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V_{cb} and V_{ub} CKM Matrix Elements

OMITTED FROM SUMMARY TABLE

DETERMINATION OF $|V_{cb}|$

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I. Introduction

In the framework of the Standard Model, the quark sector is characterized by a rich pattern of flavor-changing transitions, described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix (see CKM review [1]). This report focuses on the quark mixing parameter $|V_{cb}|$.

Two different methods have been used to extract this parameter from data: the **exclusive** measurement, where $|V_{cb}|$ is extracted by studying $B \to D^\star \ell\nu$ or $B \to D\ell\nu$ decay processes; and the **inclusive** measurement, which uses the semileptonic width of b-hadron decays ($B \to X\ell\overline{\nu}$). Theoretical estimates play a crucial role in extracting $|V_{cb}|$, and an understanding of their uncertainties is very important.

II. $|V_{cb}|$ determination from exclusive channels

The exclusive $|V_{cb}|$ determination is obtained studying $B \to D^\star\ell\nu$ or $B \to D\ell\nu$ decays, using Heavy Quark Effective Theory (HQET), an exact theory in the limit of infinite quark masses. Currently, the $B \to D\ell\nu$ transition provides a less precise value, and is used as a check.

The decay $B \to D^\star\ell\nu$ in HQET: HQET predicts that the differential partial decay width for this process, $d\Gamma/dw$, is related to $|V_{cb}|$ through:

$$\frac{d\Gamma}{dw}(B \to D^\star\ell\nu) = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\mathcal{K}(w)\mathcal{F}(w)^2, \qquad (1)$$

where w is the inner product of the B and $D^{\star}$ meson 4-velocities, $\mathcal{K}(w)$ is a known phase-space factor, and the form factor $\mathcal{F}(w)$ is generally expressed as the product of a normalization constant, $\mathcal{F}(1)$, and a function, $g(w)$, constrained by experimental studies of the helicity amplitudes characterizing this decay [2] and dispersion relations [3].

There are several different corrections to the infinite mass value $\mathcal{F}(1) = 1$ [4]:

$$\mathcal{F}(1) = \eta_{\rm QED}\eta_A \left[1 + \delta_{1/m_Q^2} + ...\right], \qquad (2)$$

here and in the following discussion of exclusive semileptonic decays, m_Q is a generic notation for m_c or m_b. By virtue of Luke's theorem [5], the first term in the non-perturbative expansion in powers of $1/m_Q$ vanishes. QED corrections up to leading-logarithmic order give $\eta_{\rm QED} \approx 1.007$ [6] and QCD radiative corrections to two loops give $\eta_A = 0.960 \pm 0.007$ [7]. Different estimates of the $1/m_Q^2$ corrections, involving terms proportional to $1/m_b^2$, $1/m_c^2$, and $1/(m_b m_c)$, have been performed in a quark model [8,10], with OPE sum rules [11], and, more recently, with an HQET based lattice gauge calculation [12]. The value from this quenched lattice HQET calculation is $\mathcal{F}(1) = 0.913^{+0.024}_{-0.017} \pm 0.016^{+0.003}_{-0.014}\,{}^{+0.000}_{-0.016}\,{}^{+0.006}_{-0.014}$. The errors quoted reflect the statistical accuracy, the matching error, the lattice finite size, the uncertainty in the quark masses, and an estimate of the error induced by the quenched approximation, respectively. The central value obtained with OPE sum rules is similar, $0.900 \pm 0.015 \pm 0.025 \pm 0.025$ [11], where the three errors parameterize different sources of theoretical uncertainty. Here we will use $\mathcal{F}(1) = 0.91 \pm 0.04$ [13], a value that is consistent with all the three determinations discussed above. We have chosen not to rely solely on the value of $\mathcal{F}(1)$

coming from the lattice, because of the difficulties of quantifying the uncertainty induced by the quenched approximation. Recent developments give confidence that this limitation will be overcome in the next few years [15]. Technical advances, such as new improved staggered discretization, may lead to precise value of some "gold-plated" lattice quantities, such as $\mathcal{F}(1)$ in $B \to D^\star \ell \overline{\nu}$. The stated theoretical accuracy will be checked by comparing predicted and measured values of a large number of non-perturbative quantities [17].

The analytical expression of $g(w)$, the universal form factor related to the Isgur-Wise function [18], is not known a-priori, and this introduces an additional uncertainty in the determination of $\mathcal{F}(1)|V_{cb}|$. First measurements of $|V_{cb}|$ were performed assuming a linear approximation for $g(w)$. It has been shown [19] that this assumption is not justified, and that linear fits systematically underestimate the extrapolation at zero recoil ($w = 1$) by about 3%. Most of this effect is related to the curvature of the form factor, and does not depend strongly upon the details of the chosen non-linear shape [19]. All recent published results use a non-linear shape for $g(w)$, approximated with an expansion near $w = 1$ [20]. $g(w)$ is parameterized in terms of the variable ρ^2, which is the slope of the form factor at zero recoil given in Ref. 20.

Experimental techniques to study the decay $B \to D^\star \ell \nu$:

The decay $B \to D^\star \ell \nu$ has been studied in experiments performed at center-of-mass energies equal to the $\Upsilon(4S)$ mass and the Z^0 mass. At the $\Upsilon(4S)$, experiments have the advantage that the w resolution is quite good. The dominant systematic uncertainties arise from background estimation and from the slow pion efficiency evaluation. This efficiency for charged pions is very low near w=1 and increases rapidly as w increases, while for neutral pions it drops slowly. CLEO [22] studies both $D^{\star +}$ and $D^{\star o}$ channels, while Belle [23] and BaBar [24] have so far presented only results based on $D^{\star +}\ell\nu$. In addition, kinematic constraints enable $\Upsilon(4S)$ experiments to identify the final state, including the $D^\star$, without a large contamination from the poorly known semileptonic decays including a hadronic system heavier than $D^\star$, commonly identified as '$D^{\star\star}$'. At LEP, B's are produced with a large momentum (about 30 GeV on average). The large boost produces a broadening in the reconstructed ν 4-momentum, needed to determine w, thus giving a relatively poor resolution and limited physics background rejection capabilities. On the other hand, LEP experiments benefit from an efficiency that is only mildly dependent upon w.

Experiments determine the product $(\mathcal{F}(1) \cdot |V_{cb}|)^2$ by fitting the measured $d\Gamma/dw$ distribution. Measurements have been published by CLEO [22], Belle [23], DELPHI [25], ALEPH [26], and OPAL [27]. Most recently, a preliminary measurement from BaBar has been presented [24]. At LEP, the dominant source of systematic error is the uncertainty on the contribution to $d\Gamma/dw$ from semileptonic B decays with final states including a hadron system heavier than the $D^\star$. This component includes both narrow orbitally excited charmed mesons and non-resonant or broad species. The existence of narrow resonant states is well established [1], and a signal of a broad resonance has been seen by CLEO [28], and, most recently, by Belle [29], but the decay characteristics of these states in b-hadron semileptonic decays have large uncertainties. The average of ALEPH [30], Belle [33], CLEO [31], and DELPHI [32] narrow state branching fractions show that the ratio $R_{\star\star} = \dfrac{\mathrm{B}(\overline{B} \to D_2^\star \ell\overline{\nu})}{\mathrm{B}(\overline{B} \to D_1 \ell\overline{\nu})}$ is smaller than one (< 0.6 at 95% C.L. [34]), in disagreement with HQET models where an infinite quark mass is assumed [35], but in agreement with models which take into account finite quark mass corrections [36]. Hence, LEP experiments use the treatment of narrow $D^{\star\star}$ proposed in [36], which accounts for $\mathcal{O}(1/m_c)$ corrections and provides several possible approximations of the form factors that depend on five different expansion schemes, and on three input parameters. To calculate the systematic errors, each proposed scheme is tested, with the relevant input parameters varied over a range consistent with the experimental limit on $R_{\star\star}$. The quoted systematic error is the maximal difference from the central value obtained with this method. Broad resonances or other non-resonant terms may not be modelled correctly with this approach.

