## $\boldsymbol{C P}$ VIOLATION IN $\boldsymbol{K}_{\boldsymbol{L}}$ DECAYS

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The symmetries $C$ (particle-antiparticle interchange) and $P$ (space inversion) hold for strong and electromagnetic interactions. After the discovery of large $C$ and $P$ violation in the weak interactions, it appeared that the product $C P$ was a good symmetry. In $1964 C P$ violation was observed in $K^{0}$ decays at a level given by the parameter $\epsilon \approx 2.3 \times 10^{-3}$.

A unified treatment of $C P$ violation in $K, D, B$, and $B_{s}$ mesons is given in "CP Violation in Meson Decays" by D. Kirkby and Y. Nir in this Review. A more detailed review including a thorough discussion of the experimental techniques used to determine $C P$ violation parameters is given in a book by K. Kleinknecht [1]. Here we give a concise summary of the formalism needed to define the parameters of $C P$ violation in $K_{L}$ decays, and a description of our fits for the best values of these parameters.

## 1. Formalism for $C P$ violation in Kaon decay:

$C P$ violation has been observed in the semi-leptonic decays $K_{L}^{0} \rightarrow \pi^{\mp} \ell^{ \pm} \nu$, and in the nonleptonic decay $K_{L}^{0} \rightarrow 2 \pi$. The experimental numbers that have been measured are

$$
\begin{align*}
A_{L} & =\frac{\Gamma\left(K_{L}^{0} \rightarrow \pi^{-} \ell^{+} \nu\right)-\Gamma\left(K_{L}^{0} \rightarrow \pi^{+} \ell^{-} \nu\right)}{\Gamma\left(K_{L}^{0} \rightarrow \pi^{-} \ell^{+} \nu\right)+\Gamma\left(K_{L}^{0} \rightarrow \pi^{+} \ell^{-} \nu\right)}  \tag{1a}\\
\eta_{+-} & =A\left(K_{L}^{0} \rightarrow \pi^{+} \pi^{-}\right) / A\left(K_{S}^{0} \rightarrow \pi^{+} \pi^{-}\right) \\
& =\left|\eta_{+-}\right| e^{i \phi_{+-}}  \tag{1b}\\
\eta_{00} & =A\left(K_{L}^{0} \rightarrow \pi^{0} \pi^{0}\right) / A\left(K_{S}^{0} \rightarrow \pi^{0} \pi^{0}\right) \\
& =\left|\eta_{00}\right| e^{i \phi_{00}} . \tag{1c}
\end{align*}
$$

$C P$ violation can occur either in the $K^{0}-\bar{K}^{0}$ mixing or in the decay amplitudes. Assuming $C P T$ invariance, the mass eigenstates of the $K^{0}-\bar{K}^{0}$ system can be written

$$
\begin{equation*}
\left|K_{S}\right\rangle=p\left|K^{0}\right\rangle+q\left|\bar{K}^{0}\right\rangle, \quad\left|K_{L}\right\rangle=p\left|K^{0}\right\rangle-q\left|\bar{K}^{0}\right\rangle \tag{2}
\end{equation*}
$$

If $C P$ invariance held, we would have $q=p$ so that $K_{S}$ would be $C P$-even and $K_{L} C P$-odd. (We define $\left|\bar{K}^{0}\right\rangle$ as $C P\left|K^{0}\right\rangle$ ).
$C P$ violation in $K^{0}-\bar{K}^{0}$ mixing is then given by the parameter $\widetilde{\epsilon}$ where

$$
\begin{equation*}
\frac{p}{q}=\frac{(1+\widetilde{\epsilon})}{(1-\widetilde{\epsilon})} \tag{3}
\end{equation*}
$$

