V_{ud}, V_{us}, THE CABIBBO ANGLE, AND CKM UNITARITY

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The Cabibbo-Kobayashi-Maskawa (CKM) [1,2] three-generation quark mixing matrix written in terms of the Wolfenstein parameters (λ, A, ρ, η) [3] nicely illustrates the orthonormality constraint of unitarity and central role played by λ.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (1)$$

That cornerstone is a carryover from the two-generation Cabibbo angle, $\lambda = sin(\theta_{\text{Cabibbo}}) = V_{us}$. Its value is a critical ingredient in determinations of the other parameters and in tests of CKM unitarity.

Unfortunately, the precise value of λ has been somewhat controversial in the past, with kaon decays suggesting [4] $\lambda \simeq 0.220$ while hyperon decays [5] and indirect determinations via nuclear β-decays imply a somewhat larger $\lambda \simeq 0.225 - 0.230$. That discrepancy is often discussed in terms of a deviation from the unitarity requirement

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (2)$$

For many years, using a value of V_{us} derived from $K \to \pi e \nu$ (K_{e3}) decays, that sum was consistently 2–2.5 sigma below unity, a potential signal [6] for new physics effects. Below, we discuss the current status of V_{ud}, V_{us}, and their associated unitarity test in Eq. (2). (Since $|V_{ub}|^2 \simeq 1 \times 10^{-5}$ is negligibly small, it is ignored in this discussion.)

V_{ud}

The value of V_{ud} has been obtained from superallowed nuclear, neutron, and pion decays. Currently, the most precise determination of V_{ud} comes from superallowed nuclear beta-decays [6] ($0^+ \to 0^+$ transitions). Measuring their half-lives, t, and Q values which give the decay rate factor, f, leads to a precise determination of V_{ud} via the master formula [7–9]

$$|V_{ud}|^2 = \frac{2984.48(5)\,\mathrm{sec}}{ft(1+\mathrm{RC})} \tag{3}$$

where RC denotes the entire effect of electroweak radiative corrections, nuclear structure, and isospin violating nuclear effects. RC is nucleus dependent, ranging from about +3.1% to +3.6% for the nine best measured superallowed decays. In Table 1, we give the ft values along with their implied V_{ud} for the nine best measured superallowed decays [6, 10]. They collectively give a weighted average (with errors combined in quadrature) of

$$V_{ud} = 0.97377(27)\ \text{(superallowed)} \tag{4}$$

which, assuming unitarity, corresponds to $\lambda = 0.2275(12)$. We note, however, that a recent remeasurement [10] of the ^{46}V Q value has significantly affected its ft and V_{ud} values, with the latter now about 2.7 sigma below the average. That recent shift may point to a potential problem with the Q values and ft values of the other superallowed beta decays. Remeasurement of all Q values using modern atomic trapping techniques is called for and in progress.

Combined measurements of the neutron lifetime, τ_n, and the ratio of axial-vector/vector couplings, $g_A \equiv G_A/G_V$, via neutron decay asymmetries can also be used to determine V_{ud}:

$$|V_{ud}|^2 = \frac{4908.7(1.9)\mathrm{sec}}{\tau_n(1+3g_A^2)}, \tag{5}$$

where the error stems from uncertainties in the electroweak radiative corrections [8] due to hadronic loop effects. Those effects have been recently updated and their error was reduced by about a factor of 2 [9], leading to a ± 0.0002 theoretical

Table 1: Values of V_{ud} implied by various precisely measured superallowed nuclear beta decays. The ft values are taken from a recent update by Savard et al. [10]. Uncertainties in V_{ud} correspond to 1) nuclear structure and $Z^2\alpha^3$ uncertainties [6, 11] added in quadrature with the ft error, 2) a common error assigned to nuclear Coulomb distortion effects [11], and 3) a common uncertainty in the radiative corrections from quantum loop effects [9]. Only the first error is used to obtain the weighted average.