To combine the published data, the central values and the errors of $\mathcal{F}(1)|V_{cb}|$ and ρ^2 are re-scaled to the same set of input parameters and their quoted uncertainties [21]. The $\mathcal{F}(1)|V_{cb}|$ values used for this average are extracted using the parametrization in Ref. 22, based on the experimental determinations of the vector and axial form factor ratios R_1 and R_2 [38]. The LEP data, which originally used theoretical values for these ratios, are re-scaled accordingly [37]. Table 1 summarizes the corrected data. The averaging procedure [37] takes into account statistical and systematic correlations between $\mathcal{F}(1)|V_{cb}|$ and ρ^2. Averaging the measurements in Table 1, we get:

$$\mathcal{F}(1)|V_{cb}| = (38.2 \pm 0.5 \pm 0.9) \times 10^{-3}$$

and

$$\rho^2 = 1.56 \pm 0.05 \pm 0.13, \tag{3}$$

with χ^2 per degree of freedom of 22/12. The error ellipses for the corrected measurements and for the world average are shown in Figure 1. They are the product between the 1 σ error of $\mathcal{F}(1)|V_{cb}|$, ρ^2, and the correlation between the two. Since this is a 2 parameter fit, the ellipses correspond to about 37% CL contours. The χ^2 per degree of freedom is 1.8. We have not rescaled the errors.

Table 1: Experimental results from $B \to D^\star \ell \nu$ analyses after the correction to common inputs and world average. The LEP numbers are corrected to use R_1 and R_2 from CLEO data. ρ^2 is the slope of the form factor at zero recoil as defined in Ref. 20. $\mathrm{Corr}_{\mathrm{stat}}$ is the statistical correlation between $\mathcal{F}(1)|V_{cb}|$ and ρ^2. (* Average of two measurements.)

Exp.	$\mathcal{F}(1)\lvert V_{cb}\rvert(\times 10^3)$	ρ^2	$\mathrm{Corr}_{\mathrm{stat}}$
ALEPH	$33.6 \pm 2.1 \pm 1.6$	$0.75 \pm 0.25 \pm 0.37$	94%
DELPHI*	$36.2 \pm 1.1 \pm 1.8$	$1.42 \pm 0.10 \pm 0.33$	92%
OPAL*	$39.1 \pm 0.9 \pm 1.8$	$1.43 \pm 0.12 \pm 0.31$	89%
Belle	$36.3 \pm 1.9 \pm 1.9$	$1.45 \pm 0.16 \pm 0.20$	91%
CLEO	$43.3 \pm 1.3 \pm 1.8$	$1.61 \pm 0.09 \pm 0.21$	87%
BaBar	$34.1 \pm 0.2 \pm 1.3$	$1.23 \pm 0.02 \pm 0.28$	92%
World average	$38.2 \pm 0.5 \pm 0.9$	$1.56 \pm 0.05 \pm 0.13$	53%

The main contributions to the $\mathcal{F}(1)|V_{cb}|$ systematic error are from the uncertainty on the $B \to D^{\star\star}\ell\nu$ shape and $\mathrm{B}(b \to B_d)$, fully correlated among the LEP experiments, the branching fraction of D and $D^\star$ decays, fully correlated among all the experiments, and the slow pion reconstruction from Belle and CLEO which are uncorrelated, The main contribution to the ρ^2 systematic error is from the uncertainties in the measured values of R_1 and R_2, fully correlated among experiments. Because of

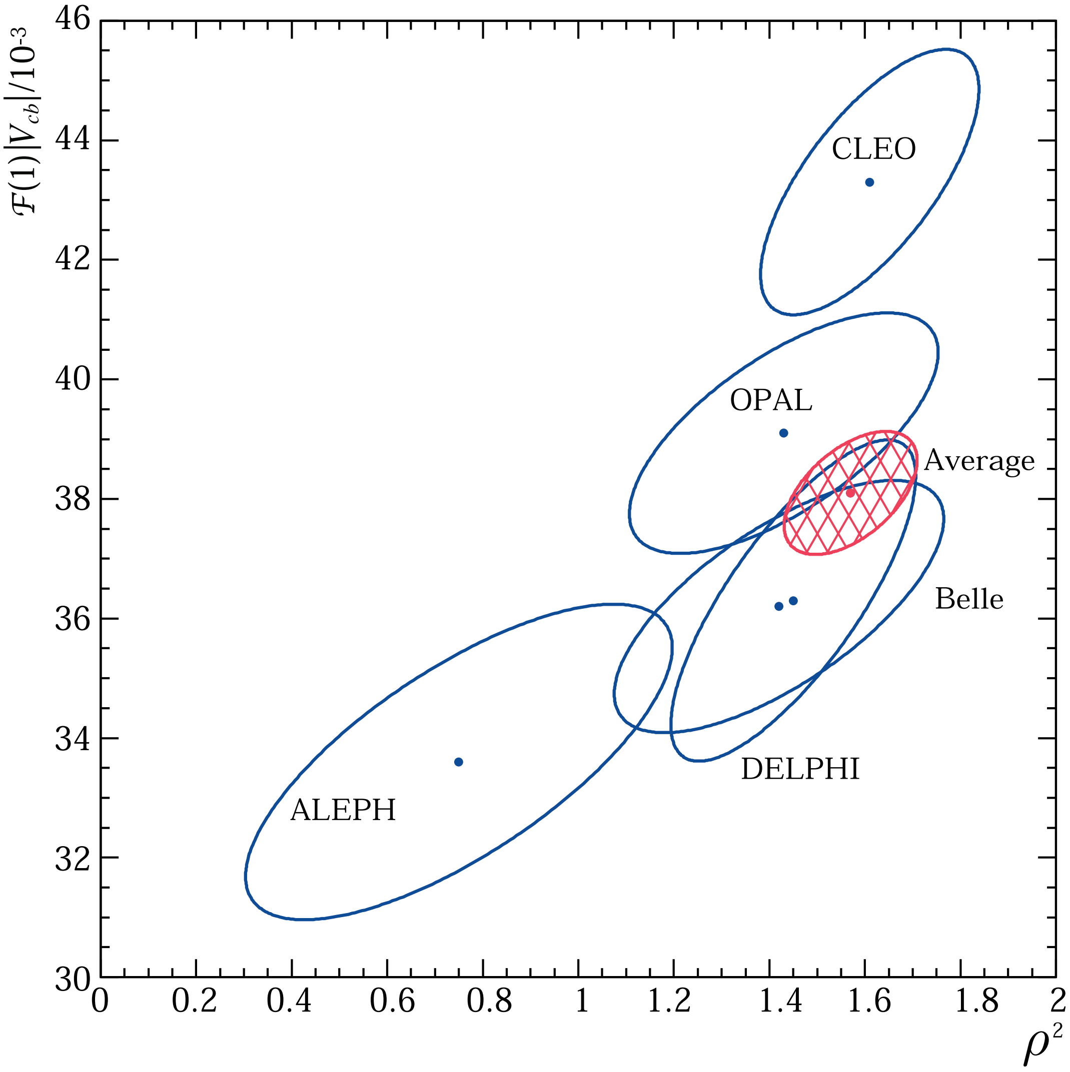

Figure 1: The error ellipses for the corrected measurements and world average for $\mathcal{F}(1)|V_{cb}|$ vs ρ^2. The ellipses are the product between the 1 σ error of $\mathcal{F}(1)|V_{cb}|$, ρ^2, and the correlation between the two. Consequently the ellipses correspond to about 37% CL. See full-color version on color pages at end of book.

the large contribution of this uncertainty to the non-diagonal terms of the covariance matrix, the averaged ρ^2 is higher than one would naively expect.

Using $\mathcal{F}(1) = 0.91 \pm 0.04$ [13], we get $|V_{cb}| = (42.0 \pm 1.1_{\rm exp} \pm 1.8_{\rm theo}) \times 10^{-3}$. The dominant error is theoretical, and there are good prospects to reduce it in the next few years [14], [17].

The decay $B \to D\ell\nu$: The study of the decay $B \to D\ell\nu$ poses new challenges both from the theoretical and experimental point of view.

The differential decay rate for $B \to D\ell\nu$ can be expressed as:

$$\frac{d\Gamma_D}{dw}(B \to D\ell\nu) = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} \mathcal{K}_{\mathcal{D}}(w) \mathcal{G}(w)^2, \tag{4}$$

where w is the inner product of the B and D meson 4-velocities, $\mathcal{K}_{\mathcal{D}}(w)$ is the phase space, and the form factor $\mathcal{G}(w)$ is generally expressed as the product of a normalization factor, $\mathcal{G}(1)$, and a function, $g_D(w)$, constrained by dispersion relations [3].

