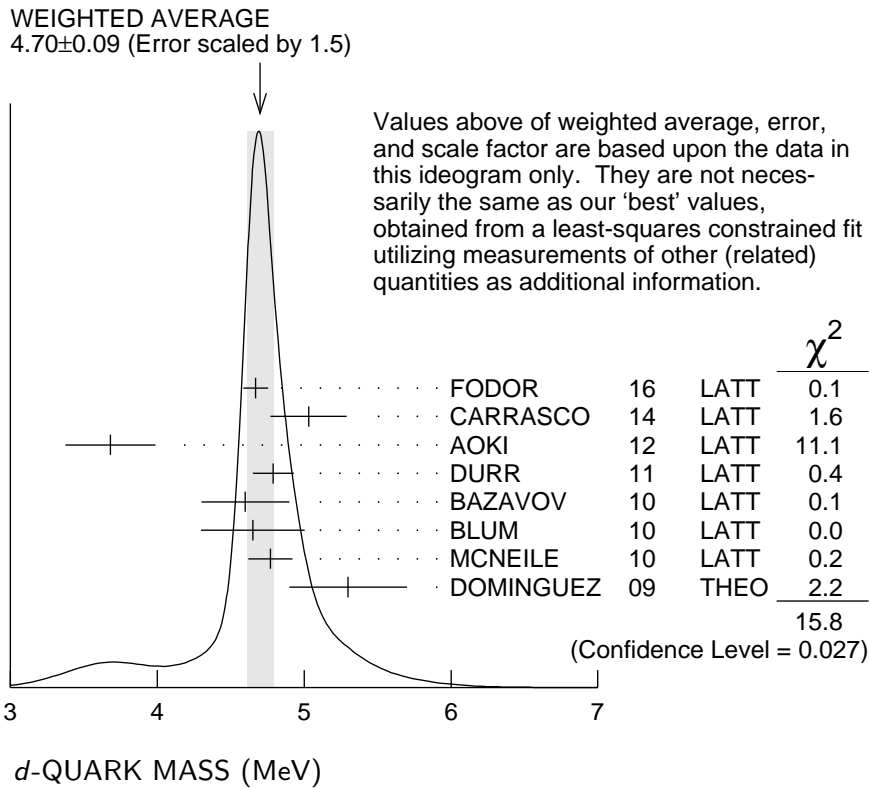
⁶ BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use $2+1$ dynamical quark flavors.

⁷ DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_d is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratios, m_s/\overline{m} and m_u/m_d .

⁸ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .

⁹ DEANDREA 08 determine $m_u - m_d$ from $\eta \rightarrow 3\pi^0$, and combine with the PDG 06 lattice average value of $m_u + m_d = 7.6 \pm 1.6$ to determine m_u and m_d .

- 10 BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- 11 JAMIN 06 determine $m_d(2 \text{ GeV})$ by combining the value of m_s obtained from the spectral function for the scalar $K\pi$ form factor with other determinations of the quark mass ratios.
- 12 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order. The quark masses m_u and m_d were determined from their $(m_u+m_d)/2$ measurement and AUBIN 04A m_u/m_d value.
- 13 NARISON 06 uses sum rules for $e^+e^- \rightarrow \text{hadrons}$ to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- 14 AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses, and one-loop perturbative renormalization constant.