Nucleus	ft (sec)	V_{ud}
^{10}C	3039.5(47)	0.97381(77)(15)(19)
$^{14}0$	3043.3(19)	0.97368(39)(15)(19)
^{26}Al	3036.8(11)	0.97406(23)(15)(19)
^{34}Cl	3050.0(12)	0.97412(26)(15)(19)
^{38}K	3051.1(10)	0.97404(26)(15)(19)
^{42}Sc	3046.8(12)	0.97330(32)(15)(19)
^{46}V	3050.7(12)	0.97280(34)(15)(19)
^{50}Mn	3045.8(16)	0.97367(41)(15)(19)
^{54}Co	3048.4(11)	0.97373(40)(15)(19)
Weighted Ave.		0.97377(11)(15)(19)

uncertainty in V_{ud} (common to all V_{ud} extractions). Using the world averages (from PDG 2004) [4]

$$\begin{aligned} \tau_n^{\text{ave}} &= 885.7(8)\text{sec} \\ g_A^{\text{ave}} &= 1.2695(29) \end{aligned} \tag{6}$$

leads to

$$V_{ud} = 0.9746(4)_{\tau_n}(18)_{g_A}(2)_{\text{RC}} \tag{7}$$

with the error dominated by g_A uncertainties (which have been expanded due to experimental inconsistencies). We note that a recent precise measurement [12] of $\tau_n = 878.5(7)(3)$ sec is also inconsistent with the 2004 world average and would lead to a considerably larger $V_{ud} = 0.9786(4)(18)(2)$. Future neutron studies are expected to resolve these inconsistencies and significantly reduce the uncertainties in g_A and τ_n, potentially making them the best way to determine V_{ud}.

The recently completed PIBETA experiment at PSI measured the very small ($\mathcal{O}(10^{-8})$) branching ratio for $\pi^+ \to \pi^o e^+ \nu_e$ with about $\pm 1/2\%$ precision. Their result gives [13]

$$V_{ud} = 0.9749(26) \left[\frac{BR(\pi^+ \to e^+\nu_e(\gamma))}{1.2352 \times 10^{-4}} \right]^{\frac{1}{2}} \tag{8}$$

which is normalized using the very precisely determined theoretical prediction for $BR(\pi^+ \to e^+\nu_e(\gamma)) = 1.2352(5) \times 10^{-4}$ [7] rather than the experimental branching ratio of [4] $1.230(4) \times 10^{-4}$ which would lower the value to $V_{ud} = 0.9728(30)$. Theoretical uncertainties in that determination are very small; however, much higher statistics would be required to make this approach competitive with others.

V_{us}

$|V_{us}|$ may be determined from kaon decays, hyperon decays, and tau decays. Previous determinations have most often used $K\ell 3$ decays:

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1 + \delta_K^\ell + \delta_{SU2}) C^2 \left| V_{us} \right|^2 f_+^2(0) I_K^\ell . \tag{9}$$

Here, ℓ refers to either e or μ, G_F is the Fermi constant, M_K is the kaon mass, S_{EW} is the short-distance radiative correction, δ_K^ℓ is the mode-dependent long-distance radiative correction, $f_+(0)$ is the calculated form factor at zero momentum transfer for the $\ell\nu$ system, and I_K^ℓ is the phase-space integral, which depends on measured semileptonic form factors. For charged kaon decays, δ_{SU2} is the deviation from one of the ratio of $f_+(0)$ for the charged to neutral kaon decay; it is zero for the neutral kaon. C^2 is 1 (1/2) for neutral (charged) kaon decays. Previous PDG determinations of $|V_{us}|$ have been based only on $K \to \pi e \nu$ decays; $K \to \pi \mu \nu$ decays have not been used because of large uncertainties in I_K^μ. The experimental measurements are the semileptonic decay widths (based on the semileptonic branching fractions and lifetime) and form factors (allowing calculation of the phase space integrals). Theory is needed for

S_{EW}, δ_K^ℓ, δ_{SU2}, and $f_+(0)$. These experimental and theoretical inputs are discussed in the following paragraphs.

