

## EXTRA DIMENSIONS

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### *I Introduction*

Proposals for a spacetime with more than three spatial dimensions date back to the 1920s, mainly through the work of Kaluza and Klein, in an attempt to unify the forces of nature [1]. Although their initial idea failed, the formalism that they and others developed is still useful nowadays. Around 1980, string theory proposed again to enlarge the number of space dimensions, this time as a requirement for describing a consistent theory of quantum gravity. The extra dimensions were supposed to be compactified at a scale close to the Planck scale, and thus not testable experimentally in the near future.

A different approach was given by Arkani-Hamed, Dimopoulos and Dvali (ADD) in their seminal paper in 1998 [2]. They showed that the weakness of gravity could be explained by postulating two or more extra dimensions in which only gravity could propagate. The size of these extra dimensions should range between roughly a millimeter and  $\sim 1/\text{TeV}$ , leading to possible observable consequences in current and future experiments. A year later, Randall and Sundrum (RS) [3] found a new possibility using a warped geometry. They postulated a five-dimensional Anti-de Sitter (AdS) spacetime with a compactification scale of order TeV. The origin of the smallness of the electroweak scale versus the Planck scale was explained by the gravitational redshift factor present in the warped AdS metric. As in the ADD model, originally only gravity was assumed to propagate in the extra dimensions, although it was soon clear that this was not necessary in the RS model and also the SM gauge fields [4] and SM fermions [5,6] could propagate in the five-dimensional space.

The physics of warped extra-dimensional models have an alternative interpretation by means of the AdS/CFT correspondence [7]. Models with warped extra dimensions are related to four-dimensional strongly-interacting theories, allowing an understanding of the properties of five-dimensional fields as those

of four-dimensional (4D) composite states [8]. This has opened new directions for tackling outstanding questions in particle physics, such as the flavor problem, grand unification, and the origin of electroweak symmetry breaking or supersymmetry breaking.

### ***1.1 Kaluza-Klein Theories***

Theories with compact extra dimensions can be written as theories in ordinary four dimensions by performing a Kaluza-Klein (KK) reduction. As an illustration, consider a simple example, namely a field theory of a complex scalar in flat five-dimensional (5D) spacetime. The action will be given by <sup>†</sup>

$$S_5 = - \int d^4x dy M_5 [|\partial_\mu \phi|^2 + |\partial_y \phi|^2 + \lambda_5 |\phi|^4] , \quad (1)$$

where  $y$  refers to the extra (fifth) dimension. A universal scale  $M_5$  has been extracted in front of the action in order to keep the 5D field with the same mass-dimension as in four dimensions. This theory is perturbative for energies  $E \lesssim \ell_5 M_5 / \lambda_5$  where  $\ell_5 = 24\pi^3$  [9].

Let us now consider that the fifth dimension is compact with the topology of a circle  $S^1$  of radius  $R$ , which corresponds to the identification of  $y$  with  $y + 2\pi R$ . In such a case, the 5D complex scalar field can be expanded in a Fourier series:

$$\phi(x, y) = \frac{1}{\sqrt{2\pi R M_5}} \sum_{n=-\infty}^{\infty} e^{iny/R} \phi^{(n)}(x) ,$$

that, inserted in Eq. (1) and integrating over  $y$ , gives

$$S_5 = S_4^{(0)} + S_4^{(n)} ,$$

where

$$S_4^{(0)} = - \int d^4x \left[ |\partial_\mu \phi^{(0)}|^2 + \lambda_4 |\phi^{(0)}|^4 \right] , \text{ and} \quad (2)$$

$$S_4^{(n)} = - \int d^4x \sum_{n \neq 0} \left[ |\partial_\mu \phi^{(n)}|^2 + \left(\frac{n}{R}\right)^2 |\phi^{(n)}|^2 \right] + \text{quartic int.}$$

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<sup>†</sup> Our convention for the metric is  $\eta_{MN} = \text{Diag}(-1, 1, 1, 1, 1)$ .

The  $n = 0$  mode self-coupling is given by

$$\lambda_4 = \frac{\lambda_5}{2\pi R M_5}. \quad (3)$$

The above action corresponds to a 4D theory with a massless scalar  $\phi^{(0)}$ , referred to as the zero-mode, and an infinite tower of massive modes  $\phi^{(n)}$ , known as KK modes. The KK reduction thus allows a treatment of 5D theories as 4D field theories with an infinite number of fields. At energies smaller than  $1/R$ , the KK modes can be neglected, leaving the zero-mode action of Eq. (2). The strength of the interaction of the zero-mode, given by Eq. (3), decreases as  $R$  increases. Thus, for a large extra dimension  $R \gg 1/M_5$ , the massless scalar is weakly coupled.

## *II Large Extra Dimensions for Gravity*

### *II.1 The ADD Scenario*

The ADD scenario [2,10,11] assumes a  $D = 4 + \delta$  dimensional spacetime, with  $\delta$  compactified spatial dimensions. The weakness of gravity arises since it propagates in the higher-dimensional space. The SM is assumed to be localized in a 4D subspace, a 3-brane, as can be found in certain string constructions [12]. Gravity is described by the Einstein-Hilbert action in  $D = 4 + \delta$  spacetime dimensions

$$S_D = -\frac{\bar{M}_D^{2+\delta}}{2} \int d^4x d^\delta y \sqrt{-g} \mathcal{R} + \int d^4x \sqrt{-g_{\text{ind}}} \mathcal{L}_{\text{SM}}, \quad (4)$$

where  $x$  labels the ordinary four coordinates,  $y$  the  $\delta$  extra coordinates,  $g$  refers to the determinant of the  $D$ -dimensional metric whose Ricci scalar is defined by  $\mathcal{R}$ , and  $\bar{M}_D$  is the reduced Planck scale of the  $D$ -dimensional theory. In the second term of Eq. (4), which gives the gravitational interactions of SM fields, the  $D$ -dimensional metric reduces to the induced metric on the 3-brane where the SM fields propagate. The extra dimensions are assumed to be flat and compactified in a volume  $V_\delta$ . As an example, consider a toroidal compactification of equal radii  $R$  and volume  $V_\delta = (2\pi R)^\delta$ . After a KK reduction, one finds that the fields that couple to the SM are the spin-2 gravitational field  $G_{\mu\nu}(x, y)$  and a tower of spin-1 KK graviscalars [13]. The graviscalars, however, only couple to SM fields through the trace