$C P$ violation can also occur in the decay amplitudes

$$
\begin{equation*}
A\left(K^{0} \rightarrow \pi \pi(I)\right)=A_{I} e^{i \delta_{I}}, \quad A\left(\bar{K}^{0} \rightarrow \pi \pi(I)\right)=A_{I}^{*} e^{i \delta_{I}} \tag{4}
\end{equation*}
$$

where $I$ is the isospin of $\pi \pi, \delta_{I}$ is the final-state phase shift, and $A_{I}$ would be real if $C P$ invariance held. The $C P$-violating observables are usually expressed in terms of $\epsilon$ and $\epsilon^{\prime}$ defined by

$$
\begin{equation*}
\eta_{+-}=\epsilon+\epsilon^{\prime}, \quad \eta_{00}=\epsilon-2 \epsilon^{\prime} . \tag{5a}
\end{equation*}
$$

One can then show [2]

$$
\begin{gather*}
\epsilon=\widetilde{\epsilon}+i\left(\operatorname{Im} A_{0} / \operatorname{Re} A_{0}\right)  \tag{5b}\\
\sqrt{2} \epsilon^{\prime}=i e^{i\left(\delta_{2}-\delta_{0}\right)}\left(\operatorname{Re} A_{2} / \operatorname{Re} A_{0}\right)\left(\operatorname{Im} A_{2} / \operatorname{Re} A_{2}-\operatorname{Im} A_{0} / \operatorname{Re} A_{0}\right)
\end{gather*}
$$

$$
\begin{equation*}
A_{L}=2 \operatorname{Re} \epsilon /\left(1+|\epsilon|^{2}\right) \approx 2 \operatorname{Re} \epsilon \tag{5c}
\end{equation*}
$$

In Eqs. (5a), small corrections [3] of order $\epsilon^{\prime} \times \operatorname{Re}\left(A_{2} / A_{0}\right)$ are neglected, and Eq. ( $5 d$ ) assumes the $\Delta S=\Delta Q$ rule.

The quantities $\operatorname{Im} A_{0}, \operatorname{Im} A_{2}$, and $\operatorname{Im} \tilde{\epsilon}$ depend on the choice of phase convention, since one can change the phases of $K^{0}$ and $\bar{K}^{0}$ by a transformation of the strange quark state $|s\rangle \rightarrow|s\rangle e^{i \alpha}$; of course, observables are unchanged. It is possible by a choice of phase convention to set $\operatorname{Im} A_{0}$ or $\operatorname{Im} A_{2}$ or $\operatorname{Im} \tilde{\epsilon}$ to zero, but none of these is zero with the usual phase conventions in the Standard Model. The choice $\operatorname{Im} A_{0}=0$ is called the Wu-Yang phase convention [4], in which case $\epsilon=\widetilde{\epsilon}$. The value of $\epsilon^{\prime}$ is independent of phase convention, and a nonzero value demonstrates $C P$ violation in the decay amplitudes, referred to as direct $C P$ violation. The possibility that direct $C P$ violation is essentially zero, and that $C P$ violation occurs only in the mixing matrix, was referred to as the superweak theory [5].

By applying $C P T$ invariance and unitarity the phase of $\epsilon$ is given approximately by

$$
\begin{gather*}
\phi_{\epsilon} \approx \tan ^{-1} \frac{2\left(m_{K_{L}}-m_{K_{S}}\right)}{\Gamma_{K_{S}}-\Gamma_{K_{L}}} \approx 43.52 \pm 0.05^{\circ}  \tag{6a}\\
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\end{gather*}
$$

while Eq. (5c) gives the phase of $\epsilon^{\prime}$ to be

$$
\begin{equation*}
\phi_{\epsilon^{\prime}}=\delta_{2}-\delta_{0}+\frac{\pi}{2} \approx 42.3 \pm 1.5^{\circ} \tag{6b}
\end{equation*}
$$