The strategy to extract $\mathcal{G}(1)|V_{cb}|$ is identical to that used for the $B \to D^\star\ell\nu$ decay. However, in this case there is no suppression of $1/m_Q$ (*i.e.*, no Luke theorem) and corrections and QCD effects on $\mathcal{G}(1)$ are calculated with less accuracy than $\mathcal{F}(1)$ [39,40]. Moreover, $d\Gamma_D/dw$ is more heavily suppressed near $w = 1$ than $d\Gamma_{D^*}/dw$, due to the helicity mismatch between initial and final states. This channel is also much more challenging from the experimental point of view as it is hard to isolate from the dominant $B \to D^\star\ell\nu$ background, as well as from fake D–ℓ combinations. Thus, the extraction of $|V_{cb}|$ from this channel is less precise than the one from the $B \to D^\star\ell\nu$ decay. Nevertheless, the $B \to D\ell\nu$ channel provides a consistency check, and allows a test of heavy-quark symmetry [40] through the measurement of the form factor $\mathcal{G}(w)$, as HQET predicts the ratio $\mathcal{G}(w)/\mathcal{F}(w)$ to be very close to one.

Belle [41] and ALEPH [26] studied the $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ channel, while CLEO [42] studied both $B^+ \to D^0\ell^+\overline{\nu}$ and $\overline{B}^0 \to D^+\ell^-\overline{\nu}$ decays. Averaging the data in Table 2 [37], we get $\mathcal{G}(1)|V_{cb}| = (41.8 \pm 3.7) \times 10^{-3}$ and $\rho_D^2 = 1.15 \pm 0.16$, where ρ_D^2 is the slope of the form factor at zero recoil given in Ref. 20.

Table 2: Experimental results after the correction to common inputs and world average. ρ_D^2 is the slope of the form factor at zero recoil given in Ref. 20.

Exp.	$\mathcal{G}(1)\|V_{cb}\|(\times 10^3)$	ρ_D^2
ALEPH	$39.3 \pm 10.0 \pm 6.5$	$0.97 \pm 0.98 \pm 0.38$
Belle	$41.8 \pm 4.4 \pm 5.2$	$1.12 \pm 0.22 \pm 0.14$
CLEO	$44.4 \pm 5.8 \pm 3.5$	$1.27 \pm 0.25 \pm 0.14$
World average	$41.8 \pm 2.5 \pm 2.7$	$1.15 \pm 0.13 \pm 0.09$

The theoretical predictions for $\mathcal{G}(1)$ are consistent: 1.03 ± 0.07 [43], and 1.02 ± 0.08 [40]. A quenched lattice calculation gives $\mathcal{G}(1) = 1.058^{+0.021}_{-0.017}$ [44], where the errors do not include the uncertainties induced by the quenching approximation and lattice spacing. An unquenched value should be available in the next few years [15]. A recent study of the decay $B \to D\ell\overline{\nu}$ in the context of heavy quark sum rules [16] argues that this channel can provide an alternative very precise determination of $|V_{cb}|$. *Ref. [16] uses heavy quark sum rules to relate exclusive form factors and inclusive semileptonic width and argues that in the approximation $\mu_\pi^2 - \mu_G^2 << \mu_\pi^2$, many power corrections vanish to all orders in $1/m_Q$. The parameter μ_π^2 represents the expectation value of the leading local heavy quark kinetic operator and μ_G^2 parameterizes the corresponding expectation value of the chromomagnetic operator. A more extensive discussion of the theoretical treatment of inclusive semileptonic decays is given in the next section.

Using $\mathcal{G}(1) = 1.04 \pm\ 0.06$, we get $|V_{cb}| = (40.2 \pm 3.6_{\rm exp} \pm 2.3_{\rm theo}) \times 10^{-3}$, consistent with the value extracted from $B \to D^\star \ell\nu$ decay, but with a larger uncertainty.

The experiments have also measured the differential decay rate distribution to extract the ratio $\mathcal{G}(w)/\mathcal{F}(w)$. The data are compatible with a universal from factor as predicted by HQET.

III. $|V_{cb}|$ determination from inclusive B semileptonic decays

Alternatively, $|V_{cb}|$ can be extracted from the inclusive semileptonic width, requiring measurements of both the B lifetimes and the semileptonic branching fraction $\mathrm{B}(B \to X_c \ell\nu)$ [45,46]. In this case, quark-hadron duality bridges the gap between theoretical calculations and experimental observables [47]. The modern theoretical formulation based on the Operator Product Expansion (OPE) determines the inclusive decay amplitudes in inverse powers of $1/m_Q$ [45]. Non-perturbative corrections to the leading term, given by the spectator decay amplitude, arise only to order $1/m_b^2$. Quark-hadron duality is an important *ab initio* assumption in these calculations [47]. As M. Shifman put it [47], "It is fair to say that (short of the full solution of QCD) understanding and controlling the accuracy of the quark-hadron duality is one of the most important and challenging problem for the QCD practitioner today." In other words, as pointed out by Shifman and Buchalla [49], "duality violation parameterize our ignorance."

Models can give estimates of the uncertainty induced by duality violations [48], [50]. These models need to have a clear physical interpretation and must be tested, in their key features, against experimental data [47]. The models quoted before imply different power suppression of duality violations. This issue needs to be resolved with further theoretical effort in defining clear and unambiguous quantitative tests of duality violations complemented by an experimental program to validate them.

The coefficients of the $1/m_b$ power terms are expectation values of operators that include non-perturbative physics. Relationships that are valid up to $1/m_b^2$ include four such parameters: the expectation value of the kinetic operator, corresponding to the average of the square of the heavy-quark momentum inside the hadron, the expectation value of the chromomagnetic operator, and the heavy-quark masses (m_b and m_c). The expectation value of the kinetic operator is introduced in the literature as μ_π^2 [45,46] or $-\lambda_1$ [51,52], and the expectation value of the chromomagnetic operator as μ_G^2 [45,46], or $3\lambda_2$ [51,52]. The two notations reflect a difference in the approach used to handle the energy scale μ used to separate long-distance from short-distance physics. HQET is most commonly renormalized in a mass-independent scheme, thus making the quark masses the pole masses of the underlying theory (QCD). The second group of authors prefer the definition of the non-perturbative operators using a mass scale $\mu \approx 1$ GeV.

The semileptonic width expression in Ref. 53 has been used to extract $|V_{cb}|$ from the semileptonic branching fraction measured by CLEO, and to measure the heavy-quark expansion (HQE) parameters $\overline{\Lambda}$ and λ_1, as discussed below. The most recent version of the alternative formulation can be found in Ref. 9.

The quark masses are related to the corresponding meson masses through [8]:

$$m_b = \overline{M}_B - \overline{\Lambda} + \frac{\lambda_1}{2\overline{M}_B}, \tag{5}$$

where $\overline{M}_B$ is the spin averaged $B - B^{\star}$ mass ($\overline{M}_B = 5.3134$ GeV/c^2). A similar equation relates m_c and $\overline{M}_D$. The parameter $\overline{\Lambda}$ represents the energy of the light quark and gluons. From the equations relating m_b and m_c to the corresponding spin-averaged meson masses, experimentalists usually derive the constraint on m_c to be used in the theoretical formulae. It has been pointed out [9] that it may be opportune to replace this constraint with an independent experimental determination of m_c.

HQE and moments in semileptonic decays:

Experimental determinations of the HQE parameters are important in several respects. In particular, redundant determinations of these parameters may uncover inconsistencies, or point to violation of some important assumptions inherent in these calculations. The parameter λ_2 can be extracted from the $B^{\star} - B$ mass splitting, whereas the other parameters need more elaborate measurements.

The CLEO collaboration determines the parameter $\overline{\Lambda}$ from the first moment of the γ energy in the decay $b \to s\gamma$, which gives the average energy of the γ emitted in this transition. Using the formalism of Ref. 53, they obtain $\overline{\Lambda} = 0.35 \pm 0.07 \pm 0.10$ GeV [54].

The parameter λ_1 can be determined from of the first moment of the mass M_X of the hadronic system recoiling against the $\ell - \overline{\nu}$ pair. The relationship between the first moment of $M_1 \equiv < M_X^2 - M_D^2 >$ and the parameters $\overline{\Lambda}$ and λ_1 is given in Ref. 55.