of the energy-momentum tensor, resulting in weaker couplings to the SM fields. The Fourier expansion of the spin-2 field is given by

$$G_{\mu\nu}(x, y) = G_{\mu\nu}^{(0)}(x) + \frac{1}{\sqrt{V_\delta}} \sum_{\vec{n} \neq 0} e^{i\vec{n} \cdot \vec{y}/R} G_{\mu\nu}^{(\vec{n})}(x), \quad (5)$$

where  $\vec{y} = (y_1, y_2, \dots, y_\delta)$  are the extra-dimensional coordinates and  $\vec{n} = (n_1, n_2, \dots, n_\delta)$ . Eq. (5) contains a massless state, the 4D graviton, and its KK tower with masses  $m_{\vec{n}}^2 = |\vec{n}|^2/R^2$ . At energies below  $1/R$  the action is that of the zero-mode

$$S_4^{(0)} = -\frac{\bar{M}_D^{2+\delta}}{2} \int d^4x V_\delta \sqrt{-g^{(0)}} \mathcal{R}^{(0)} + \int d^4x \sqrt{-g_{\text{ind}}^{(0)}} \mathcal{L}_{\text{SM}},$$

where we can identify the 4D reduced Planck mass,  $M_P \equiv G_N/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$  GeV, as a function of the  $D$ -dimensional parameters:

$$M_P^2 = V^\delta \bar{M}_D^{2+\delta} \equiv R^\delta M_D^{2+\delta}. \quad (6)$$

Fixing  $M_D$  at around the electroweak scale  $M_D \sim \text{TeV}$  to avoid introducing a new mass-scale in the model, Eq. (6) gives a prediction for  $R$ :

$$\delta = 1, 2, \dots, 6 \rightarrow R \sim 10^9 \text{ km}, 0.5 \text{ mm}, \dots, 0.1 \text{ MeV}^{-1}. \quad (7)$$

The option  $\delta = 1$  is clearly ruled out. However this is not the case for  $\delta \geq 2$ , and possible observable consequences can be sought in present and future experiments.

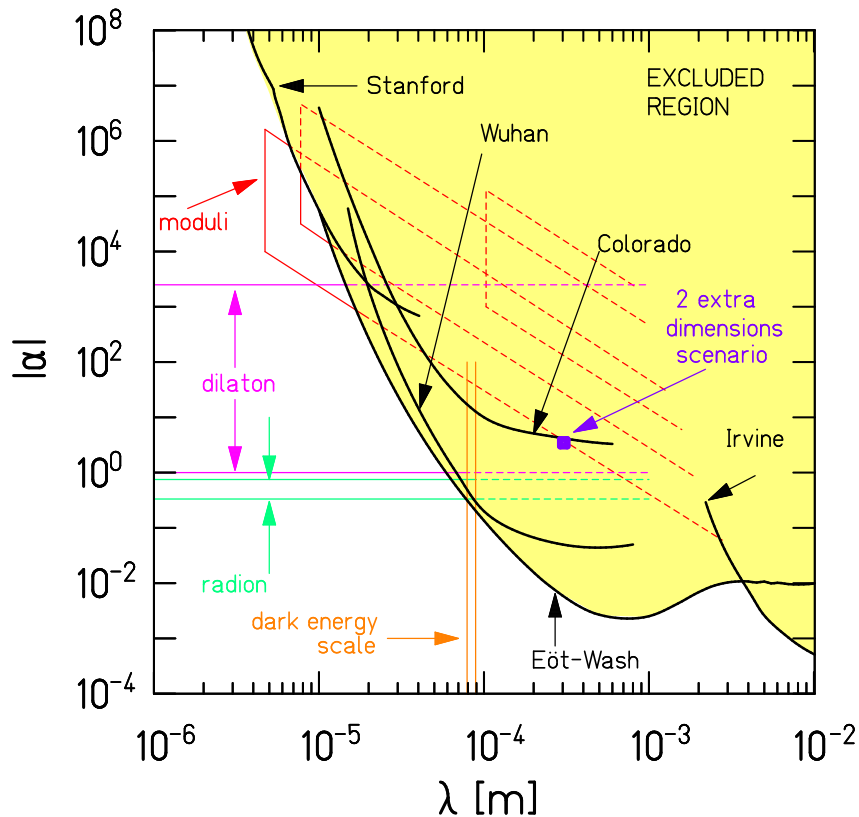
Consistency of the model requires a stabilization mechanism for the radii of the extra dimensions, to the values shown in Eq. (7). The fact that we need  $R \gg 1/M_D$  leads to a new hierarchy problem, the solution of which might require imposing supersymmetry in the extra-dimensional bulk [14].

## ***II.2 Tests of the Gravitational Force Law at Sub-mm Distances***

The KK modes of the graviton give rise to deviations from Newton's law of gravitation for distances  $\lesssim R$ . Such deviations are usually parametrized by a modified Newtonian potential of the form

$$V(r) = -G_N \frac{m_1 m_2}{r} \left[ 1 + \alpha e^{-r/\lambda} \right]. \quad (8)$$

For a 2-torus compactification,  $\alpha = 16/3$  and  $\lambda = R$ . Searches for deviations from Newton’s law of gravitation have been performed in several experiments. Fig. 1, taken from Ref. [15], gives the present constraints. We find  $R < 37\mu\text{m}$  at 95% CL for  $\delta = 2$ , corresponding to  $M_D > 3.6$  TeV.



**Figure 1:** Experimental limits (from Ref. [15]) on  $\alpha$  and  $\lambda$  of Eq. (8), which parametrize deviations from Newton’s law of gravitation.