where the numerical value is based on an analysis of $\pi-\pi$ scattering using chiral perturbation theory [6]. The approximation in Eq. (6a) depends on the assumption that direct $C P$ violation is very small in all $K^{0}$ decays. This is expected to be good to a few tenths of a degree, as indicated by the small value of $\epsilon^{\prime}$ and of $\eta_{+-0}$ and $\eta_{000}$, the $C P$-violation parameters in the decays $K_{S} \rightarrow \pi^{+} \pi^{-} \pi^{0}[7]$, and $K_{S} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ [8]. The relation in Eq. (6a) is exact in the superweak theory, so this is sometimes called the superweak-phase $\phi_{\mathrm{SW}}$. An important point for the analysis is that $\cos \left(\phi_{\epsilon^{\prime}-} \phi_{\epsilon}\right) \simeq 1$. The consequence is that only two real quantities need be measured, the magnitude of $\epsilon$ and the value of $\left(\epsilon^{\prime} / \epsilon\right)$, including its sign. The measured quantity $\left|\eta_{00} / \eta_{+-}\right|^{2}$ is very close to unity so that we can write

$$
\begin{gather*}
\left|\eta_{00} / \eta_{+-}\right|^{2} \approx 1-6 \operatorname{Re}\left(\epsilon^{\prime} / \epsilon\right) \approx 1-6 \epsilon^{\prime} / \epsilon  \tag{7a}\\
\operatorname{Re}\left(\epsilon^{\prime} / \epsilon\right) \approx \frac{1}{3}\left(1-\left|\eta_{00} / \eta_{+-}\right|\right) \tag{7b}
\end{gather*}
$$

From the experimental measurements in this edition of the Review, and the fits discussed in the next section, one finds

$$
\begin{gather*}
|\epsilon|=(2.228 \pm 0.011) \times 10^{-3}  \tag{8a}\\
\phi_{\epsilon}=(43.5 \pm 0.5)^{\circ}  \tag{8b}\\
\operatorname{Re}\left(\epsilon^{\prime} / \epsilon\right) \approx \epsilon^{\prime} / \epsilon=(1.66 \pm 0.23) \times 10^{-3}  \tag{8c}\\
\phi_{+-}=(43.4 \pm 0.5)^{\circ}  \tag{8d}\\
\phi_{00^{-}} \phi_{+-}=(0.34 \pm 0.32)^{\circ}  \tag{8e}\\
A_{L}=(3.32 \pm 0.06) \times 10^{-3} \tag{8f}
\end{gather*}
$$

Direct $C P$ violation, as indicated by $\epsilon^{\prime} / \epsilon$, is expected in the Standard Model. However, the numerical value cannot be reliably predicted because of theoretical uncertainties [9]. The value of $A_{L}$ agrees with Eq. (5d). The values of $\phi_{+-}$and $\phi_{00}-\phi_{+-}$are used to set limits on CPT violation [see "Tests of Conservation Laws"].

## 2. Fits for $K_{L}^{0} C P-v i o l a t i o n ~ p a r a m e t e r s: ~$

In recent years, $K_{L}^{0} C P$-violation experiments have improved our knowledge of $C P$-violation parameters, and their consistency with the expectations of $C P T$ invariance and unitarity. To determine the best values of the $C P$-violation parameters in $K_{L}^{0} \rightarrow \pi^{+} \pi^{-}$and $\pi^{0} \pi^{0}$ decay, we make two types of fits, one for the phases $\phi_{+-}$and $\phi_{00}$ jointly with $\Delta m$ and $\tau_{S}$, and the other for the amplitudes $\left|\eta_{+-}\right|$and $\left|\eta_{00}\right|$ jointly with the $K_{L}^{0} \rightarrow \pi \pi$ branching fractions.

Fits to $\phi_{+-}, \phi_{00}, \Delta \phi, \Delta m$, and $\tau_{S}$ data: These are joint fits to the data on $\phi_{+-}, \phi_{00}$, the phase difference $\Delta \phi=\phi_{00}-\phi_{+-}$, the $K_{L}^{0}-K_{S}^{0}$ mass difference $\Delta m$, and the $K_{S}^{0}$ mean life $\tau_{S}$, including the effects of correlations.