The measured value for $< M_X^2 - M_D^2 >$ [55] is 0.251 ± 0.066 GeV^2. This constraint, combined with the measurement of the mean photon energy in $b \to s\gamma$, implies a value of $\lambda_1 = -0.24 \pm 0.11$ GeV^2, to order $1/M_B^3$ and $\beta_0 \alpha_s^2$ in ($\overline{\mathrm{MS}}$).

The shape of the lepton spectrum provides further constraints on OPE. Moments of the lepton momentum with a cut $p_\ell^{CM} \geq 1.5$ GeV/c have been measured by the CLEO collaboration [62]. The two approaches give consistent results, although the technique used to extract the OPE parameters has still relatively large uncertainties associated with the $1/m_b^3$ form factors. The sensitivity to $1/m_b^3$ corrections depends upon which moments are considered. Bauer and Trott [61] have performed an extensive study of the sensitivity of lepton energy moments to non-perturbative effects. In particular, they have proposed "duality moments," very insensitive to neglected higher order terms. The comparison between the CLEO measurement of these moments [62] and the predicted values shows a very impressive agreement:

$$D_3 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{0.7} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{1.5} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.5190 \pm 0.0007 & (\mathrm{T}) \\ 0.5193 \pm 0.0008 & (\mathrm{E}) \end{cases}$$

$$D_4 \equiv \frac{\int_{1.6\,\mathrm{GeV}} E_\ell^{2.3} \frac{d\Gamma}{dE_\ell}\, dE_\ell}{\int_{1.5\,\mathrm{GeV}} E_\ell^{2.9} \frac{d\Gamma}{dE_\ell}\, dE_\ell} = \begin{cases} 0.6034 \pm 0.0008 & (\mathrm{T}) \\ 0.6036 \pm 0.0006 & (\mathrm{E}) \end{cases} \qquad (6)$$

(where "T" and "E" denote theory and experiment, respectively).

More recently, both CLEO and BaBar explored the moments of the hadronic mass M_X^2 with lower lepton momentum cuts. In order to identify the desired semileptonic decay from background processes including cascade decays, continuum leptons and fake leptons, CLEO performs a fit for the contributions of signal and backgrounds to the full three-dimensional differential decay rate distribution as a function of the reconstructed quantities q^2, M_X^2, $\cos\theta_{W\ell}$. The signal includes the components $B \to D\ell\overline{\nu}$, $B \to D^{\star}\ell\overline{\nu}$, $B \to D^{\star\star}\ell\overline{\nu}$, $B \to X_c\ell\overline{\nu}$ non-resonant, and $B \to X_u\ell\overline{\nu}$. The backgrounds considered are: secondary leptons, continuum leptons and fake leptons. BaBar uses a sample where the hadronic decay of one B is fully reconstructed and the charged lepton from the other B is identified. In this case the main sources of systematic errors are the uncertainties related to the detector modelling and reconstruction.

Figure 2 shows the extracted $< M_X^2 - \overline{M}_D^2 >$ moments as a function of the minimum lepton momentum cut from these two measurements, as well as the original measurement with $p_\ell \geq 1.5$ GeV/c. The results are compared with theory bands that reflect experimental errors, $1/m_b^3$ correction uncertainties and uncertainties in the higher order QCD radiative corrections [56]. The CLEO and BaBar results are consistent and show an improved agreement with theoretical predictions with respect to earlier preliminary results [57]. Moments of the M_X distribution without an explicit lepton momentum cut have been extracted from preliminary DELPHI data [58] and give consistent results.

Experimental determination of the semileptonic branching fraction:

The value of B($B \to X_c\ell\nu$) has been measured both at the $\Upsilon(4S)$ and LEP.

Experiments taking data at the Υ(4S) center-of-mass energy determine the inclusive semileptonic branching fraction

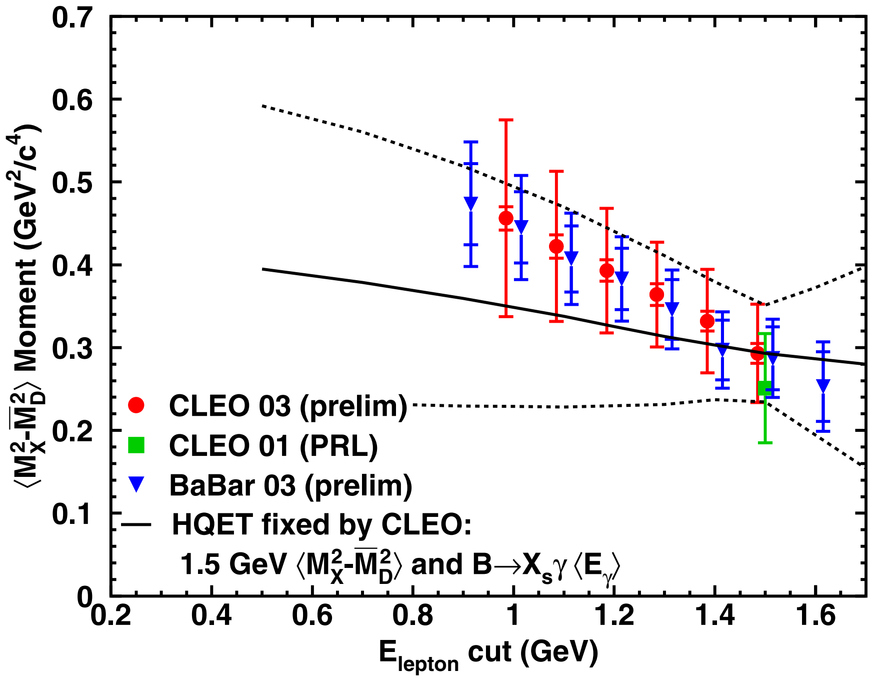

Figure 2: The results of the recent CLEO analysis [59] compared to previous measurements [55,60] and the HQET prediction. The theory bands shown in the figure reflect the variation of the experimental errors on the two constraints, the variation of the third-order HQET parameters by the scale $(0.5 \text{ GeV})^3$, and variation of the size of the higher order QCD radiative corrections [56]. See full-color version on color pages at end of book.

through a lepton tagged sample. In this approach, a di-lepton sample is studied, and the charge correlation between the two leptons is used to disentangle leptons coming from the direct decay $B \to X_c \ell \nu$ and the dominant background at low lepton momenta, the cascade decay $B \to X_c \to X_s \ell \nu$. This method was pioneered by the ARGUS collaboration [63] to measure the electron spectrum from $B \to X_c \ell\nu$ down to 0.6 GeV/c. Thus, it reduces the model dependence of the extracted semileptonic branching fraction very substantially. Experimental data are summarized in Table 3. The systematic error is dominated by experimental uncertainties: lepton identification efficiency, fake rate determination, and tracking efficiencies contribute to 3% of this overall error. The remaining error is a sum of several small corrections associated with the uncertainty in the mixing parameter, and additional background estimates [64]. BaBar [65] and Belle [66] have studied the inclusive electron spectrum with the same technique.

Table 3: B($b \to \ell$) measurement from experiments at $\Upsilon(4S)$ center-of-mass energy and their average. The errors quoted reflect statistical, and systematic uncertainties. These measurements are largely model independent.

Experiment	B($b \to \ell\nu$)%
ARGUS	$9.75 \pm 0.50 \pm 0.39$
CLEO	$10.49 \pm 0.17 \pm 0.43$
Belle	$10.96 \pm 0.12 \pm 0.50$
BaBar	$10.91 \pm 0.18 \pm 0.29$
Υ(4S) Average	10.73 ± 0.28

Combining $\Upsilon(4S)$ results [1], we obtain: B($b \to X\ell\nu$) = (10.73 ± 0.28)%. Upon subtracting B($b \to u\ell\nu$) = (0.17 ± 0.05)%, we get: B($b \to X_c\ell\nu$) = (10.56 ± 0.28)%. Using τ_{B^+}, τ_{B^0} [1], and the ratio between charged and neutral B pair production $f_{+-}/f_{00} = 1.044 \pm 0.05$ [21], we obtain the semileptonic width $\Gamma(b \to X_c\ell\nu) = (0.434 \pm 0.011 \pm 0.003) \times 10^{-10}$ MeV, where the second error includes the uncertainties from $\mathrm{B}(b \to u\ell\nu)$, and the model dependence. A common value for the ratio f_{+-}/f_{00} between B^+B^- and $B^0\overline{B}^0$ final states produced at the $\Upsilon(4S)$ is used here. This parameter is very sensitive to the precise value of the center-of-mass energy and beam energy spread [70] and thus as more precise data become available, it is important to check that it is appropriate to average f_{+-}/f_{00} at different machines.