### II.3 Astrophysical and Cosmological Constraints

The light KK gravitons could be copiously produced in stars, carrying away energy. Ensuring that the graviton luminosity is low enough to preserve the agreement of stellar models with observations provides powerful bounds on the scale  $M_D$ . The most stringent arises from supernova SN1987A, giving  $M_D > 27$  (2.4) TeV for  $\delta = 2$  (3) [16]. After a supernova explosion, most of the KK gravitons stay gravitationally trapped in the remnant neutron star. The requirement that neutron

stars are not excessively heated by KK decays into photons leads to  $M_D > 1700$  (76) TeV for  $\delta = 2$  (3) [17].

Cosmological constraints are also quite stringent [18]. To avoid overclosure of the universe by relic gravitons one needs  $M_D > 7$  TeV for  $\delta = 2$ . Relic KK gravitons decaying into photons contribute to the cosmic diffuse gamma radiation, from which one can derive the bound  $M_D > 100$  TeV for  $\delta = 2$ .

We must mention however that bounds coming from the decays of KK gravitons into photons can be reduced if we assume that KK gravitons decay mainly into other non-SM states. This could happen, for example, if there are other 3-branes with hidden sectors residing on them [10].

## ***II.4 Collider Signals***

### ***II.4a Graviton and Other Particle Production***

Although each KK graviton has a purely gravitational coupling, suppressed by  $1/M_P$ , inclusive processes in which one sums over the almost continuous spectrum of available gravitons have cross sections suppressed only by powers of  $M_D$ . Processes involving gravitons are therefore detectable in collider experiments if  $M_D \sim$  TeV. A number of experimental searches for evidence of large extra dimensions have been performed at colliders, and interpreted in the context of the ADD model.

One signature arises from direct graviton emission. By making a derivative expansion of Einstein gravity, one can construct an effective theory, valid for energies much lower than  $M_D$ , and use it to make predictions for graviton-emission processes at colliders [13,19,20]. Gravitons produced in the final state would escape detection, giving rise to missing transverse energy ( $\cancel{E}_T$ ). The results quoted below are 95% CL lower limits on  $M_D$  for a range of values of  $\delta$  between 2 and 6, with more stringent limits corresponding to lower  $\delta$  values.

A combined  $\gamma + \cancel{E}_T$  result from LEP yields limits of  $M_D > 0.66 - 1.60$  TeV [21], and less stringent results also exist from LEP for the  $Z + \cancel{E}_T$  final state. At hadron colliders, experimentally sensitive channels include the  $j + \cancel{E}_T$  and  $\gamma + \cancel{E}_T$  final states. The most stringent limits from the Tevatron include CDF results of  $M_D > 0.94 - 1.94$  TeV [22], using the combined  $j/\gamma + \cancel{E}_T$  final state. DØ results limit  $M_D > 0.80 -$

0.88 TeV [23] from  $\gamma + \cancel{E}_T$ , and  $M_D > 0.63 - 0.89$  TeV [24] from  $j + \cancel{E}_T$ . At the LHC, using a dataset of  $1 \text{ fb}^{-1}$  and assuming leading order (LO) signal cross sections, CMS sets limits of  $M_D > 2.25 - 3.67$  TeV [25] from analyzing the  $j + \cancel{E}_T$  final state, and  $M_D > 1.03 - 1.21$  TeV [26] from  $\gamma + \cancel{E}_T$ . To account for next-to-leading order (NLO) signal enhancements, sizeable k-factors, in the range  $1.3 - 2$  depending on  $\delta$ , would increase these limits by typically  $\approx 10\%$  if applied. ATLAS  $j + \cancel{E}_T$  results with  $1 \text{ fb}^{-1}$  provide limits of  $M_D > 1.68 - 3.16$  TeV [27], using LO cross sections. For the  $j + \cancel{E}_T$  analyses, the LHC experiments handle somewhat differently the issue that the effective theory is only valid for energies much less than  $M_D$ : CMS suppresses the graviton cross section by a factor  $M_D^2/\hat{s}$  for  $\sqrt{\hat{s}} > M_D$ , where  $\sqrt{\hat{s}}$  is the parton-level center-of-mass energy of the hard collision. ATLAS simply truncates the differential cross section to remove the contribution from events where  $\sqrt{\hat{s}} > M_D$ , and points out that the effect of the truncation grows with  $\delta$ , from a negligible impact for  $\delta = 2$  up to a 16% reduction in the limit for  $\delta = 6$ .

In models in which the ADD scenario is embedded in a string theory at the TeV scale [12], we expect the string scale  $M_s$  to be smaller than  $M_D$ , and therefore production at the LHC of string resonances [28]. Analysis of the dijet invariant mass distribution have been interpreted by CMS for a  $1 \text{ fb}^{-1}$  dataset to exclude at 95% CL string excitations of quarks and gluons that decay predominantly to  $q + g$  with masses less than 4 TeV [29].

#### ***II.4b Virtual graviton effects***

One can also search for virtual graviton effects, the calculation of which however depends on the ultraviolet cut-off of the theory and is therefore very model dependent. In the literature, several different formulations exist [13,20,30] for the dimension-eight operator for gravity exchange at tree level:

$$\mathcal{L}_8 = \pm \frac{4}{M_{TT}^4} \left( T_{\mu\nu} T^{\mu\nu} - \frac{1}{\delta + 2} T_\mu^\mu T_\nu^\nu \right), \quad (9)$$

where  $T_{\mu\nu}$  is the energy-momentum tensor and  $M_{TT}$  is related to  $M_D$  by some model-dependent coefficient [31]. The relations with the parametrizations of Refs. [30] and [13] are,