Branching Fractions. Recent measurements of the $K \to \pi e\nu$ branching fractions are significantly different from previous PDG averages, probably as a result of inadequate treatment of radiation in older experiments. We therefore choose to base averages on recent measurements where the treatment of radiation is clear.

For the K_L branching fractions, we consider the following experimental inputs:

- KTeV measured the following 5 partial width ratios [14, 15]: $\Gamma(K_L \to \pi^\pm\mu^\mp\nu)/\Gamma(K_L \to \pi^\pm e^\mp\nu)$, $\Gamma(K_L \to \pi^+\pi^-\pi^0)/\Gamma(K_L \to \pi^\pm e^\mp\nu)$, $\Gamma(K_L \to \pi^0\pi^0\pi^0)/\Gamma(K_L \to \pi^\pm e^\mp\nu)$, $\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_L \to \pi^\pm e^\mp\nu)$, and $\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_L \to \pi^0\pi^0\pi^0)$. Since the six decay modes listed above account for more than 99.9% of the total decay rate, the five partial width ratios may be converted into measurements of the branching fractions for the six decay modes.
- KLOE uses a tagged K_L sample to measure the 4 largest K_L branching fractions [16].
- NA48 measures the following 2 ratios: $\Gamma_{Ke3}/\Gamma(2\ track)$ [17] and $\Gamma_{000}/\Gamma(K_S \to \pi^0\pi^0)$ [18]. These ratios may be used to determine $B(K_{e3})$.

A fit to all of these measurements, accounting for correlations, gives the K_L semileptonic branching fractions in Table 2. Figure 1 shows a comparison of the new experimental measurements, the best fit values, and the 2002 PDG fit values [19]. Note that the new measurements are consistent with each other, but are shifted significantly from the PDG fit.

For $K_S \to \pi e\nu$, we use the new KLOE measurement: $B(K_S \to \pi e\nu) = (7.06 \pm 0.06 \pm 0.04) \times 10^{-4}$.

For $K^\pm \to \pi^0 e^\pm\nu$, we use the BNL E865 [20] measurement of $B(K^\pm \to \pi^0 e^\pm\nu) = (5.13\pm0.1)\%$. Preliminary measurements from NA48, KLOE, and ISTRA+ are consistent with this result.

Kaon Lifetime. KLOE has performed two new measurements of the K_L lifetime: one based on exploiting the lifetime

Table 2: Average K_L semileptonic branching fractions and widths based on fit to new measurements from KTeV, KLOE, and NA48. The partial width measurements use the average K_L lifetime quoted in Table 3.

Decay Mode	Branching fraction	Γ_i $(10^7 s^{-1})$
$K_L \to \pi^{\pm} e^{\mp} \nu$	0.4040 ± 0.0008	0.7908 ± 0.0032
$K_L \to \pi^{\pm} \mu^{\mp} \nu$	0.2699 ± 0.0008	0.5283 ± 0.0023

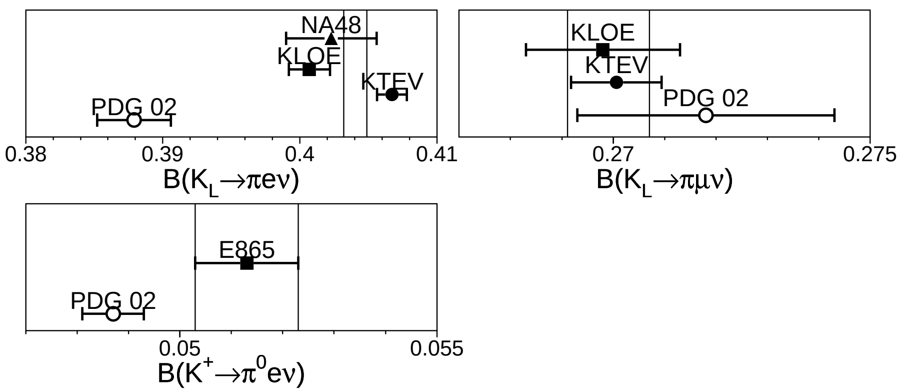

Figure 1: Recent $K_L \to \pi e \nu$, $K_L \to \pi \mu \nu$, and $K^{\pm} \to \pi^0 e^{\pm} \nu$ branching fraction measurements (solid points) compared to PDG 2002 fit (open circles). The vertical lines indicate the $\pm 1\sigma$ bounds from a fit to all recent measurements (from KTeV, KLOE, NA48, and E865).