At LEP, B^0, B^-, B_s, and b baryons are produced, so the measured inclusive semileptonic branching ratio is an average over the different hadron species. Assuming that the semileptonic widths of all b hadrons are equal, the following relation holds:

$$\begin{aligned}
\mathrm{B}(b \to X_c l\nu)_{\mathrm{LEP}} = \\
& f_{B^0}\frac{\Gamma(B^0 \to X_c l\nu)}{\Gamma(B^0)} + f_{B^-}\frac{\Gamma(B^- \to X_c l\nu)}{\Gamma(B^-)} \\
& + f_{B_s}\frac{\Gamma(B_s \to X_c l\nu)}{\Gamma(B_s)} + f_{\Lambda_b}\frac{\Gamma(\Lambda_b \to X_c l\nu)}{\Gamma(\Lambda_b)} \\
& = \Gamma(B \to X_c \ell\nu)\tau_b \ , \qquad (7)
\end{aligned}$$

where τ_b is the average b-hadron lifetime. Taking into account the present precision of LEP measurements of b-baryon semileptonic branching ratios and lifetimes, the estimate uncertainty for a possible difference for the width of b baryons is 0.13%. The average LEP value for $\mathrm{B}(b \to X\ell\nu) = (10.59 \pm 0.30)\%$ is taken from a fit [71], which combines the semileptonic branching ratios, the B^0–$\overline{B}^0$ mixing parameter $\overline{\chi}_b$, and $R_b = \Gamma(Z \to b\overline{b})/\Gamma(Z \to \mathrm{had})$. Ref. 72 shows that the main contribution to the model uncertainty is the composition of the semileptonic width, including the narrow, wide and non-resonant $D^{\star\star}$ states. B_s and b baryons are about 20% of the total signal, and their contribution to the uncertainty of the spectrum is small. In this average, we use the modelling error quoted by Ref. 72, rather than the error from the combined fit, as the ALEPH procedure is based on more recent information. The dominant errors in the combined branching fraction are the modelling of semileptonic decays (2.6%) and the detector related items (1.3%).

Subtracting $\mathrm{B}(b \to u\ell\nu)$ from the LEP semileptonic branching fraction, we get: $\mathrm{B}(b \to X_c\ell\nu) = (10.52 \pm 0.32)\%$, and using τ_b [1]: $\Gamma(b \to X_c\ell\nu) = (0.439 \pm 0.010 \pm 0.011) \times 10^{-10}$ MeV, where the systematic error 0.011×10^{-10}MeV reflects the $\mathrm{B}(b \to u\ell\nu)$ uncertainty and the model dependence.

Combining the LEP and the $\Upsilon(4S)$ semileptonic widths, we get: $\Gamma(b \to X_c\ell\nu) = (0.44 \pm 0.01) \times 10^{-10}$ MeV, which is used in the formula of Ref. 55 to get:

$$|V_{cb}|_{\mathrm{incl}} = (41.0 \pm 0.5_{\mathrm{exp}} \pm 0.5_{\lambda_1,\overline{\Lambda}} \pm 0.8_{\mathrm{theo}}) \times 10^{-3} \ , \qquad (8)$$

where the first error is experimental, and the second is from the measured value of λ_1 and $\overline{\Lambda}$, assumed to be universal up to higher orders. The third error is from $1/m_b^3$ corrections and from the ambiguity in the α_s scale definition. The error on the average b-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching ratio.

IV. Conclusions

The values of $|V_{cb}|$ obtained both from the inclusive and exclusive method agree within errors. The value of $|V_{cb}|$ obtained from the analysis of the $B \to D^\star\ell\nu$ decay is:

$$|V_{cb}|_{\mathrm{exclusive}} = (42.0 \pm 1.1_{\mathrm{exp}} \pm 1.9_{\mathrm{theo}}) \times 10^{-3} \ , \qquad (9)$$

where the first error is experimental and the second error is from the $1/m_Q^2$ corrections to $\mathcal{F}(1)$. The value of $|V_{cb}|$, obtained from inclusive semileptonic branching fractions is:

$$|V_{cb}|_{\rm incl} = (41.0 \pm 0.5_{\rm exp} \pm 0.5_{\lambda_1,\overline{\Lambda}} \pm 0.8_{\rm theo}) \times 10^{-3}. \qquad (10)$$

In addition, non-quantified uncertainties are associated with a possible quark-hadron duality violations when using the inclusive method. A first conservative assessment of these uncertainties may be obtained from the difference between the two values of $|V_{cb}|$ extracted from $B \to D^{\star}\ell\overline{\nu}$ and from inclusive measurements. These data imply about 6% uncertainty for non-quantified assumptions in the inclusive determination. This result is largely affected by the quantified theoretical errors in the two determinations and thus does not give a very stringent bound.

High precision tests of lattice gauge theory calculations and more refined experimental assessments of quark-hadron duality in inclusive semileptonic decays are needed to achieve the ultimate accuracy in our knowledge of V_{cb}.

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V_{cb} MEASUREMENTS

For the discussion of V_{cb} measurements, which is not repeated here, see the review on "Determination of $|V_{cb}|$."

The CKM matrix element $|V_{cb}|$ can be determined by studying the rate of the semileptonic decay $B \to D^{(*)}\ell\nu$ as a function of the recoil kinematics of $D^{(*)}$ mesons. Taking advantage of theoretical constraints on the normalization and a linear ω dependence of the form factors provided by Heavy Quark Effective Theory (HQET), the $|V_{cb}|\times F(\omega)$ and ρ^2 (a^2) can be simultaneously extracted from data, where ω is the scalar product of the two-meson four velocities, $F(1)$ is the form factor at zero recoil (ω=1) and ρ^2 is the slope, sometimes denoted as a^2. Using the theoretical input of $F(1)$, a value of $|V_{cb}|$ can be obtained.

"OUR EVALUATION" is an average using rescaled values of the data listed below. The average and rescaling were performed by the Heavy Flavor Averaging Group (HFAG) and are described at http://www.slac.stanford.edu/xorg/hfag/. The averaging/rescaling procedure takes into account corrections between the measurements.

$|V_{cb}| \times F(1)$ (from $B^0 \to D^{*-}\ell^+\nu$)

VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
0.0381±0.0011 OUR EVALUATION with ρ^2=1.55 ± 0.15 and a correlation 0.88. The fitted χ^2 is 21.9 for 10 degrees of freedom.				
0.0371±0.0019 OUR AVERAGE Error includes scale factor of 1.7. See the ideogram below.				
0.0431±0.0013±0.0018	[1] ADAM	03	CLE2	$e^+e^- \to \Upsilon(4S)$
0.0354±0.0019±0.0018	[2] ABE	02F	BELL	$e^+e^- \to \Upsilon(4S)$
$0.0355\pm0.0014^{+0.0023}_{-0.0024}$	[3] ABREU	01H	DLPH	$e^+e^- \to Z$
0.0371±0.0010±0.0020	[4] ABBIENDI	00Q	OPAL	$e^+e^- \to Z$
0.0319±0.0018±0.0019	[5] BUSKULIC	97	ALEP	$e^+e^- \to Z$

• • • We do not use the following data for averages, fits, limits, etc. • • •

$0.0431 \pm 0.0013 \pm 0.0018$	[6] BRIERE	02	CLE2	$e^+ e^- \rightarrow \Upsilon(4S)$
$0.0328 \pm 0.0019 \pm 0.0022$	ACKERSTAFF	97G	OPAL	Repl. by ABBIENDI 00Q
$0.0350 \pm 0.0019 \pm 0.0023$	[7] ABREU	96P	DLPH	Repl. by ABREU 01H
$0.0351 \pm 0.0019 \pm 0.0020$	[8] BARISH	95	CLE2	Repl. by ADAM 03
$0.0314 \pm 0.0023 \pm 0.0025$	BUSKULIC	95N	ALEP	Repl. by BUSKULIC 97

[1] Average of the $B^0 \rightarrow D^*(2010)^- \ell^+ \nu$ and $B^+ \rightarrow \overline{D}^*(2007))\ell^+ \nu$ modes with $\rho^2 = 1.61 \pm 0.09 \pm 0.21$ and $f_{+-} = 0.521 \pm 0.012$.