respectively,  $M_{TT} = M_S$  and  $M_{TT} = (2/\pi)^{1/4}\Lambda_T$ . The experimental results below are given as 95% CL lower limits on  $M_{TT}$ , including in some cases the possibility of both constructive or destructive interference, depending on the sign chosen in Eq. (9). Results from  $e^\pm p \rightarrow e^\pm X$  at HERA limit  $M_{TT} \gtrsim 800$  GeV [32]. The most sensitive limits from LEP arise from the  $e^+e^- \rightarrow ee$  and  $e^+e^- \rightarrow \gamma\gamma$  final states, with limits for the case of constructive (destructive) interference corresponding to  $M_{TT} > 1.1$  (1.0) TeV [33] and  $M_{TT} > 1.0$  (0.9) TeV [34], respectively. The most stringent results, for constructive (destructive) interference, from the Tevatron include limits of  $M_{TT} > 1.48$  (1.37) TeV [35] from  $p\bar{p} \rightarrow ee/\gamma\gamma + X$ , and  $M_{TT} > 1.48$  (1.34) TeV [36] from an analysis of the angular distributions in dijet events. Results from the LHC extend the sensitivity to higher scales. CMS has reported  $1 \text{ fb}^{-1}$  results in the diphoton and dimuon final states. For constructive interference and using NLO cross sections, the CMS  $\gamma\gamma$  and  $\mu\mu$  results correspond approximately to limits of  $M_{TT} > 2.8$  TeV [37] and 2.4 TeV [38], respectively. A  $2 \text{ fb}^{-1}$  update of the ATLAS  $\gamma\gamma$  analysis in Ref. [39] provides limits of  $M_{TT} > 2.7$  TeV (2.3 TeV) for constructive (destructive) interference.

At the one-loop level, gravitons can also generate dimension-six operators with coefficients that are also model dependent. Experimental bounds on these operators can also give stringent constraints on  $M_D$  [31].

### **II.4c Black Hole Production**

The physics at energies  $\sqrt{s} \sim M_D$  is sensitive to the details of the quantum theory of gravity. Nevertheless, in the transplanckian regime,  $\sqrt{s} \gg M_D$ , one can rely on a semiclassical description of gravity to obtain predictions. An interesting feature of transplanckian physics is the creation of black holes [40]. A black hole is expected to be formed in a collision in which the impact parameter is smaller than the Schwarzschild radius [41]:

$$R_S = \frac{1}{M_D} \left[ \frac{2^\delta \pi^{(\delta-3)/2}}{\delta+2} \Gamma\left(\frac{\delta+3}{2}\right) \frac{M_{BH}}{M_D} \right]^{1/(\delta+1)}, \quad (10)$$

where  $M_{BH}$  is the mass of the black hole, which would roughly correspond to the total energy in the collision. The cross section



for black hole production can be estimated to be of the same order as the geometric area  $\sigma \sim \pi R_S^2$ . For  $M_D \sim \text{TeV}$ , this gives a production of  $\sim 10^7$  black holes at the  $\sqrt{s} = 14 \text{ TeV}$  LHC with an integrated luminosity of  $30 \text{ fb}^{-1}$  [40]. A black hole would provide a striking experimental signature since it is expected to thermally radiate with a Hawking temperature  $T_H = (\delta + 1)/(4\pi R_S)$ , and therefore would evaporate democratically into all SM states.

At the LHC, one starts to be able to access a regime where the energies are large compared to (or at least comparable to)  $M_D$  values in the TeV range. However, given the present constraints on  $M_D$ , the LHC will not be able to reach energies much above  $M_D$ . This implies that predictions based on the semiclassical approximation could receive sizeable modifications from model-dependent quantum-gravity effects.

The LHC experiments have performed searches for microscopic black holes by looking for excesses above the SM background in final states with multiple high  $p_T$  objects [42,43], high  $p_T$  jets [44], high  $p_T$  leptons and jets [45], and in same-sign dimuon events [46]. No excesses have been observed. The results are usually quoted as model-independent limits on the cross section for new physics in the final state and kinematic region analyzed. These results can then be used to provide constraints on models of low-scale gravity and weakly-coupled string theory. In addition, limits are sometimes quoted on particular implementations of models, which are used as benchmarks to illustrate the sensitivity. For example, using  $1 \text{ fb}^{-1}$ , CMS provides limits in the mass range of 4-5 TeV [42] for semiclassical black holes, and ATLAS gets similar results [45]. ATLAS have also searched for black holes via a  $36 \text{ pb}^{-1}$  analysis of the invariant mass and angular distributions in dijet events, excluding  $M_D$  values in the range from 0.75 up to 3.26 (3.69) TeV [47] for the case of 2 (6) extra dimensions.

In weakly-coupled string models the semiclassical description of gravity fails in the energy range between  $M_s$  and  $M_s/g_s^2$  since stringy effects are important. In this regime one expects, instead of black holes, the formation of string balls, made of highly excited long strings, that could be copiously produced at

the LHC for  $M_s \sim \text{TeV}$  [48], and would evaporate thermally at the Hagedorn temperature giving rise to high-multiplicity events. CMS has interpreted their  $1 \text{ fb}^{-1}$  multi-object search to exclude the production of string balls with a minimum mass in the range of 4.1-4.5 TeV [42], depending on the details of the model. ATLAS gets similar results [45] for their  $1 \text{ fb}^{-1}$  search for an excess of events with high  $p_T$  leptons and jets.

### *III TeV-Scale Extra Dimensions*

#### *III.1 Warped Extra Dimensions*

The RS model [3] is the most attractive setup of warped extra dimensions at the TeV scale, since it provides an alternative solution to the hierarchy problem. The RS model is based on a 5D theory with the extra dimension compactified on an orbifold,  $S^1/Z_2$ , a circle  $S^1$  with the extra identification of  $y$  with  $-y$ . This corresponds to the segment  $y \in [0, \pi R]$ , a manifold with boundaries at  $y = 0$  and  $y = \pi R$ . Let us now assume that this 5D theory has a cosmological constant in the bulk  $\Lambda$ , and on the two boundaries  $\Lambda_0$  and  $\Lambda_{\pi R}$ :

$$S_5 = - \int d^4x dy \left\{ \sqrt{-g} \left[ \frac{1}{2} M_5^3 \mathcal{R} + \Lambda \right] + \sqrt{-g_0} \delta(y) \Lambda_0 + \sqrt{-g_{\pi R}} \delta(y - \pi R) \Lambda_{\pi R} \right\}, \quad (11)$$