Measurements of $\phi_{+-}$and $\phi_{00}$ are highly correlated with $\Delta m$ and $\tau_{S}$. Some measurements of $\tau_{S}$ are correlated with $\Delta m$. The correlations are given in the footnotes of the $\phi_{+-}$and $\phi_{00}$ sections of the $K_{L}^{0}$ Listings, and the $\tau_{S}$ section of the $K_{S}^{0}$ Listings.

In most cases, the correlations are quoted as $100 \%$, i.e., with the value and error of $\phi_{+-}$or $\phi_{00}$ given at a fixed value of $\Delta m$ and $\tau_{S}$, with additional terms specifying the dependence of the value on $\Delta m$ and $\tau_{S}$. These cases lead to diagonal bands in Figs. 1 and 2. The KTeV experiment [10] quotes its results as values of $\Delta m, \tau_{S}, \phi_{\epsilon}, \operatorname{Re}\left(\epsilon^{\prime} / \epsilon\right)$, and $\operatorname{Im}\left(\epsilon^{\prime} / \epsilon\right)$ with correlations, leading to the ellipses labeled "b." The correlations for the KTeV measurements are given in the $\operatorname{Im}\left(\epsilon^{\prime} / \epsilon\right)$ section of the $K_{L}^{0}$ Listings. For small $\left|\epsilon^{\prime} / \epsilon\right|, \phi_{+-} \approx \phi_{\epsilon}+\operatorname{Im}\left(\epsilon^{\prime} / \epsilon\right)$.

The data on $\tau_{S}, \Delta m$, and $\phi_{+-}$shown in Figs. 1 and 2 are combined with data on $\phi_{00}$ and $\phi_{00}-\phi_{+-}$in two fits, one without assuming $C P T$, and the other with this assumption. The results without assuming $C P T$ are shown as ellipses labeled "a." These ellipses are seen to be in good agreement with the superweak phase

$$
\begin{equation*}
\phi_{\mathrm{SW}}=\tan ^{-1}\left(\frac{2 \Delta m}{\Delta \Gamma}\right)=\tan ^{-1}\left(\frac{2 \Delta m \tau_{S} \tau_{L}}{\hbar\left(\tau_{L}-\tau_{S}\right)}\right) \tag{9}
\end{equation*}
$$

In Figs. 1 and 2, $\phi_{\text {SW }}$ is shown as narrow bands labeled "j."


Figure 1: $\phi_{+-}$vs $\Delta m$ for experiments which do not assume $C P T$ invariance. $\Delta m$ measurements appear as vertical bands spanning $\Delta m \pm$ $1 \sigma$, cut near the top and bottom to aid the eye. Most $\phi_{+-}$measurements appear as diagonal bands spanning $\phi_{+-} \pm \sigma_{\phi}$. Data are labeled by letters: "b"-FNAL KTeV, "c"-CERN CPLEAR, "d"-FNAL E773, "e"-FNAL E731, "f"-CERN, " g "-CERN NA31, and are cited in Table 1. The narrow band " j " shows $\phi_{\mathrm{SW}}$. The ellipse "a" shows the $\chi^{2}=1$ contour of the fit result.

Table 2 column 2, "Fit w/o CPT," gives the resulting fitted parameters, while Table 3 gives the correlation matrix for this fit. The white ellipses labeled "a" in Fig. 1 and Fig. 2 are the $\chi^{2}=1$ contours for this fit.

For experiments which have dependencies on unseen fit parameters, that is, parameters other than those shown on the x or y axis of the figure, their band positions are evaluated using the fit results and their band widths include the fitted

Table 1: References, Document ID's, and sources corresponding to the letter labels in the figures. The data are given in the $\phi_{+-}$and $\Delta m$ sections of the $K_{L}$ Listings, and the $\tau_{S}$ section of the $K_{S}$ Listings.

| Label Source | PDG Document ID | Ref. |  |
| :---: | :--- | :--- | :---: |
| a | this Review | OUR FIT |  |
| b | FNAL KTeV | ABOUZAID 11 | $[10]$ |
| c | CERN CPLEAR | APOSTOLAKIS 99C | $[11]$ |
| d | FNAL E773 | SCHWINGENHEUER 95 | $[12]$ |
| e | FNAL E731 | GIBBONS 93,93C | $[13,14]$ |
| f | CERN | GEWENIGER 74B,74C | $[15,16]$ |
| g | CERN NA31 | CAROSI 90 | $[17]$ |
| h | CERN NA48 | LAI 02C | $[18]$ |
| i | CERN NA31 | BERTANZA 97 | $[19]$ |
| j | this Review | SUPERWEAK 16 |  |

uncertainty in the unseen parameters. This is also true for the $\phi_{\text {SW }}$ bands.