$$\bar{m} = (m_u + m_d)/2$$

See the comments for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

\overline{MS} MASS (MeV)	DOCUMENT ID	TECN
3.5 $\begin{smallmatrix} +0.7 \\ -0.3 \end{smallmatrix}$	OUR EVALUATION	See the ideogram below.
3.70 ± 0.17	¹ CARRASCO 14	LATT
3.45 ± 0.12	² ARTHUR 13	LATT
3.59 ± 0.21	³ AOKI 11A	LATT
3.469 $\pm 0.047 \pm 0.048$	⁴ DURR 11	LATT
3.6 ± 0.2	⁵ BLOSSIER 10	LATT
3.39 ± 0.06	⁶ MCNEILE 10	LATT
4.1 ± 0.2	⁷ DOMINGUEZ 09	THEO
3.72 ± 0.41	⁸ ALLTON 08	LATT
3.55 $\begin{smallmatrix} +0.65 \\ -0.28 \end{smallmatrix}$	⁹ ISHIKAWA 08	LATT
4.25 ± 0.35	¹⁰ BLUM 07	LATT
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●		
3.40 ± 0.07	⁶ DAVIES 10	LATT
3.85 $\pm 0.12 \pm 0.4$	¹¹ BLOSSIER 08	LATT
\geq 4.85 ± 0.20	¹² DOMINGUEZ...08B	THEO
4.026 ± 0.048	¹³ NAKAMURA 08	LATT
4.08 $\pm 0.25 \pm 0.42$	¹⁴ GOCKELER 06	LATT
4.7 $\pm 0.2 \pm 0.3$	¹⁵ GOCKELER 06A	LATT
3.2 ± 0.3	¹⁶ MASON 06	LATT
3.95 ± 0.3	¹⁷ NARISON 06	THEO
2.8 ± 0.3	¹⁸ AUBIN 04	LATT
4.29 $\pm 0.14 \pm 0.65$	¹⁹ AOKI 03	LATT
3.223 ± 0.3	²⁰ AOKI 03B	LATT
4.4 $\pm 0.1 \pm 0.4$	²¹ BECIREVIC 03	LATT
4.1 $\pm 0.3 \pm 1.0$	²² CHIU 03	LATT

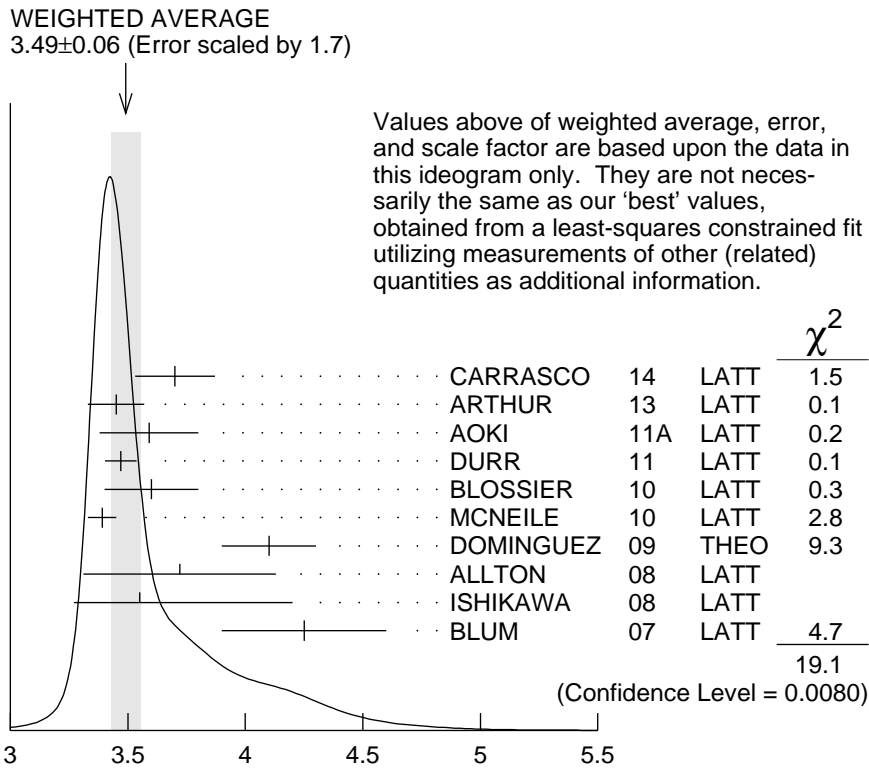
¹ CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

² ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions. Masses at $\mu = 3$ GeV have been converted to $\mu = 2$ GeV using conversion factors given in their paper.

³ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.

⁴ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.