where  $g_0$  and  $g_{\pi R}$  are the values of the determinant of the induced metric on the two respective boundaries. Einstein's equations can be solved, giving in this case the metric

$$ds^2 = a(y)^2 dx^\mu dx^\nu \eta_{\mu\nu} + dy^2, \quad a(y) = e^{-ky}, \quad (12)$$

where  $k = \sqrt{-\Lambda/(6M_5^3)}$ . Consistency of the solution requires  $\Lambda_0 = -\Lambda_{\pi R} = -\Lambda/k$ . The metric in Eq. (12) corresponds to a 5D AdS space. The factor  $a(y)$  is called the ‘‘warp’’ factor and determines how 4D scales change as a function of the position in the extra dimension. In particular, this implies that energy scales for 4D fields localized at the boundary at  $y = \pi R$  are red-shifted by a factor  $e^{-k\pi R}$  with respect to those localized at  $y = 0$ . For this reason, the boundaries at  $y = 0$  and  $y = \pi R$

are usually referred to as the ultraviolet (UV) and infrared (IR) boundaries, respectively.

As in the ADD case, we can perform a KK reduction and obtain the low-energy effective theory of the 4D massless graviton. In this case we obtain

$$M_P^2 = \int_0^{\pi R} dy e^{-2ky} M_5^3 = \frac{M_5^3}{2k} (1 - e^{-2k\pi R}). \quad (13)$$

Taking  $M_5 \sim k \sim M_P$ , we can generate an IR-boundary scale of order  $ke^{-k\pi R} \sim \text{TeV}$  for an extra dimension of radius  $R \simeq 11/k$ . Mechanisms to stabilize  $R$  to this value have been proposed [49] that, contrary to the ADD case, do not require introducing any new small or large parameter. Therefore a natural solution to the hierarchy problem can be achieved in this framework if the Higgs field, whose vacuum expectation value (VEV) is responsible for electroweak symmetry breaking, is localized at the IR-boundary where the effective mass scales are of order TeV.

In the original RS model [3], all the SM fields were assumed to be localized on the IR-boundary. Nevertheless, for the hierarchy problem, only the Higgs field has to be localized there. SM gauge bosons and fermions can propagate in the 5D bulk [4,5,6,50]. By performing a KK reduction from the 5D action of a gauge boson, we find [4]

$$\frac{1}{g_4^2} = \int_0^{\pi R} dy \frac{1}{g_5^2} = \frac{\pi R}{g_5^2},$$

where  $g_D$  ( $D = 4, 5$ ) is the gauge coupling in  $D$ -dimensions. Therefore the 4D gauge couplings can be of order one, as is the case of the SM, if we demand  $g_5^2 \sim \pi R$ . Using  $kR \sim 10$  and  $g_4 \sim 0.5$ , we obtain the 5D gauge coupling

$$g_5 \sim 4/\sqrt{k}. \quad (14)$$

Boundary kinetic terms for the gauge bosons can modify this relation, allowing for larger values of  $g_5\sqrt{k}$ .

Fermions propagating in the RS extra dimension have 4D massless zero-modes with wavefunctions which vary as  $f_0 \sim \text{Exp}[(1/2 - c_f)ky]$ , where  $c_fk$  is their 5D mass [51,6].

Depending on the free parameter  $c_f k$ , fermions can be localized either towards the UV-boundary ( $c_f > 1/2$ ) or IR-boundary ( $c_f < 1/2$ ). Since the Higgs is localized on the IR-boundary, we can generate exponentially suppressed Yukawa couplings by having the fermion zero-modes localized towards the UV-boundary, generating naturally the light SM fermion spectrum [6]. A large overlap with the wavefunction of the Higgs is needed for the top quark, in order to generate its large mass, thus requiring it to be localized towards the IR-boundary. In conclusion, the large mass hierarchies present in the SM fermion spectrum can be easily obtained in warped models via suitable choices of the order-one parameters  $c_f$  [52]. In these scenarios deviations in flavor physics from the SM predictions are expected to arise from flavor-changing KK gluon couplings [53]. These put certain constraints on the parameters of the models and predicts new physics effects to be observed in  $B$ -physics processes [54].

The masses of the KK states can also be calculated. One finds [6]

$$m_n \simeq \left( n + \frac{\alpha}{2} - \frac{3}{4} \right) \pi k e^{-\pi k R}, \quad (15)$$

where  $n = 1, 2, \dots$  and  $\alpha = \{|c_f + 1/2|, 1, 2\}$  for KK fermions, KK gauge bosons and KK gravitons, respectively. Their masses are of order  $k e^{-\pi k R} \sim \text{TeV}$ ; the first KK state of fermions with  $-1/2 < c_f < 1/2$  (such as the top quark) would be the lightest, while gravitons are expected to be the heaviest.

### ***III.1a Models of Electroweak Symmetry Breaking***

Theories in warped extra dimensions can be used to implement symmetry breaking at low energies by boundary conditions [55]. For example, for a  $U(1)$  gauge symmetry in the 5D bulk, this can be easily achieved by imposing a Dirichlet boundary condition on the IR-boundary,  $A_\mu|_{y=\pi R} = 0$ . This makes the zero-mode gauge boson get a mass, given by  $m_A = g_4 \sqrt{2k/g_5^2} e^{-\pi k R}$ , that can be smaller than the KK masses for  $g_5^2 \gg 1/k$ . A very different situation occurs if the Dirichlet boundary condition is imposed on the UV-boundary,  $A_\mu|_{y=0} = 0$ . In this case the zero-mode gauge boson disappears from the spectrum. Finally, if a Dirichlet boundary condition

is imposed on the two boundaries, we obtain a massless 4D scalar corresponding to the fifth component of the 5D gauge boson,  $A_5$ . Thus, different scenarios can be implemented by appropriately choosing the 5D bulk gauge symmetry,  $\mathcal{G}_5$ , and the symmetries to which it reduces on the UV and IR-boundary,  $\mathcal{H}_{UV}$  and  $\mathcal{H}_{IR}$  respectively. In all cases the KK spectrum comes in representations of the group  $\mathcal{G}_5$ .