[2] Measured using exclusive $B^0 \rightarrow D^*(892)^- e^+ \nu$ decays with $\rho^2 = 1.35 \pm 0.17 \pm 0.19$ and a correlation of 0.91.

[3] ABREU 01H measured using about 5000 partial reconstructed D^* sample with a $\rho^2 = 1.34 \pm 0.14 ^{+0.24}_{-0.22}$.

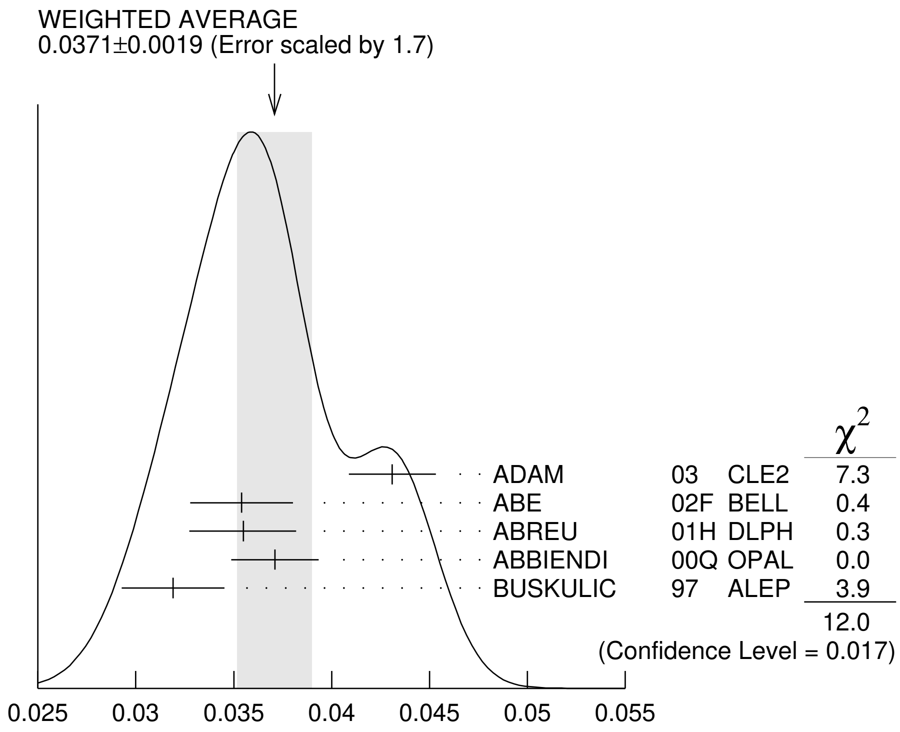
[4] ABBIENDI 00Q: measured using both inclusively and exclusively reconstructed $D^{*\pm}$ samples with a $\rho^2 = 1.21 \pm 0.12 \pm 0.20$. The statistical and systematic correlations between $|V_{cb}| \times F(1)$ and ρ^2 are 0.90 and 0.54 respectively.

[5] BUSKULIC 97: measured using exclusively reconstructed $D^{*\pm}$ with a $a^2 = 0.31 \pm 0.17 \pm 0.08$. The statistical correlation is 0.92.

[6] BRIERE 02 result is based on the same analysis and data sample reported in ADAM 03.

[7] ABREU 96P: measured using both inclusively and exclusively reconstructed $D^{*\pm}$ samples.

[8] BARISH 95: measured using both exclusive reconstructed $B^0 \rightarrow D^{*-} \ell^+ \nu$ and $B^+ \rightarrow D^{*0} \ell^+ \nu$ samples. They report their experiment's uncertainties $\pm 0.0019 \pm 0.0018 \pm 0.0008$, where the first error is statistical, the second is systematic, and the third is the uncertainty in the lifetimes. We combine the last two in quadrature.



$|V_{cb}| \times F(1)$ (from $B^0 \rightarrow D^{*-} \ell^+ \nu$)

$|V_{cb}| \times F(1)$ (from $B \to D^- \ell^+ \nu$)

VALUE	DOCUMENT ID		TECN	COMMENT
0.0418±0.0037 OUR EVALUATION with ρ^2=1.15 ± 0.16 and a correlation of 0.93. The fitted χ^2 is 0.3 for 4 degrees of freedom.				
0.039 ±0.004 OUR AVERAGE				
0.0411±0.0044±0.0052	[9] ABE	02E	BELL	$e^+e^- \to \Upsilon(4S)$
0.0416±0.0047±0.0037	[10] BARTELT	99	CLE2	$e^+e^- \to \Upsilon(4S)$
0.0278±0.0068±0.0065	[11] BUSKULIC	97	ALEP	$e^+e^- \to Z$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$0.0337 \pm 0.0044 ^{+0.0072}_{-0.0049}$	[12] ATHANAS	97	CLE2	Repl. by BARTELT 99

[9] Using the missing energy and momentum to extract kinematic information about the undetected neutrino in the $B^0 \to D^- \ell^+ \nu$ decay.

[10] BARTELT 99: measured using both exclusive reconstructed $B^0 \to D^- \ell^+ \nu$ and $B^+ \to D^0 \ell^+ \nu$ samples.

[11] BUSKULIC 97: measured using exclusively reconstructed $D^\pm$ with a $a^2 = -0.05 \pm 0.53 \pm 0.38$. The statistical correlation is 0.99.

[12] ATHANAS 97: measured using both exclusive reconstructed $B^0 \to D^- \ell^+ \nu$ and $B^+ \to D^0 \ell^+ \nu$ samples with a $\rho^2 = 0.59 \pm 0.22 \pm 0.12^{+0.59}_{-0}$. They report their experiment's uncertainties $\pm 0.0044 \pm 0.0048^{+0.0053}_{-0.0012}$, where the first error is statistical, the second is systematic, and the third is the uncertainty due to the form factor model variations. We combine the last two in quadrature.

DETERMINATION OF V_{ub}

Updated December 2003 by M. Battaglia (University of California, Berkeley and LBNL) and L. Gibbons (Cornell University, Ithaca) .

The precise determination of the magnitude of V_{ub} with a robust, well-understood uncertainty remains one of the key goals of the heavy flavor physics programs, both experimentally and theoretically. Because $|V_{ub}|$, the smallest element in the CKM mixing matrix, provides a bound on the upper vertex of one of the triangles representing the unitarity property of the CKM matrix, it plays a crucial role in the examination of the unitarity constraints and the fundamental questions on which the constraints can bear (see the minireviews on the CKM matrix [1] and on CP violation [2] for details). Investigation of these issues require measurements that are precise and that have well-understood uncertainties.

The charmless semi-leptonic (s.l.) decay channel $b \to u\ell\overline{\nu}$ provides the cleanest path for the determination of $|V_{ub}|$. However, the theory for the heavy-to-light $b \to u$ transition cannot be as well constrained as that for the heavy-to-heavy $b \to c$ transition used in the determination of $|V_{cb}|$ (see the $|V_{cb}|$ minireview [3]). The extraction of $|V_{ub}|$ and the interplay between experimental measurements and their theoretical interpretation are further complicated by the large background from $b \to c\ell\overline{\nu}$ decay, which has a rate about 60 times higher than that for charmless s.l. decay. Measurements based both on exclusive decay channels and on inclusive techniques have been, and are being, pursued.

The last several years have seen significant developments in both the theoretical framework and the experimental techniques used in the study of $b \to u\ell\overline{\nu}$. The inclusive theory has progressed significantly in categorization of the corrections to the base theory still needed, in their relative importance in different regions of phase space, and in the determination of some of them. Recent work on exclusive processes bolsters confidence in the current uncertainties for the form factor calculations needed to extract $|V_{ub}|$. Experimentally, we have new inclusive and exclusive measurements that minimize dependence on detailed modeling of the signal process to separate signal from the $b \to c\ell\overline{\nu}$ background, have well-defined sensitivities in particular regions of phase space and have improved signal-to-background ratios. These improvements provide us with a first opportunity to develop a method for a robust determination of $|V_{ub}|$ with complete error estimates, including constraints on hitherto unquantified contributions. We review the current determinations of $|V_{ub}|$, focusing primarily on these recent developments. An average of the inclusive information from all regions of phase space remains, unfortunately, beyond our reach because of the potentially sizable corrections for which we lack estimates. Rather, we combine the inclusive results to obtain a central value and, in particular, a more complete evaluation of the uncertainty than has been possible in the past.