⁵ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.



$$\bar{m} = (m_u + m_d) / 2 \text{ (MeV)}$$

- ⁶ DAVIES 10 and MCNEILE 10 determine $\bar{m}_c(\mu) / \bar{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass \bar{m} is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10 and the BAZAVOV 10 values for the light quark mass ratio, m_s / \bar{m} .
- ⁷ DOMINGUEZ 09 use QCD finite energy sum rules for the two-point function of the divergence of the axial vector current computed to order α_s^4 .
- ⁸ ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- ⁹ ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of $\mathcal{O}(a)$ improved Wilson quarks, and one-loop perturbative renormalization.
- ¹⁰ BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- ¹¹ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- ¹² DOMINGUEZ-CLARIMON 08B obtain an inequality from sum rules for the scalar two-point correlator.
- ¹³ NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- ¹⁴ GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\bar{m}(2 \text{ GeV}) = 4.08 \pm 0.25 \pm 0.19 \pm 0.23 \text{ MeV}$, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.

- ¹⁵ GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.
- ¹⁶ MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order.
- ¹⁷ NARISON 06 uses sum rules for $e^+ e^- \rightarrow$ hadrons to order α_s^3 to determine m_s combined with other determinations of the quark mass ratios.
- ¹⁸ AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- ¹⁹ AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory.
- ²⁰ The errors given in AOKI 03B were ${}^{+0.046}_{-0.069}$. We changed them to ± 0.3 for calculating the overall best values. AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.
- ²¹ BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization.
- ²² CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.

m_u/m_d MASS RATIO

<u>VALUE</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>COMMENT</u>
0.38–0.58 OUR EVALUATION	See the ideogram below.		
0.485 ± 0.011 ± 0.016	¹ FODOR	16	LATT
0.4482 ${}^{+0.0173}_{-0.0206}$	² BASAK	15	LATT
0.470 ± 0.056	³ CARRASCO	14	LATT
0.698 ± 0.051	⁴ AOKI	12	LATT
0.42 ± 0.01 ± 0.04	⁵ BAZAVOV	10	LATT
0.4818 ± 0.0096 ± 0.0860	⁶ BLUM	10	LATT
0.550 ± 0.031	⁷ BLUM	07	LATT
• • • We do not use the following data for averages, fits, limits, etc. • • •			
0.43 ± 0.08	⁸ AUBIN	04A	LATT
0.410 ± 0.036	⁹ NELSON	03	LATT
0.553 ± 0.043	¹⁰ LEUTWYLER	96	THEO Compilation

¹ FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

² BASAK 15 is a lattice computation using 2+1 dynamical quark flavors.

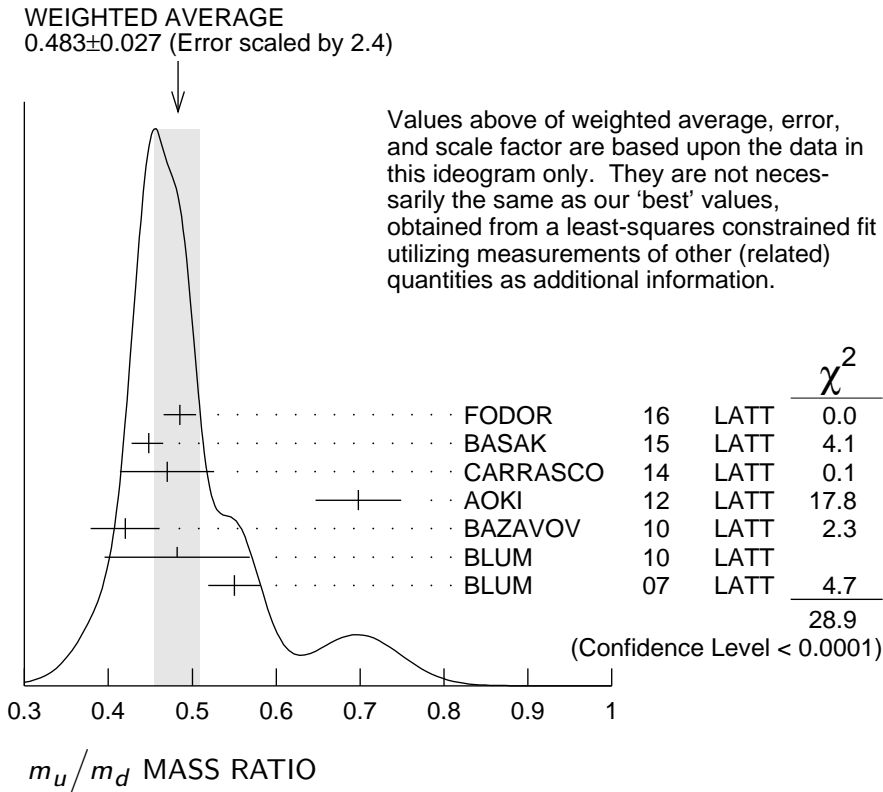
³ CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