If $C P T$ invariance and unitarity are assumed, then by Eq. (6a), the phase of $\epsilon$ is constrained to be approximately equal to
$\phi_{\mathrm{SW}}=(43.50258 \pm 0.00021)^{\circ}+54.1(\Delta m-0.5289)^{\circ}+32.0\left(\tau_{S}-0.89564\right)$
where we have linearized the $\Delta m$ and $\tau_{S}$ dependence of Eq. (9). The error $\pm 0.00021$ is due to the uncertainty in $\tau_{L}$. Here $\Delta m$ has units $10^{10} \hbar \mathrm{~s}^{-1}$ and $\tau_{S}$ has units $10^{-10} \mathrm{~s}$.

If in addition we use the observation that $\operatorname{Re}\left(\epsilon^{\prime} / \epsilon\right) \ll 1$ and $\cos \left(\phi_{\epsilon^{\prime}}-\phi_{\epsilon}\right) \simeq 1$, as well as the numerical value of $\phi_{\epsilon^{\prime}}$ given in Eq. (6b), then Eqs. (5a), which are sketched in Fig. 3, lead to the constraint

$$
\begin{align*}
\phi_{00}-\phi_{+-} & \approx-3 \operatorname{Im}\left(\frac{\epsilon^{\prime}}{\epsilon}\right) \\
& \approx-3 \operatorname{Re}\left(\frac{\epsilon^{\prime}}{\epsilon}\right) \tan \left(\phi_{\epsilon^{\prime}}-\phi_{\epsilon}\right) \\
& \approx 0.006^{\circ} \pm 0.008^{\circ}, \tag{11}
\end{align*}
$$

so that $\phi_{+-} \approx \phi_{00} \approx \phi_{\epsilon} \approx \phi_{\mathrm{SW}}$.


Figure 2: $\phi_{+-}$vs $\tau_{S} . \tau_{S}$ measurements appear as vertical bands spanning $\tau_{S} \pm 1 \sigma$, some of which are cut near the top and bottom to aid the eye. Most $\phi_{+-}$measurements appear as diagonal or horizontal bands spanning $\phi_{+-} \pm \sigma_{\phi}$. Data are labeled by letters: "b"-FNAL KTeV, "c"-CERN CPLEAR, "d"-FNAL E773, "e"FNAL E731, "f"-CERN, "g"-CERN NA31, "h"-CERN NA48, "i"-CERN NA31, and are cited in Table 1. The narrow band " j " shows $\phi_{\text {SW }}$. The ellipse "a" shows the fit result's $\chi^{2}=1$ contour.

In the fit assuming $C P T$, we constrain $\phi_{\epsilon}=\phi_{\mathrm{SW}}$ using the linear expression in Eq. (10), and constrain $\phi_{00}-\phi_{+-}$using Eq. (11). These constraints are inserted into the Listings with the Document ID of SUPERWEAK 16. Some additional data for which the authors assumed $C P T$ are added to this fit or substitute for other less precise data for which the authors did not make this assumption. See the Listings for details.