dependence of the detector acceptance to find the K_L lifetime required to make the sum of branching fractions equal to 1 [16], and another based on the $K_L \to 3\pi^0$ decay distribution [21]. These new results and the old PDG average are listed in Table 3. The new average value, which we use for the results quoted below, is $\tau_L = (50.98 \pm 0.21) ns$.

Combining the K_L branching fractions with the new lifetime gives the partial decay widths quoted in Table 2. Note that correlations between the KLOE branching fractions and the "indirect" KLOE lifetime determination have been taken into account.

For the K_S and K^+ lifetimes, we use the PDG average values.

Table 3: K_L lifetime measurements.

Source	Lifetime (ns)
PDG 2004 Average	51.5 ± 0.4
KLOE (sum of branching fractions)	50.72 ± 0.35
KLOE ($3\pi^0$ distribution)	50.87 ± 0.31
New Average	50.98 ± 0.21

Phase Space Integrals. Recent experiments have also remeasured the semileptonic form factors needed to calculate the phase space integrals. These recent measurements of the semileptonic form factors are much more precise than previous averages, making it possible to use both the muon and electron decay modes for K_L.

We use the KTeV quadratic form factor results [22] for neutral kaon decays and the ISTRA+ quadratic form factor measurements [23] for charged kaons. For both charged and neutral decays, we include an additional 0.7% uncertainty in the phase space integrals, as suggested by KTeV [22], to account for differences between the quadratic and pole model form factor parametrizations, both of which give acceptable fits to the data. The resulting phase space integrals are $I^e_{K0} = 0.1535 \pm 0.0011$, $I^\mu_{K0} = 0.10165 \pm 0.0008$, and $I^e_{K+} = 0.1591 \pm 0.0012$.

Theoretical Inputs. We use the following theoretical inputs to calculate $f_+(0)|V_{us}|$ from Eq. (9).

- Short-distance radiative correction [7, 24]: $S_{EW} = 1.023$;
- Long-distance radiative corrections [25, 26]: $\delta^e_{K0} = 0.0104 \pm 0.002$, $\delta^\mu_{K0} = 0.019 \pm 0.003$, $\delta^e_{K+} = 0.0006 \pm 0.002$;
- SU2 breaking correction [25,27] $\delta_{SU2} = 0.046 \pm 0.004$.

$K_{\ell 3}$ results for $|V_{us}|$. Figure 2 shows a comparison of the PDG and the averages of recent measurements for $|V_{us}|f_+(0)$ for $K^\pm$, K_L, and K_S. The average of all recent measurements gives

$$f_+^{K^0\pi^-}(0)|V_{us}| = 0.2169 \pm 0.0009. \tag{10}$$

The figure also shows $f_+(0)(1 - |V_{ud}|^2 - |V_{ub}|^2)^{1/2}$, the expectation for $f_+(0)|V_{us}|$ assuming unitarity, based on $|V_{ud}| = 0.9738 \pm 0.0003$, $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, and the Leutwyler-Roos calculation of $f_+(0) = 0.961 \pm 0.008$ [27]. Using the result in Eq. (10) with the Leutwyler-Roos calculation of $f_+(0)$ gives

$$|V_{us}| = \lambda = 0.2257 \pm 0.0021. \tag{11}$$

A similar result for $f_+(0)$ was recently obtained from a quenched lattice gauge theory calculation [28]. Other calculations of $f_+(0)$ result in $|V_{us}|$ values that differ by as much as 2% from the result in Eq. (11). For example, a recent chiral perturbation theory calculation [29, 30] gives $f_+(0) = 0.974 \pm 0.012$, which implies a lower value of $|V_{us}| = 0.2227 \pm 0.0029$ [31].