⁴ AOKI 12 is a lattice computation using 1 + 1 + 1 dynamical quark flavors.

⁵ BAZAVOV 10 is a lattice computation using 2+1 dynamical quark flavors.

⁶ BLUM 10 is a lattice computation using 2+1 dynamical quark flavors.

- 7 BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- 8 AUBIN 04A perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with continuum estimate of electromagnetic effects in the kaon masses.
- 9 NELSON 03 computes coefficients in the order p^4 chiral Lagrangian using a lattice calculation with three dynamical flavors. The ratio m_u/m_d is obtained by combining this with the chiral perturbation theory computation of the meson masses to order p^4 .
- 10 LEUTWYLER 96 uses a combined fit to $\eta \rightarrow 3\pi$ and $\psi' \rightarrow J/\psi (\pi, \eta)$ decay rates, and the electromagnetic mass differences of the π and K .



s-QUARK MASS

See the comment for the u quark above.

We have normalized the \overline{MS} masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35.

<u>\overline{MS} MASS (MeV)</u>	<u>DOCUMENT ID</u>	<u>TECN</u>
96 $\pm \frac{8}{4}$ OUR EVALUATION	See the ideogram below.	
87.6 ± 6.0	1 ANANTHANA..16	THEO
93.6 ± 0.8	2 CHAKRABOR..15	LATT
99.6 ± 4.3	3 CARRASCO 14	LATT
94.4 ± 2.3	4 ARTHUR 13	LATT
94 ± 9	5 BODENSTEIN 13	THEO
102 ± 3 ± 1	6 FRITZSCH 12	LATT

96.2 ± 2.7	7 AOKI	11A	LATT
$95.5 \pm 1.1 \pm 1.5$	8 DURR	11	LATT
95 ± 6	9 BLOSSIER	10	LATT
$97.6 \pm 2.9 \pm 5.5$	10 BLUM	10	LATT
107.3 ± 11.7	11 ALLTON	08	LATT
102 ± 8	12 DOMINGUEZ	08A	THEO
$90.1^{+17.2}_{-6.1}$	13 ISHIKAWA	08	LATT

• • • We do not use the following data for averages, fits, limits, etc. • • •

92.4 ± 1.5	14 DAVIES	10	LATT
92.2 ± 1.3	14 MCNEILE	10	LATT
$105 \pm 3 \pm 9$	15 BLOSSIER	08	LATT
105.6 ± 1.2	16 NAKAMURA	08	LATT
119.5 ± 9.3	17 BLUM	07	LATT
$105 \pm 6 \pm 7$	18 CHETYRKIN	06	THEO
$111 \pm 6 \pm 10$	19 GOCKELER	06	LATT
$119 \pm 5 \pm 8$	20 GOCKELER	06A	LATT
92 ± 9	21 JAMIN	06	THEO
87 ± 6	22 MASON	06	LATT
104 ± 15	23 NARISON	06	THEO
$\geq 71 \pm 4, \leq 151 \pm 14$	24 NARISON	06	THEO
$96^{+5}_{-3}^{+16}_{-18}$	25 BAIKOV	05	THEO
81 ± 22	26 GAMIZ	05	THEO
125 ± 28	27 GORBUNOV	05	THEO
93 ± 32	28 NARISON	05	THEO
76 ± 8	29 AUBIN	04	LATT
$116 \pm 6 \pm 0.65$	30 AOKI	03	LATT
$84.5^{+12}_{-1.7}$	31 AOKI	03B	LATT
$106 \pm 2 \pm 8$	32 BECIREVIC	03	LATT
$92 \pm 9 \pm 16$	33 CHIU	03	LATT
117 ± 17	34 GAMIZ	03	THEO
103 ± 17	35 GAMIZ	03	THEO