Table 2: Fit results for $\phi_{+-}, \Delta m, \tau_{S}, \phi_{00}$, $\Delta \phi=\phi_{00}-\phi_{+-}$, and $\phi_{\epsilon}$ without and with the $C P T$ assumption.

| Quantity(units) | Fit w/o $C P T$ | Fit w/ $C P T$ |
| :--- | :---: | :---: |
| $\phi_{+-}\left({ }^{\circ}\right)$ | $43.4 \pm 0.5(\mathrm{~S}=1.2)$ | $43.51 \pm 0.05(\mathrm{~S}=1.2)$ |
| $\Delta m\left(10^{10} \hbar \mathrm{~s}^{-1}\right)$ | $0.5289 \pm 0.0010$ | $0.5293 \pm 0.0009(\mathrm{~S}=1.3)$ |
| $\tau_{S}\left(10^{-10} \mathrm{~s}\right)$ | $0.89564 \pm 0.00033$ | $0.8954 \pm 0.0004(\mathrm{~S}=1.1)$ |
| $\phi_{00}\left({ }^{\circ}\right)$ | $43.7 \pm 0.6(\mathrm{~S}=1.2)$ | $43.52 \pm 0.05(\mathrm{~S}=1.3)$ |
| $\Delta \phi\left({ }^{\circ}\right)$ | $0.34 \pm 0.32$ | $0.006 \pm 0.014(\mathrm{~S}=1.7)$ |
| $\phi_{\epsilon}\left({ }^{\circ}\right)$ | $43.5 \pm 0.5(\mathrm{~S}=1.3)$ | $43.52 \pm 0.05(\mathrm{~S}=1.2)$ |
| $\chi^{2}$ | 16.4 | 20.0 |
| $\#$ Deg. Free. | 14 | 16 |



Figure 3: Sketch of Eqs. (5a). Not to scale.
The results of this fit are shown in Table 2, column 3, "Fit w/CPT," and the correlation matrix is shown in Table 4. The $\Delta m$ precision is improved by the $C P T$ assumption.

Table 3: Correlation matrix for the results of the fit without the $C P T$ assumption

|  | $\phi_{+-}$ | $\Delta m$ | $\tau_{S}$ | $\phi_{00}$ | $\Delta \phi$ | $\phi_{\epsilon}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $\phi_{+-}$ | 1.000 | 0.596 | -0.488 | 0.827 | -0.040 | 0.976 |
| $\Delta m$ | 0.596 | 1.000 | -0.572 | 0.487 | -0.035 | 0.580 |
| $\tau_{S}$ | -0.488 | -0.572 | 1.000 | -0.423 | -0.014 | -0.484 |
| $\phi_{00}$ | 0.827 | 0.487 | -0.423 | 1.000 | 0.529 | 0.929 |
| $\Delta \phi$ | -0.040 | -0.035 | -0.014 | 0.529 | 1.000 | 0.178 |
| $\phi_{\epsilon}$ | 0.976 | 0.580 | -0.484 | 0.929 | 0.178 | 1.000 |

Table 4: Correlation matrix for the results of the fit with the $C P T$ assumption

|  | $\phi_{+-}$ | $\Delta m$ | $\tau_{S}$ | $\phi_{00}$ | $\Delta \phi$ | $\phi_{\epsilon}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $\phi_{+-}$ | 1.000 | 0.972 | -0.311 | 0.957 | -0.105 | 0.995 |
| $\Delta m$ | 0.972 | 1.000 | -0.509 | 0.958 | -0.007 | 0.977 |
| $\tau_{S}$ | -0.311 | -0.509 | 1.000 | -0.306 | 0.004 | -0.312 |
| $\phi_{00}$ | 0.957 | 0.958 | -0.306 | 1.000 | 0.189 | 0.981 |
| $\Delta \phi$ | -0.105 | -0.007 | 0.004 | 0.189 | 1.000 | -0.006 |
| $\phi_{\epsilon}$ | 0.995 | 0.977 | -0.312 | 0.981 | -0.006 | 1.000 |