**Higgsless models:** One of the most interesting applications of warped extra dimensions is for models of electroweak symmetry breaking. To guarantee the relation  $M_W^2 \simeq M_Z^2 \cos^2 \theta_W$ , a custodial  $SU(2)_V$  symmetry is needed in the bulk and IR-boundary [56]. For this reason the minimal symmetry pattern is [57]

$$\begin{aligned}\mathcal{G}_5 &= SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \\ \mathcal{H}_{IR} &= SU(3)_c \times SU(2)_V \times U(1)_X \\ \mathcal{H}_{UV} &= G_{SM}\end{aligned}$$

where  $G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$  is the SM gauge group with the identification of hypercharge as  $Y = T_3^R + X$ . In this theory the  $W$  and  $Z$  bosons are massive, there is no Higgs boson, and the first KK mode associated to the  $W$  boson has a mass given by

$$m_{KK} \simeq \frac{3\pi g_5 \sqrt{k}}{4g_4} M_W. \quad (16)$$

Using Eq. (14), one obtains  $m_{KK} \simeq 1.2$  TeV.

**Composite Higgs models:** Alternatively, warped extra dimensions can give rise to scenarios, often called gauge-Higgs unified models, where the Higgs appears as the fifth component of a 5D gauge boson,  $A_5$ . The Higgs mass is protected by the 5D gauge invariance and can only get a nonzero value from non-local one-loop effects [58]. As in the Higgsless case, a custodial symmetry is needed. The simplest realization [59] has

$$\begin{aligned}\mathcal{G}_5 &= SU(3)_c \times SO(5) \times U(1)_X \\ \mathcal{H}_{IR} &= SU(3)_c \times SO(4) \times U(1)_X \\ \mathcal{H}_{UV} &= G_{SM}\end{aligned}$$

The Higgs gets a potential at the one-loop level that triggers a VEV, breaking the electroweak symmetry. In these models

there is a light Higgs with mass  $100 - 200$  GeV that, as will be explained in Sec. III.3, behaves as a composite state. The lightest KK modes of the model are color fermions with charges  $Q = -1/3, 2/3$  and  $5/3$  [60].

### **III.1b Constraints from Electroweak Precision Tests**

Models in which the SM gauge bosons propagate in 1/TeV-sized extra dimensions give generically large corrections to electroweak observables. When the SM fermions are confined on a boundary these corrections are universal and can be parametrized by four quantities:  $\widehat{S}$ ,  $\widehat{T}$ ,  $W$  and  $Y$ , as defined in Ref. [61]. For warped models, where the 5D gauge coupling of Eq. (14) is large, the most relevant parameter is  $\widehat{T}$ , which gives the bound  $m_{KK} \gtrsim 10$  TeV [50]. When a custodial symmetry is imposed [56], the main constraint comes from the  $\widehat{S}$  parameter, requiring  $m_{KK} \gtrsim 3$  TeV independently of the value of  $g_5$ . Notice that this bound, when applied to 5D Higgsless models where Eq. (16) holds, constrains the coupling  $g_5^2 k$  to be close to its nonperturbative value [62]. Also corrections to the  $Zb_L\bar{b}_L$  coupling can be important [50], especially in warped models for electroweak symmetry breaking as the ones described above.

### **III.1c Kaluza-Klein Searches**

The main prediction of 1/TeV-sized extra dimensions is the presence of a discretized KK spectrum, with masses around the TeV scale, associated with the SM fields that propagate in the extra dimension.

In the original RS model, only gravity propagates in the 5D bulk. Experimental searches have been performed for the lightest RS graviton through its decay to a variety of SM particle-antiparticle pairs. The results are usually interpreted in the plane of the dimensionless coupling  $k/M_P$  versus  $m_1$ , where  $M_P$  is the reduced Planck mass defined previously and  $m_1$  is the mass of the lightest KK excitation of the graviton. Since the AdS curvature  $\sim k$  cannot exceed the cut-off scale of the model, which is estimated to be  $\ell_5^{1/3} M_5$  [31], we must demand  $k \ll \sqrt{2\ell_5} M_P$ . The results quoted below are 95% CL lower limits on the KK graviton mass for a coupling  $k/M_P = 0.1$ .

Tevatron limits exist for the diphoton, dielectron and dimuon final states, and are typically near the kinematic limit

of order 1 TeV. The most stringent results from the Tevatron are the CDF limit of  $m_1 > 1.11$  TeV [64] from combining  $G \rightarrow \gamma\gamma/ee/\mu\mu$ , and the DØ limit of  $m_1 > 1.05$  TeV [65] using  $G \rightarrow \gamma\gamma/ee$ . The LHC experiments can probe higher masses, with recent results available using  $\approx 1 \text{ fb}^{-1}$  of 7 TeV data. No published predictions exist for NLO cross sections for 7 TeV  $pp$  collisions, and CMS and ATLAS adopted different conventions with regard to k-factors, meaning some care is required when comparing their results. CMS used a k-factor value of  $\approx 1.6$  for their combined  $ee/\mu\mu$  result of  $m_1 > 1.78$  TeV [66], and for their preliminary result of  $m_1 > 1.72$  TeV [37] using the  $\gamma\gamma$  final state. The ATLAS limit of  $m_1 > 1.63$  TeV [67] from the  $ee/\mu\mu$  final state combination assumes the signal cross section as obtained with LO\* PDFs; this choice corresponds to an effective k-factor of  $\approx 1.1$  for masses near the obtained limit. A  $2 \text{ fb}^{-1}$  update of the ATLAS  $\gamma\gamma$  analysis in Ref. [39] assumes a k-factor of 1.75 and provides a limit of  $m_1 > 1.95$  TeV when combined with the  $1 \text{ fb}^{-1}$   $ee/\mu\mu$  results.

