

## REVIEW OF D-MESON DALITZ PLOT ANALYSES

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The formalism of Dalitz-plot analysis is reviewed in the preceding note. Recent studies of multi-body decays of charm mesons probe a variety of physics, including  $\gamma/\phi_3$ ,  $D^0-\bar{D}^0$  mixing, searches for  $CP$  violation, doubly Cabibbo-suppressed decays, and properties of S-wave  $\pi\pi$ ,  $K\pi$ , and  $K\bar{K}$  resonances. In the following, we discuss: (1)  $D^0 \rightarrow K_S^0\pi^+\pi^-$ ; (2) doubly Cabibbo-suppressed decays; and (3)  $CP$  violation. The properties of the light meson resonances determined in D-meson Dalitz-plot analyses are reported in the light unflavored meson section of this *Review*.

$D^0 \rightarrow K_S^0\pi^+\pi^-$ : Several experiments have analyzed  $D^0 \rightarrow K_S^0\pi^+\pi^-$  decay. A CLEO analysis [1] included ten resonances:  $K_S^0\rho^0$ ,  $K_S^0\omega$ ,  $K_S^0f_0(980)$ ,  $K_S^0f_2(1270)$ ,  $K_S^0f_0(1370)$ ,  $K^*(892)^-\pi^+$ ,  $K_0^*(1430)^-\pi^+$ ,  $K_2^*(1430)^-\pi^+$ ,  $K^*(1680)^-\pi^+$ , and the doubly Cabibbo-suppressed (DCS) mode  $K^*(892)^+\pi^-$ . The CLEO model does not provide a good description of higher-statistics BABAR and Belle data samples. An improved description is obtained in three ways: First, by adding more Breit-Wigner resonances. Second, following the methodology of FOCUS [2], by applying a  $K$ -matrix model [3–5] to the  $\pi\pi$  S-wave [6,7]. Third, by adding a parameterization to the  $K\pi$  S-wave motivated by the LASS experiment [8].

A BABAR analysis [7,9,10] added to the CLEO model the  $K^*(1410)^-\pi^+$ ,  $K_S^0\rho^0(1450)$ , the DCS modes  $K_0^*(1430)^+\pi^-$  and  $K_2^*(1430)^+\pi^-$ , and two Breit-Wigner  $\pi\pi$  S-wave contributions. A Belle analysis [11–13] included all the components of BABAR and added two more DCS modes,  $K^*(1410)^+\pi^-$  and  $K^*(1680)^+\pi^-$ . Recently, BABAR has modeled the  $\pi\pi$  S-wave using a  $K$ -matrix model for the  $\pi\pi$  and  $K\pi$  S-waves [14].

The primary motivation for the analysis of the decay  $D^0 \rightarrow K_S^0\pi^+\pi^-$  is to study  $D^0 - \bar{D}^0$  oscillations and the CKM angles. The quasi-two-body intermediate states include both  $CP$ -even and  $CP$ -odd eigenstates as well as doubly Cabibbo-suppressed channels. Time-dependent analyses of the Dalitz plot from CLEO [15] and Belle [6] simultaneously determined the strong

transition amplitudes and phases, the mixing parameters  $x$  and  $y$  without phase or sign ambiguity, and the  $CP$ -violating parameter  $|q/p|$  and  $\text{Arg}(q/p)$ . See the note on “ $D^0 - \bar{D}^0$  Mixing” for a discussion.

The CKM angle  $\gamma/\phi_3$  [16] and the quark-mixing parameter  $\cos 2\beta/\phi_1$  [17] can be determined using the decays  $B^- \rightarrow D^{(*)}K^{(*)-}$  and  $\bar{B}^0 \rightarrow Dh^0$ , respectively, followed by the decay  $D \rightarrow K_S^0\pi^+\pi^-$ . The Belle and BABAR experiments measured  $\gamma/\phi_3$  (Belle [11–13] and BABAR [7,9,10,14,18] and  $\cos 2\beta/\phi_1$  (Belle [19], BABAR [20]). In these analyses, a large systematic uncertainty in the relative phase between the  $D^0$  and  $\bar{D}^0$  amplitudes point by point across the Dalitz plot remains to be fully understood.

The quantum entangled production of  $D^0\bar{D}^0$  pairs from  $\psi(3770)$  enables a model-independent determination of the  $D^0/\bar{D}^0$  relative phase. Studying  $CP$ -tagged Dalitz plots [21,22] provides sensitivity to the cosine of the relative phase, while studying double-tagged Dalitz plots [22] probes both the cosine and sine of the  $D^0/\bar{D}^0$  phase difference. CLEO analyzed [23] the  $D^0 \rightarrow K_S^0\pi^+\pi^-$  and  $D^0 \rightarrow K_L^0\pi^+\pi^-$  samples using the  $CP$ -even tag modes  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $K_L^0\pi^0$  (vs.  $K_S^0\pi^+\pi^-$  only), the  $CP$ -odd tag modes  $K_S^0\pi^0$ ,  $K_S^0\eta$ , and the double-tag modes  $(K_S^0\pi^+\pi^-)^2$  and  $(K_S^0\pi^+\pi^-)(K_L^0\pi^+\pi^-)$ . These measurements can reduce the model uncertainty on  $\gamma/\phi_3$  to about  $3^\circ$ .