¹ ANANTHANARAYAN 16 determine $\overline{m}_s(2 \text{ GeV}) = 106.70 \pm 9.36 \text{ MeV}$ and $74.47 \pm 7.77 \text{ MeV}$ from fits to ALEPH and OPAL τ decay data, respectively. We have used the weighted average of the two.

² CHAKRABORTY 15 is a lattice QCD computation that determines m_c and m_c/m_s using pseudoscalar mesons masses tuned on gluon field configurations with 2+1+1 dynamical flavors of HISQ quarks with u/d masses down to the physical value.

³ CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.

⁴ ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions. Masses at $\mu = 3 \text{ GeV}$ have been converted to $\mu = 2 \text{ GeV}$ using conversion factors given in their paper.

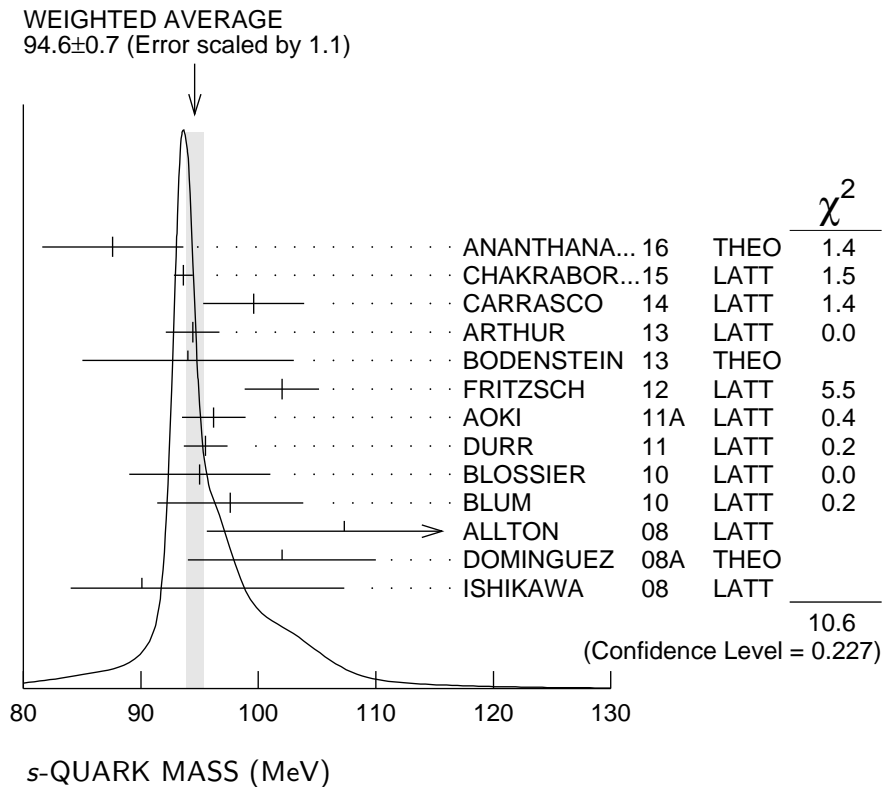
⁵ BODENSTEIN 13 determines m_s from QCD finite energy sum rules, and the perturbative computation of the pseudoscalar correlator to five-loop order.