Less stringent limits on the KK graviton mass come from the  $WW/ZZ$  final states at the Tevatron. By combining searches with one, two, or three charged leptons, DØ sets a limit in the  $WW$  final state of  $m_1 > 754$  GeV [68]. An earlier CDF result in the  $e + \cancel{E}_T + jj$  final state limits  $m_1 > 606$  GeV [69]. A preliminary  $ZZ$  result from CDF [70] reports 4 events compatible with a mass of  $\approx 325$  GeV in the final state with four charged leptons (electrons or muons), and they state that the probability to observe such a distribution of events given the SM background lies in the range  $(2.7 - 10.5) \times 10^{-5}$ . However, CDF analyses with two charged leptons and either a pair of jets or  $\cancel{E}_T$  do not confirm a signal for a new heavy particle decaying to a pair of  $Z$  bosons. The combined result is quoted as a limit of 0.26 pb on the production cross section of a 325 GeV RS Graviton decaying to  $ZZ$ . A DØ measurement of the  $ZZ$  production cross section in the four charged lepton final state [71] also shows no evidence of an enhancement at a  $ZZ$  invariant mass near 325 GeV.

If the SM fields propagate in the 5D bulk, the couplings of the KK graviton to  $ee/\mu\mu/\gamma\gamma$  are suppressed [72], and

the above bounds would not apply. In this case the graviton is the heaviest KK state, so experimental searches for KK gauge bosons and fermions are more appropriate discovery channels in these scenarios. For the scenarios discussed above in which only the Higgs and the top quark are localized towards the IR-boundary, the KK gauge bosons mainly decay into top quarks, longitudinal  $W/Z$  bosons, and Higgs bosons. Couplings to light SM fermions are suppressed by a factor  $g_4/\sqrt{g_5^2 k} \sim 0.2$  [6]. Searches have been made for evidence of the lightest KK excitation of the gluon, through its decay to  $t\bar{t}$  pairs. The production rates are quite insensitive to the values of  $g_5$  since these are proportional to the KK gluon couplings to light fermions or gluons ( $\propto g_4/\sqrt{g_5^2 k}$ ) and to top quarks ( $\propto \sqrt{g_5^2 k}$ ). An ATLAS analysis of the lepton-plus-jets final state using  $0.2 \text{ fb}^{-1}$  of data excludes KK gluons with masses below 650 GeV [73], while a  $1 \text{ fb}^{-1}$  analysis of the dilepton final state yields a lower limit of 0.84 TeV [74]. A  $0.9 \text{ fb}^{-1}$  CMS analysis of the fully hadronic final state searches uses “top tagging” techniques to identify events where one or both of the top quarks is highly boosted and reconstructed as a single merged jet; this analysis claims to exclude KK gluons with a mass lying between 1 and 1.5 TeV [75], though the paper states that the large width expected for the KK gluon was not taken into account in determining the result. A gauge KK excitation could be also sought through its decay to longitudinal  $W/Z$  bosons. While, as reported elsewhere in this volume, searches for  $WZ$  resonances have been used to set limits on sequential SM  $W'$  bosons or other models, as yet no  $WZ$  experimental results have been interpreted in the context of warped extra-dimensions.

The lightest KK states are usually the partners of the top quark. In warped models of electroweak symmetry breaking, these are color states with charges  $Q = -1/3, 2/3$  and  $5/3$ , and masses between 0.5 and 1.5 TeV [60]. They can be either singly- or pair-produced, and mainly decay into a combination of  $W/Z$  with top/bottom. Of particular note, the  $Q = 5/3$  state decays mainly into  $W^+t \rightarrow W^+W^+b$ , giving a pair of same-sign leptons in the final state [76].  $D\bar{O}$  searched with  $5.4 \text{ fb}^{-1}$



for single production of heavy vector-like quarks decaying to  $W/Z + j$  [77], and set limits on their production cross sections that could be reinterpreted to provide some exclusion on KK fermions decaying to  $W/Z + b$ , though the sensitivity might be less than optimal since  $b$ -tagging was not used in the analysis. CMS performed a  $1 \text{ fb}^{-1}$  search for pair-production of heavy vector-like quarks  $T$ , which excludes such quarks with masses below 475 GeV [78], assuming a 100% branching ratio for the decay  $T \rightarrow tZ$ .

### ***III.2 Connection with Strongly-Coupled Models via the AdS/CFT Correspondence***

The AdS/CFT correspondence [7] provides a connection between warped extra-dimensional models and strongly-coupled theories in ordinary 4D. Although the exact connection is only known for certain cases, the AdS/CFT techniques have been very useful to obtain, at the qualitative level, a 4D holographic description of the various phenomena in warped extra-dimensional models [8].

The connection goes as follows. The physics of the bulk AdS<sub>5</sub> models can be interpreted as that of a 4D conformal field theory (CFT) which is strongly-coupled. The extra-dimensional coordinate  $y$  plays the role of the renormalization scale  $\mu$  of the CFT by means of the identification  $\mu \equiv ke^{-ky}$ . Therefore the UV-boundary corresponds in the CFT to a UV cut-off scale at  $\Lambda_{UV} = k \sim M_P$ , breaking explicitly conformal invariance, while the IR-boundary can be interpreted as a spontaneous breaking of the conformal symmetry at energies  $ke^{-k\pi R} \sim \text{TeV}$ . Fields localized on the UV-boundary are elementary fields external to the CFT, while fields localized on the IR-boundary and KK states corresponds to composite resonances of the CFT. Furthermore, local gauge symmetries in the 5D models,  $\mathcal{G}_5$ , correspond to global symmetries of the CFT, while the UV-boundary symmetry can be interpreted as a gauging of the subgroup  $\mathcal{H}_{UV}$  of  $\mathcal{G}_5$  in the CFT. Breaking gauge symmetries by IR-boundary conditions corresponds to the spontaneous breaking  $\mathcal{G}_5 \rightarrow \mathcal{H}_{IR}$  in the CFT at energies  $\sim ke^{-k\pi R}$ . Using this correspondence we can easily derive the 4D massless spectrum

of the compactified AdS<sub>5</sub> models. We also have the identification  $k^3/M_5^3 \approx 16\pi^2/N^2$  and  $g_5^2 k \approx 16\pi^2/N^r$  ( $r = 1$  or  $2$  for CFT fields in the fundamental or adjoint representation of the gauge group, respectively), where  $N$  plays the role of the number of colors of the CFT. Therefore the weak-coupling limit in AdS<sub>5</sub> corresponds to a large- $N$  expansion in the CFT.