Inclusive measurements of $b \to u\ell\overline{\nu}$:

Theoretically, issues regarding the calculation of the total semileptonic partial width $\Gamma(B \to X_u \ell \nu)$ via the operator product expansion (OPE) are well-understood [4–9]. The OPE is both a nonperturbative power series in $1/m_b$ and a perturbative expansion in α_s. At order $1/m_b^2$, it predicts

$$\begin{aligned}\Gamma(B \to X_u \ell \nu) = {} & \frac{G_F^2 |V_{ub}|^2}{192\pi^3}\, m_b^5 \\ & \times \left[1 - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \ldots - \mathcal{O}(\frac{\alpha_s}{\pi})\right] \ ,\end{aligned} \tag{1}$$

where λ_2 parameterizes the hyperfine interaction between the heavy quark and the light degrees of freedom and λ_1 is related to the Fermi momentum of the heavy quark. The perturbative corrections are known to order α_s^2 [10]. The OPE is alternatively expressed in terms of the nonperturbative parameters μ_π^2 and μ_G^2, which are closely related to λ_1 and λ_2, respectively, but differ significantly in their infrared treatment. Within the OPE, the importance of a proper field theoretic treatment of the parameters is paramount both for the total rate and for the restricted phase space studies discussed below. The treatment of the quark mass and its associated uncertainties are particularly important given the strong mass dependance of the width. Such considerations have led to useful definitions like the kinematic mass, which are discussed in detail in Ref. 11 and Ref. 12.

The error induced by uncertainties in the nonperturbative parameters $\lambda_{1,2}$ is relatively small, and an evaluation [13] by the LEP VUB working group yielded

$$|V_{ub}| = 0.00445 \left(\frac{\mathrm{B}(b \to u\ell\overline{\nu})}{0.002} \frac{1.55\mathrm{ps}}{\tau_b} \right)^{1/2} \times (1 \pm 0.020_{\mathrm{OPE}} \pm 0.052_{m_b}) \ . \quad (2)$$

The quoted uncertainty is dominated by the uncertainty in the b quark mass, for which $m_b^{1S}(1\ \mathrm{GeV}) = 4.58 \pm 0.09$ GeV was assumed. The value and uncertainty are in good agreement with a recent survey [14]. No weak annihilation uncertainties (discussed below) are included in the quoted OPE error. Use of the quark-level OPE for prediction of moments of the true inclusive spectra has generated concern regarding potential violation of the underlying assumption of global quark-hadron duality. This concern has been confronted by theoretical wisdom [15] supporting global duality for the inclusive $b \to u\ell\overline{\nu}$ transition, and new data both support this assumption and allow placement of quantitative limits on the violation. In particular, the exclusive and inclusive extractions of $|V_{cb}|$ agree to $(0.8 \pm 1.6) \times 10^{-3}$ [3]. Taking the uncertainty as an upper bound on the global duality violation for $|V_{ub}|$ shows that it should not exceed $\simeq$4%. Using this as bound as an uncertainty estimate for duality effects in the partial width prediction brings the total uncertainty on $|V_{ub}|$ to 6.8%. The agreement of the OPE parameters extracted using moments of different distributions in s.l. decays further supports the small scale for duality violation effects.

While theoretically the total inclusive rate would allow determination of $|V_{ub}|$ to better than 10%, experimentally the much more copious $b \to c\ell\overline{\nu}$ process makes a measurement over the full phase space unrealizable. To overcome this background, inclusive $b \to u\ell\overline{\nu}$ measurements utilize restricted regions of phase space in which the $b \to c\ell\overline{\nu}$ process is kinematically highly suppressed. The background is forbidden in the regions of large charged lepton energy $E_\ell > (M_B^2 - M_D^2)/(2M_B)$ (the endpoint), low hadronic mass $M_X < M_D$ and large dilepton mass $q^2 > (M_B - M_D)^2$. Extraction of $|V_{ub}|$ from such a measurement requires knowledge of the fraction of the total $b \to u\ell\overline{\nu}$ rate that lies within the utilized region of phase space, which complicates the theoretical issues and uncertainty considerably. Ref. 16 and Ref. 17 discuss the issues in detail.

CLEO [18], BaBar [19], and Belle [20] have all presented recent measurements of the $b \to u\ell\overline{\nu}$ rate near the endpoint. The results, which are for integrated ranges in the $\Upsilon(4S)$ rest frame, are summarized in Table 1. Experimentally, these measurements must contend with a large background from continuum e^+e^- annihilation processes. Suppression of these backgrounds introduces significant efficiency variation with the q^2 of the decay, which introduces model dependence. Greater awareness of this issue has resulted in more sophisticated suppression methods in these recent measurements and thus in over a factor of three reduction in the model dependence of the measured rates relative to earlier measurements [21, 22]. Future measurements, either using fully-reconstructed B-tag samples that would remove the problem or a modest q^2 binning, would essentially eliminate the remaining model dependence.

BaBar [23] and Belle [24] have presented new analyses of the low M_X region [25–28]. They also utilize a moderate (only $\sim 10\%$ loss) $p_\ell > 1.0$ GeV/c requirement. This technique was pioneered by Delphi at LEP [29], but the achievable resolution and signal-to-background ratio were lower compared to those obtained at the B factories. Because of experimental resolution on M_X, the $b \to c\ell\overline{\nu}$ background smears below its theoretical

Table 1: Partial branching fractions for $b \to u\ell\overline{\nu}$ within the charged lepton momentum range ($\Upsilon(4S)$ frame) from 2.6 GeV/c down to the indicated minimum. The estimated fraction f_E of the total $b \to u\ell\overline{\nu}$ rate expected to lie in that range is also given. The dagger (†) indicates the quantity that received the QED radiative correction appropriate to the indicated mix of electrons and muons, which has not always been treated self-consistently in the literature.

$p_\ell^{\rm min}$ (GeV/c)	$\Delta\mathcal{B}_u(p)$ (10^{-4})	f_E	
2.0	$4.22 \pm 0.33 \pm 1.78$	$\dagger 0.266 \pm 0.041 \pm 0.024$	CLEO (e,μ)
2.1	$3.28 \pm 0.23 \pm 0.73$	$\dagger 0.198 \pm 0.035 \pm 0.020$	CLEO (e,μ)
2.2	$2.30 \pm 0.15 \pm 0.35$	$\dagger 0.130 \pm 0.024 \pm 0.015$	CLEO (e,μ)
2.3	$1.43 \pm 0.10 \pm 0.13$	$\dagger 0.074 \pm 0.014 \pm 0.009$	CLEO (e,μ)
	$\dagger 1.52 \pm 0.14 \pm 0.14$	$0.078 \pm 0.015 \pm 0.009$	BaBar (e)
	$1.19 \pm 0.11 \pm 0.10$	$\dagger 0.072 \pm 0.014 \pm 0.008$	Belle (e)
2.4	$0.64 \pm 0.07 \pm 0.05$	$\dagger 0.037 \pm 0.007 \pm 0.003$	CLEO (e,μ)

lower limit of $M_X = M_D$, so experiments must impose more stringent M_X requirements which are theoretically more problematic. The BaBar and Belle analyses are based on "B-tag" samples of fully reconstructed hadronic decays and $D^{(*)}\ell\nu$ decays, respectively. In both cases, M_X is calculated directly from the particles remaining after removal of tag and lepton contributions. The BaBar analysis, in particular, reveals a beautiful $b \to u\ell\overline{\nu}$ signal with an unsurpassed signal to background ratio of about 2:1 in the region $M_X < 1.55$ GeV/c, which rivals that of current exclusive analyses. This analysis demonstrates the anticipated power of a large fully reconstructed sample, both in the signal to background levels and in the excellent resolution that can be achieved. The efficiency versus M_X appears reasonably uniform, and the signal yield fitting procedure minimizes the dependence of the extracted rate on the modeling of the detailed $b \to u\ell\overline{\nu}$ dynamics. Both allow for improved determination of $|V_{ub}|$ as theory advances.