⁶ FRITZSCH 12 determine m_s using a lattice computation with $N_f = 2$ dynamical flavors.

⁷ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.

- 8 DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.
- 9 BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.
- 10 BLUM 10 determines light quark masses using a QCD plus QED lattice computation of the electromagnetic mass splittings of the low-lying hadrons. The lattice simulations use 2+1 dynamical quark flavors.
- 11 ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- 12 DOMINGUEZ 08A make determination from QCD finite energy sum rules for the pseudoscalar two-point function computed to order α_s^4 .
- 13 ISHIKAWA 08 use a lattice computation of the light meson spectrum with 2+1 dynamical flavors of $\mathcal{O}(a)$ improved Wilson quarks, and one-loop perturbative renormalization.
- 14 DAVIES 10 and MCNEILE 10 determine $\overline{m}_c(\mu)/\overline{m}_s(\mu) = 11.85 \pm 0.16$ using a lattice computation with $N_f = 2 + 1$ dynamical fermions of the pseudoscalar meson masses. Mass m_s is obtained from this using the value of m_c from ALLISON 08 or MCNEILE 10.
- 15 BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- 16 NAKAMURA 08 do a lattice computation using quenched domain wall fermions and non-perturbative renormalization.
- 17 BLUM 07 determine quark masses from the pseudoscalar meson masses using a QED plus QCD lattice computation with two dynamical quark flavors.
- 18 CHETYRKIN 06 use QCD sum rules in the pseudoscalar channel to order α_s^4 .
- 19 GOCKELER 06 use an unquenched lattice computation of the axial Ward Identity with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization, to obtain $\overline{m}_s(2 \text{ GeV}) = 111 \pm 6 \pm 4 \pm 6 \text{ MeV}$, where the first error is statistical, the second and third are systematic due to the fit range and force scale uncertainties, respectively. We have combined the systematic errors linearly.
- 20 GOCKELER 06A use an unquenched lattice computation of the pseudoscalar meson masses with $N_f = 2$ dynamical light quark flavors, and non-perturbative renormalization.
- 21 JAMIN 06 determine $\overline{m}_s(2 \text{ GeV})$ from the spectral function for the scalar $K\pi$ form factor.
- 22 MASON 06 extract light quark masses from a lattice simulation using staggered fermions with an improved action, and three dynamical light quark flavors with degenerate u and d quarks. Perturbative corrections were included at NNLO order.
- 23 NARISON 06 uses sum rules for $e^+ e^- \rightarrow$ hadrons to order α_s^3 .
- 24 NARISON 06 obtains the quoted range from positivity of the spectral functions.
- 25 BAIKOV 05 determines $\overline{m}_s(M_\tau) = 100^{+5+17}_{-3-19}$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^3 , with an estimate of the α_s^4 terms. We have converted the result to $\mu = 2 \text{ GeV}$.
- 26 GAMIZ 05 determines $\overline{m}_s(2 \text{ GeV})$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^2 , with an estimate of the α_s^3 terms.
- 27 GORBUNOV 05 use hadronic tau decays to N³LO, including power corrections.
- 28 NARISON 05 determines $\overline{m}_s(2 \text{ GeV})$ from sum rules using the strange spectral function in τ decay. The computations were done to order α_s^3 .
- 29 AUBIN 04 perform three flavor dynamical lattice calculation of pseudoscalar meson masses, with one-loop perturbative renormalization constant.
- 30 AOKI 03 uses quenched lattice simulation of the meson and baryon masses with degenerate light quarks. The extrapolations are done using quenched chiral perturbation theory. Determines $m_s = 113.8 \pm 2.3^{+5.8}_{-2.9}$ using K mass as input and $m_s = 142.3 \pm 5.8^{+22}_{-0}$ using ϕ mass as input. We have performed a weighted average of these values.