Following the above AdS/CFT dictionary we can understand the RS solution to the hierarchy problem from a 4D viewpoint. The equivalent 4D model is a CFT with a TeV mass-gap and a Higgs emerging as a composite state. The AdS/CFT correspondence also shows that the 5D Higgsless models described above should share similar physics as Technicolor models [79]. Indeed, the lowest KK  $SU(2)_L$ -gauge boson behaves as the Techni-rho  $\rho_T$  with a coupling to longitudinal  $W/Z$  bosons given by  $g_5\sqrt{k} \approx g_{\rho_T}$ , while the coupling to elementary fermions is  $g_4^2/\sqrt{g_5^2 k} \approx g^2 F_{\rho_T}/M_{\rho_T}$ . Also, models with the Higgs identified as  $A_5$  correspond to models, similar to those proposed in Ref. [80], where the Higgs is a composite pseudo-Goldstone boson arising from the spontaneous breaking  $\mathcal{G}_5 \rightarrow \mathcal{H}_{IR}$  in the CFT.

Fermions in compactified AdS<sub>5</sub> also have a simple 4D holographic interpretation. The 4D massless mode described in sec. III.1 corresponds to an external fermion  $\psi_i$  linearly coupled to a fermionic CFT operator  $\mathcal{O}_i$ :  $\mathcal{L}_{\text{int}} = \lambda_i \bar{\psi}_i \mathcal{O}_i + h.c.$ . The dimension of the operator  $\mathcal{O}_i$  is related to the 5D fermion mass according to  $\text{Dim}[\mathcal{O}_i] = |c_f + 1/2| - 1$ . Therefore, by varying  $c_f$  we vary  $\text{Dim}[\mathcal{O}_i]$ , making the coupling  $\lambda_i$  irrelevant ( $c_f > 1/2$ ), marginal ( $c_f = 1/2$ ) or relevant ( $c_f < 1/2$ ). When irrelevant, the coupling is exponentially suppressed at low energies, and then the coupling of  $\psi_i$  to the CFT (and eventually to the composite Higgs) is very small. When relevant, it grows towards the IR and become as large as  $g_5$  (in units of  $k$ ), meaning that the fermion is as strongly coupled as the CFT states [59]. In this latter case  $\psi_i$  behaves as a composite fermion.

### III.3 Flat Extra Dimensions

Models with quantum-gravity at the TeV scale, as in the ADD scenario, can have extra (flat) dimensions of 1/TeV size, as it happens in string scenarios [81]. All SM fields may

propagate in these extra dimensions, leading to the possibility of observing their corresponding KK states.

A simple example is to assume that the SM gauge bosons propagate in a flat five-dimensional orbifold  $S^1/Z_2$  of radius  $R$ , with the fermions localized on a 4D boundary. The KK gauge bosons behave as sequential SM gauge bosons with a coupling to fermions enhanced by a factor  $\sqrt{2}$  [81]. The experimental limits on such sequential gauge bosons could therefore be recast as limits on KK gauge bosons. Bounds from LEP2 require  $1/R \gtrsim 6$  TeV [61].

An alternative scenario, known as Universal Extra Dimensions (UED) [82], assumes that all SM fields propagate universally in a flat orbifold  $S^1/Z_2$  with an extra  $Z_2$  parity, called KK-parity, that interchanges the two boundaries. In this case, the lowest KK state is stable becoming a Dark Matter candidate. At colliders, the KK particles would have to be created in pairs, and would then cascade decay to the lightest KK particle (LKP), which would be stable and escape detection. Experimental signatures, such as jets or leptons and  $\cancel{E}_T$ , would be similar to those of typical  $R$ -parity conserving SUSY searches. However, few experimental searches have as yet been interpreted in the UED scenario. DØ and ATLAS have both examined a specific UED model in which the KK parity is violated by gravitational interactions [83]. In this case the LKP can decay via  $\gamma^* \rightarrow \gamma + G$ , where  $G$  represents one of a tower of eV-spaced KK graviton states. Beginning with strong production of a pair of KK quarks and/or gluons [84], the final state would be  $\gamma\gamma + \cancel{E}_T + X$ , where  $\cancel{E}_T$  results from the escaping gravitons and  $X$  represents SM particles emitted during the cascade decays. The experimental analyses treat  $R$ , the UED compactification radius, as a free parameter and follow the theory calculation [85] that sets  $\Lambda$ , the cut-off used in the calculation of radiative corrections to the KK masses, such that  $\Lambda R = 20$ . The gravitational decay widths of the KK particles are set by the values of  $\delta$  and of  $M_D$ , the Planck scale in the  $(4 + \delta)$ -dimensional theory. Setting  $\delta = 6$  and  $M_D = 5$  TeV, and provided  $1/R < 2$  TeV, only the LKP has an appreciable gravitational decay, with a sizeable branching

ratio for  $\gamma^* \rightarrow \gamma + G$ . In this scenario, DØ set a 95% CL limit that  $1/R > 477$  GeV [86]. The initial ATLAS result used only  $3 \text{ pb}^{-1}$  to increase this limit to  $1/R > 729$  GeV; a recent ATLAS  $1 \text{ fb}^{-1}$  update extends it further to  $1/R > 1.23$  TeV [87].

Finally, realistic models of electroweak symmetry breaking can also be constructed with flat extra spatial dimensions, similarly to those in the warped case, requiring, however, the presence of sizeable boundary kinetic terms [88]. There is also the possibility of breaking supersymmetry by boundary conditions [89]. Models of this type could explain naturally the presence of a Higgs boson lighter than  $M_D$  [90].

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