***Doubly Cabibbo-Suppressed Decays:*** There are two classes of multibody doubly Cabibbo-suppressed (DCS) decays of  $D$  mesons. The first consists of those in which the DCS and corresponding Cabibbo-favored (CF) decays populate distinct Dalitz plots; the pairs  $D^0 \rightarrow K^+\pi^-\pi^0$  and  $D^0 \rightarrow K^-\pi^+\pi^0$ , or  $D^+ \rightarrow K^+\pi^+\pi^-$  and  $D^+ \rightarrow K^-\pi^+\pi^+$ , are examples. Our average of three measurements of  $\Gamma(D^0 \rightarrow K^+\pi^-\pi^0)/\Gamma(D^0 \rightarrow K^-\pi^+\pi^0)$  is  $(2.20 \pm 0.10) \times 10^{-3}$ . Our average of four measurements of  $\Gamma(D^+ \rightarrow K^+\pi^-\pi^+)/\Gamma(D^+ \rightarrow K^-\pi^+\pi^+)$  is  $(5.77 \pm 0.22) \times 10^{-3}$ ; see the Particle Listings.

The second class consists of decays in which the DCS and CF modes populate the same Dalitz plot; for example,  $D^0 \rightarrow K^{*-}\pi^+$  and  $D^0 \rightarrow K^{*+}\pi^-$  both contribute to  $D^0 \rightarrow K_S^0\pi^+\pi^-$ . In this class, the potential for interference of DCS and CF

amplitudes increases the sensitivity to the DCS amplitude and allows direct measurement of the relative strong phases between amplitudes. CLEO [1] and Belle [6] have measured the relative phase between  $D^0 \rightarrow K^*(892)^+\pi^-$  and  $D^0 \rightarrow K^*(892)^-\pi^+$  to be  $(189 \pm 10 \pm 3_{-5}^{+15})^\circ$  and  $(171.9 \pm 1.3)^\circ$  (statistical error only). These results are close to the  $180^\circ$  expected from Cabibbo factors and a small strong phase.

In addition, Belle [6] has results for both the relative phase (statistical errors only) and ratio  $R$  (central values only) of the DCS fit fraction relative to the CF fit fractions for  $K^*(892)^+\pi^-$ ,  $K_0^*(1430)^+\pi^-$ ,  $K_2^*(1430)^+\pi^-$ ,  $K^*(1410)^+\pi^-$ , and  $K^*(1680)^+\pi^-$ . The systematic uncertainties on  $R$  must be evaluated. The values for  $R$  in units of  $\tan^4\theta_c$  are  $2.94 \pm 0.12$ ,  $22.0 \pm 1.6$ ,  $34 \pm 4$ ,  $87 \pm 13$ , and  $500 \pm 500$ . For  $K^+\pi^-$ , the corresponding value for  $R_D$  is  $(1.28 \pm 0.02) \times \tan^4\theta_c$ . Similarly, BABAR [7] has reported central values for  $R$  for  $K^*(892)^+\pi^-$ ,  $K_0^*(1430)^+\pi^-$ , and  $K_2^*(1430)^+\pi^-$ . The values for  $R$  in units of  $\tan^4\theta_c$  are  $3.45 \pm 0.31$ ,  $7.7 \pm 3.0$ , and  $1.7 \pm 1.7$ , respectively. Recently, BABAR [14] has used a K-matrix formalism to describe the  $\pi\pi$  S-wave in  $K_S^0\pi^+\pi^-$ . The reported values for  $R$  in units of  $\tan^4\theta_c$  are  $2.78 \pm 0.11$ ,  $0.5 \pm 0.2$ , and  $1.4 \pm 0.5$ , respectively. The large differences in  $R$  among these final states could point to an interesting role for hadronic effects.

There are other ways, not involving DCS decays, in which  $D^0$  and  $\bar{D}^0$  singly Cabibbo-suppressed decays can populate the same Dalitz plot. Examples are  $D^0$  and  $\bar{D}^0$  decays to  $K_S^0K^+\pi^-$ , or to  $K_S^0K^-\pi^+$ . These final states can be used to study  $D^0$ - $\bar{D}^0$  mixing and the CKM angle  $\gamma/\phi_3$ .