- 31 AOKI 03B uses lattice simulation of the meson and baryon masses with two dynamical light quarks. Simulations are performed using the $\mathcal{O}(a)$ improved Wilson action.
- 32 BECIREVIC 03 perform quenched lattice computation using the vector and axial Ward identities. Uses $\mathcal{O}(a)$ improved Wilson action and nonperturbative renormalization. They also quote $\bar{m}/m_s = 24.3 \pm 0.2 \pm 0.6$.
- 33 CHIU 03 determines quark masses from the pion and kaon masses using a lattice simulation with a chiral fermion action in quenched approximation.
- 34 GAMIZ 03 determines m_s from SU(3) breaking in the τ hadronic width. The value of V_{us} is chosen to satisfy CKM unitarity.
- 35 GAMIZ 03 determines m_s from SU(3) breaking in the τ hadronic width. The value of V_{us} is taken from the PDG.



OTHER LIGHT QUARK MASS RATIOS

m_s/m_d MASS RATIO

VALUE	DOCUMENT ID	TECN	COMMENT
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17-22 OUR EVALUATION

• • • We do not use the following data for averages, fits, limits, etc. • • •

20.0	1 GAO	97	THEO
18.9±0.8	2 LEUTWYLER	96	THEO Compilation
21	3 DONOGHUE	92	THEO
18	4 GERARD	90	THEO
18 to 23	5 LEUTWYLER	90B	THEO

- ¹ BAZAVOV 14A is a lattice computation using 4 dynamical flavors of HISQ fermions.
- ² CARRASCO 14 is a lattice QCD computation of light quark masses using 2 + 1 + 1 dynamical quarks, with $m_u = m_d \neq m_s \neq m_c$. The u and d quark masses are obtained separately by using the K meson mass splittings and lattice results for the electromagnetic contributions.
- ³ ARTHUR 13 is a lattice computation using 2+1 dynamical domain wall fermions.
- ⁴ AOKI 11A determine quark masses from a lattice computation of the hadron spectrum using $N_f = 2 + 1$ dynamical flavors of domain wall fermions.
- ⁵ DURR 11 determine quark mass from a lattice computation of the meson spectrum using $N_f = 2 + 1$ dynamical flavors. The lattice simulations were done at the physical quark mass, so that extrapolation in the quark mass was not needed.
- ⁶ BLOSSIER 10 determines quark masses from a computation of the hadron spectrum using $N_f=2$ dynamical twisted-mass Wilson fermions.
- ⁷ ALLTON 08 use a lattice computation of the π , K , and Ω masses with 2+1 dynamical flavors of domain wall quarks, and non-perturbative renormalization.
- ⁸ BLOSSIER 08 use a lattice computation of pseudoscalar meson masses and decay constants with 2 dynamical flavors and non-perturbative renormalization.
- ⁹ OLLER 07A use unitarized chiral perturbation theory to order p^4 .
- ¹⁰ Three flavor dynamical lattice calculation of pseudoscalar meson masses.

Q MASS RATIO

$$Q \equiv \sqrt{(m_s^2 - \bar{m}^2)/(m_d^2 - m_u^2)}; \quad \bar{m} \equiv (m_u + m_d)/2$$

VALUE	DOCUMENT ID	TECN
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• • • We do not use the following data for averages, fits, limits, etc. • • •

22.0 ± 0.7	¹ COLANGELO 17	
23.4 ± 0.4 ± 0.5	² FODOR 16	LATT
22.8 ± 0.4	³ MARTEMYA... 05	THEO
22.7 ± 0.8	⁴ ANISOVICH 96	THEO

¹ COLANGELO 17 obtain Q from a dispersive analysis of KLOE collaboration data on $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays and chiral perturbation theory input.

² FODOR 16 is a lattice simulation with $N_f = 2 + 1$ dynamical flavors and includes partially quenched QED effects.

³ MARTEMYANOV 05 determine Q from $\eta \rightarrow 3\pi$ decay.

⁴ ANISOVICH 96 find Q from $\eta \rightarrow \pi^+ \pi^- \pi^0$ decay using dispersion relations and chiral perturbation theory.

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