Fits for $\epsilon^{\prime} / \epsilon,\left|\eta_{+-}\right|,\left|\eta_{00}\right|$, and $\mathrm{B}\left(K_{L} \rightarrow \pi \pi\right)$
We list measurements of $\left|\eta_{+-}\right|,\left|\eta_{00}\right|,\left|\eta_{00} / \eta_{+-}\right|$, and $\epsilon^{\prime} / \epsilon$. Independent information on $\left|\eta_{+-}\right|$and $\left|\eta_{00}\right|$ can be obtained from measurements of the $K_{L}^{0}$ and $K_{S}^{0}$ lifetimes $\left(\tau_{L}, \tau_{S}\right)$, and branching ratios (B) to $\pi \pi$, using the relations

$$
\begin{align*}
\left|\eta_{+-}\right| & =\left[\frac{\mathrm{B}\left(K_{L}^{0} \rightarrow \pi^{+} \pi^{-}\right)}{\tau_{L}} \frac{\tau_{S}}{\mathrm{~B}\left(K_{S}^{0} \rightarrow \pi^{+} \pi^{-}\right)}\right]^{1 / 2}  \tag{12a}\\
\left|\eta_{00}\right| & =\left[\frac{\mathrm{B}\left(K_{L}^{0} \rightarrow \pi^{0} \pi^{0}\right)}{\tau_{L}} \frac{\tau_{S}}{\mathrm{~B}\left(K_{S}^{0} \rightarrow \pi^{0} \pi^{0}\right)}\right]^{1 / 2} \tag{12b}
\end{align*}
$$

For historical reasons, the branching ratio fits and the $C P$-violation fits are done separately, but we want to include the influence of $\left|\eta_{+-}\right|,\left|\eta_{00}\right|,\left|\eta_{00} / \eta_{+-}\right|$, and $\epsilon^{\prime} / \epsilon$ measurements on $\mathrm{B}\left(K_{L}^{0} \rightarrow \pi^{+} \pi^{-}\right)$and $\mathrm{B}\left(K_{L}^{0} \rightarrow \pi^{0} \pi^{0}\right)$ and vice versa. We
approximate a global fit to all of these measurements by first performing two independent fits: 1) BRFIT, a fit to the $K_{L}^{0}$ branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the $\left|\eta_{+-}\right|,\left|\eta_{00}\right|,\left|\eta_{+-} / \eta_{00}\right|$, and $\epsilon^{\prime} / \epsilon$ measurements. The results from fit 1 , along with the $K_{S}^{0}$ values from this edition, are used to compute values of $\left|\eta_{+-}\right|$and $\left|\eta_{00}\right|$, which are included as measurements in the $\left|\eta_{00}\right|$ and $\left|\eta_{+-}\right|$sections with a document ID of BRFIT 16. Thus, the fit values of $\left|\eta_{+-}\right|$and $\left|\eta_{00}\right|$ given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct $|\eta|$ measurements in the branching ratio fit. The results from fit 2 above (before including BRFIT 16 values) are used along with the $K_{L}^{0}$ and $K_{S}^{0}$ mean lives and the $K_{S}^{0} \rightarrow \pi \pi$ branching fractions to compute the $K_{L}^{0}$ branching ratio $\Gamma\left(K_{L}^{0} \rightarrow\right.$ $\left.\pi^{0} \pi^{0}\right) / \Gamma\left(K_{L}^{0} \rightarrow \pi^{+} \pi^{-}\right)$. This branching ratio value is included as a measurement in the branching ratio section with a document ID of ETAFIT 16. Thus, the $K_{L}^{0}$ branching ratio fit values in this edition include the results of the direct measurement of $\left|\eta_{00} / \eta_{+-}\right|$and $\epsilon^{\prime} / \epsilon$. Most individual measurements of $\left|\eta_{+-}\right|$and $\left|\eta_{00}\right|$ enter our fits directly via the corresponding measurements of $\Gamma\left(K_{L}^{0} \rightarrow \pi^{+} \pi^{-}\right) / \Gamma($ total $)$ and $\Gamma\left(K_{L}^{0} \rightarrow \pi^{0} \pi^{0}\right) / \Gamma$ (total), and those that do not have too large errors to have any influence on the fitted values of these branching ratios. A more detailed discussion of these fits is given in the 1990 edition of this Review [20].

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