***CP Violation:*** In the limit of  $CP$  conservation, charge conjugate decays will have the same Dalitz-plot distribution. The  $D^{*\pm}$  tag enables the discrimination between  $D^0$  and  $\bar{D}^0$ . The integrated  $CP$  violation across the Dalitz plot is determined in two ways. The first uses

$$\mathcal{A}_{CP} = \int \left( \frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} \right) dm_{ab}^2 dm_{bc}^2 \bigg/ \int dm_{ab}^2 dm_{bc}^2, \quad (1)$$

where  $\mathcal{M}$  and  $\overline{\mathcal{M}}$  have the same normalization and represent the  $D^0$  and  $\bar{D}^0$  Dalitz-plot amplitudes for the three-body decay

$D \rightarrow abc$ , and  $m_{ab}$  ( $m_{bc}$ ) is the invariant mass of  $ab$  ( $bc$ ). The second uses the asymmetry in the efficiency-corrected  $D^0$  and  $\overline{D}^0$  yields,

$$\mathcal{A}_{CP} = \frac{N_{D^0} - N_{\overline{D}^0}}{N_{D^0} + N_{\overline{D}^0}}. \quad (2)$$

These expressions are less sensitive to  $CP$  violation than are the individual resonant submodes [24–26]. Our Particle Listings give limits on  $CP$  violation for 12  $D^+$ , 52  $D^0$ , and 13  $D_S^+$  decay modes. No evidence of  $CP$  violation has been observed in  $D$ -meson decays.

The possibility of interference between  $CP$ -conserving and  $CP$ -violating amplitudes provides a more sensitive probe of  $CP$  violation. The constraints on the square of the  $CP$ -violating amplitudes obtained in the resonant submodes of  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  range from  $3.5 \times 10^{-4}$  to  $28.4 \times 10^{-4}$  at 95% confidence level [24]. A similar analysis has been performed by CLEO [25] searching for  $CP$  violation in  $D^+ \rightarrow K^+ K^- \pi^+$ . The constraints on the square of the  $CP$ -violating amplitudes in the resonant submodes range from  $4 \times 10^{-4}$  to  $51 \times 10^{-4}$  at 95%. BABAR finds no evidence for  $CP$ -violating amplitudes in the resonant submodes of  $D^0 \rightarrow K^+ K^- \pi^0$  and  $D^0 \rightarrow \pi^+ \pi^- \pi^0$  [26].

## References

1. H. Muramatsu *et al.* (CLEO Collab.), Phys. Rev. Lett. **89**, 251802 (2002).
2. J.M. Link *et al.* (FOCUS Collab.), Phys. Lett. **B585**, 200 (2004).
3. E. P. Wigner, Phys. Rev. **70**, 15 (1946).
4. S. U. Chung *et al.*, Annalen Phys. **4**, 404 (1995).
5. I. J. R. Aitchison, Nucl. Phys. **A189**, 417 (1972).
6. L.M. Zhang *et al.* (Belle Collab.), Phys. Rev. Lett. **99**, 131803 (2007).
7. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. Lett. **95**, 121802 (2005).
8. D. Aston *et al.* (LASS Collab.), Nucl. Phys. **B296**, 493 (1988).
9. B. Aubert *et al.* (BABAR Collab.), hep-ex/0507101.
10. B. Aubert *et al.* (BABAR Collab.), arXiv:hep-ex/0607104.

11. A. Poluektov *et al.* (Belle Collab.), Phys. Rev. **D70**, 072003 (2004).
12. K. Abe *et al.* (Belle Collab.), [hep-ex/0411049](#).
13. A. Poluektov *et al.* (Belle Collab.), Phys. Rev. **D73**, 112009 (2006).
14. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. **D78**, 034023 (2008).
15. D.M. Asner *et al.* (CLEO Collab.), Phys. Rev. **D72**, 012001 (2005).
16. A. Giri *et al.*, Phys. Rev. **D68**, 054018 (2003).
17. A. Bondar, T. Gershon, and P. Krokovny, Phys. Lett. **B624**, 1 (2005).
18. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. **D79**, 072003 (2009).
19. P. Krokovny *et al.* (Belle Collab.), Phys. Rev. Lett. **97**, 081801 (2006).
20. B. Aubert *et al.* (BABAR Collab.), [arXiv:hep-ex/0607105](#).
21. A. Bondar and A. Poluektov, Eur. Phys. J. **C47**, 347 (2006).
22. A. Bondar and A. Poluektov, [arXiv:hep-ph/0703267](#).
23. R. A. Briere *et al.* (CLEO Collab.), Phys. Rev. **D80**, 032002 (2009).
24. D.M. Asner *et al.* (CLEO Collab.), Phys. Rev. **D70**, 091101R (2004).
25. P. Rubin *et al.* (CLEO Collab.), Phys. Rev. **D78**, 072003 (2008).
26. B. Aubert *et al.* (BABAR Collab.), Phys. Rev. **D78**, 051102 (2008).