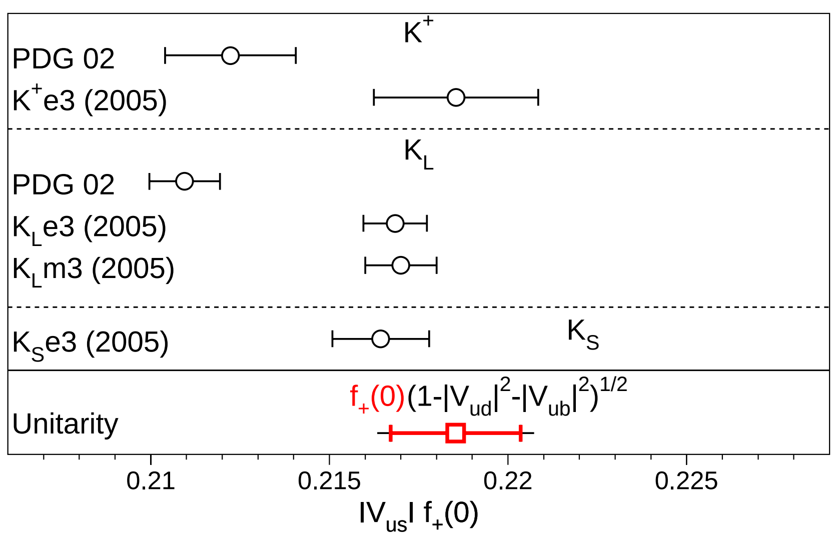

Figure 2: Comparison of determinations of $|V_{us}|f_+(0)$ from this review (labeled 2005), from the PDG 2002, and with the prediction from unitarity using $|V_{ud}|$ and the Leutwyler-Roos calculation of $f_+(0)$ [27]. For $f_+(0)(1 - |V_{ud}|^2 - |V_{ub}|^2)^{1/2}$, the inner error bars are from the quoted uncertainty in $f_+(0)$; the total uncertainties include the $|V_{ud}|$ and $|V_{ub}|$ errors.

A value of V_{us} can also be obtained from a comparison of the radiative inclusive decay rates for $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ combined with a lattice gauge theory calculation of f_K/f_π via [32]

$$\frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2387(4)\left[\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}\right]^{\frac{1}{2}} \tag{12}$$

with the small error coming from electroweak radiative corrections. Employing

$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} = 1.3383(46), \tag{13}$$

which incorporates the KLOE result [33], $B(K \to \mu\nu(\gamma)) =$ 63.66(9)(15)% and [34, 35]

$$f_K/f_\pi = 1.198(3)(+16/-5) \tag{14}$$

along with the value of V_{ud} in Eq. (4) leads to

$$|V_{us}| = 0.2245(5)(1.198 f_\pi/f_K). \tag{15}$$

It should be mentioned that hyperon decay fits suggest [5]

$$|V_{us}| = 0.2250(27) \text{ Hyperon Decays} \tag{16}$$

modulo SU(3) breaking effects that could shift that value up or down. We note that a recent representative effort [36] that incorporates SU(3) breaking found $V_{us} = 0.226(5)$. Similarly, strangeness changing tau decays give [37]

$$|V_{us}| = 0.2208(34) \text{ Tau Decays} \tag{17}$$

where the central value depends on the strange quark mass.

Employing the value of V_{ud} in Eq. (4) and V_{us} in Eq. (11) leads to the unitarity consistency check

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9992(5)(9), \tag{18}$$

where the first error is the uncertainty from $|V_{ud}|^2$ and the second error is the uncertainty from $|V_{us}|^2$. The result is in good agreement with unitarity. Averaging the direct determination of λ (V_{us}) with the determination derived from unitarity and

V_{ud} gives $\lambda = 0.227(1)$. Although unitarity now seems well established, issues regarding the Q values in superallowed nuclear β-decays, $\tau_n, g_A, f_+(0)$ and f_K/f_π must still be resolved before a definitive confirmation is possible.

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