Determination of the fraction of the $b \to u\ell\overline{\nu}$ rate in the p_ℓ endpoint or the low M_X region requires resummation of the OPE to all orders in $E_X\Lambda_{\rm QCD}/M_X^2$ [30–34]. The resummation results, at leading-twist order, in a nonperturbative shape function $f(k_+)$, where $k_+ = k^0 + k_{||}$ and $k^\mu = p_b^\mu - m_b v^\mu$ is the residual b quark momentum after the "mechanical" portion of momentum is subtracted off. Spatial components $k_{||}$ and $k_\perp$ are defined relative to the $m_b v^\mu - q^\mu$ (roughly the recoiling u quark) direction. At this order, effects such as the "jiggling" of $k_\perp$ are ignored and the differential partial width is given by the convolution of the shape function with the parton level differential distribution. Because the shape function depends only on parameters of the B meson, this leading order description holds for any B decay to a light quark. It holds, in particular, for $B \to s\gamma$, which can provides an estimation of $f(k_+)$ via the shape of the photon energy (E_γ) spectrum [31, 32]. In addition to the increased uncertainty on $|V_{ub}|$ from the m_b and b quark kinetic energy contributions that results from the restriction of phase space, higher twist contributions and unknown power corrections of order $\Lambda_{\rm QCD}/M_B$ [35, 36] also contribute to the uncertainty, as will be discussed further below.

Ideally, $|V_{ub}|$ would be determined without introduction of an intermediate extracted shape function through the use of appropriately weighted spectra [32,37–41]. This would avoid introduction of an element of model dependence. For the lepton spectrum, for example, one would take

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}\, K_{\rm pert}\frac{\widehat{\Gamma}_u(E_0)}{\widehat{\Gamma}_s(E_0)} + O(\Lambda_{\rm QCD}/M_B)\ , \qquad (3)$$

where $K_{\rm pert}$ is a calculable perturbative kernel, and $\widehat{\Gamma}_u(E_0)$ and $\widehat{\Gamma}_s(E_0)$ are appropriately weighted integrals over, respectively, the lepton energy and photon energy spectra above the minimum cutoff energy E_0. Practical application of this approach awaits measurement of the lepton momentum spectrum in the B, not the $\Upsilon(4S)$, rest frame, which B-tag methods will permit in the future. Similar expressions exist for integration over the low hadronic mass region [38,41], so, in principle, current M_X analyses should be able to take such an approach. Experimental efficiency and lepton momentum cutoffs must, however, be incorporated into the integrals. To date, experiments have instead introduced intermediate shape functions of a variety of forms into the inclusive analyses, as discussed below.

A third way to isolate the charmless s.l. signal is to use a selection based on the q^2 of the leptonic system. Restriction of phase space to regions of large q^2 also restores the validity of the OPE [42,43] and suppresses shape function effects. Taking only the region kinematically forbidden to $b \to c\ell\overline{\nu}$, $q^2 > (M_B - M_D)^2$, unfortunately introduces a low mass scale [44,45] into the OPE and the $1/m^3$ uncertainties blow up to be of order $(\Lambda_{\rm QCD}/m_c)^3$. However, a combination of M_X with looser q^2 requirements can suppress both $b \to c\ell\overline{\nu}$ background experimentally and shape function effects theoretically. Furthermore, the q^2 requirement moves the parton level pole away from the experimentally feasible M_X requirement. The shape function effects, while suppressed, cannot be neglected.

One drawback of the q^2 requirement is the elimination of higher energy hadronic final states, which may exacerbate duality concerns.

A recent Belle analysis [46] has been performed in this region. Belle employs a $p_\ell > 1.2$ GeV/c requirement in the $\Upsilon(4S)$ rest frame, and an "annealing" procedure to separate reconstructed particles into signal and "other B" halves. They then examine the integrated rate in the region $M_X < 1.7$ GeV and $q^2 > 8$ GeV^2 to extract $|V_{ub}|$, which again has the desired effect of minimizing dependence of the analysis on detailed $b \to u\ell\overline{\nu}$ modeling. The signal to background ratio of the annealing technique, about 1:6 for the Belle analysis, is significantly degraded relative to that of the hadronic B-tag technique. As we mentioned in the previous review, control of background subtractions of this size requires extreme care and careful scrutiny of the associated systematic issues. Belle finds the rate $\Delta\mathcal{B}$ in that region of phase space to be

$$\Delta\mathcal{B} = (7.37 \pm 0.89_{\mathrm{stat}} \pm 1.12_{\mathrm{sys}} \pm 0.55_{c\ell\nu} \pm 0.24_{u\ell\nu}) \times 10^{-4} \ . \quad (4)$$

An analysis of this restricted region of phase space, for which the shape function influence is significantly reduced [42], with the significantly cleaner B tag technique should be a priority for both B experiments.

Each analysis discussed here has relied on an intermediate shape function to evaluate the fraction of the inclusive rate that lies in its restricted region of phase space. The endpoint analyses have used rate fractions [18] based on intermediate shape functions derived from the CLEO $b \to s\gamma$ photon spectrum. Several two-parameter ansaetze [47,48], $F[\Lambda^{SF}, \lambda_1^{SF}]$, were implemented as the form of the shape function. These parameters are related to the HQET parameters of similar name, and play a similar role in evaluation of the rates. At this time, however, we do not know the precise relationship between the shape function parameters, or the moments of the shape function, and the HQET nonperturbative parameters $\overline{\Lambda}$ and λ_1 [49,50]. The fact that Λ^{SF} and λ_1^{SF} depend on the functional ansatz while the HQET parameters depend on the renormalization scheme underscores the current ambiguity.

With the limited E_γ statistics available, there exists a strong correlation between the two parameters because of the interplay between the effective b quark mass (controlled by Λ^{SF}) and the effective b quark kinetic energy (controlled by λ_1^{SF}) in determining the *mean* of the E_γ spectrum. No external constraints that could break the correlation, such as $m_b^{(1S)}$ or measured $b \to c\ell\overline{\nu}$ moments, have been input into the $b \to s\gamma$ fits because of their unknown relationship to the shape function parameters. The resulting effective m_b mass range contributing to the uncertainties (± 200 MeV) is therefore much larger than the current m_b uncertainty. Given the current independence of the shape function and m_b determinations and the broad effective m_b range sampled in the shape function, we do not consider it necessary to treat the E_γ-derived phase space fractions and the partial width (Eq. (2)) as positively correlated [46]. As the data statistics increase, it will become possible to constrain the shape function parameters directly from distributions in $b \to u\ell\overline{\nu}$, such as the M_X spectrum, thus removing the uncertainties introduced from their derivation from another class of decays. Furthermore, once the renormalization behaviour of the shape function and the relationship of its moments to the HQET parameters becomes known, the powerful constraints from the kinematic mass of the b quark and from moments information in the $b \to c\ell\nu$ system can be incorporated into a shape function derivation based either on $b \to s\gamma$ or $b \to u\ell\overline{\nu}$.

Two alternate approaches to the shape function evaluation have been taken in experimental studies so far. In their low-M_X analysis [23], BaBar has evaluated the phase space fraction using the same $f(k_+)$ parameterizations noted above, but has substituted HQET parameters derived from studies of spectral moments of the $b \to s\gamma$ and $b \to c\ell\overline{\nu}$ processes. The Belle M_X-q^2 analysis [46](discussed below) uses the calculation of Bauer *et al.* [42] based on a form with a single parameter $a = \Lambda^{SF}/\lambda_1^{SF}$, which was estimated from the $m_b^{(1S)}$ mass and from typical estimates for λ_1. The uncertainties in the different rate fractions in the momentum endpoint, the low M_X and the M_X-q^2 regions are strongly correlated, and the values and uncertainties are sensitive to the theoretical assumptions made. Hence, a common theoretical scheme must be chosen for meaningful comparison of the extracted values of $|V_{ub}|$. Given the *ad hoc* nature of the association of shape function parameters with the HQET parameters in the $\overline{MS}$ or the $\Upsilon(1S)$ mass scheme [7], and the difficulty in evaluating the uncertainty in such an association, we have chosen to extract $|V_{ub}|$ from all of the measurements discussed using the $b \to s\gamma$-derived shape function.

The full set of inclusive $|V_{ub}|$ results is summarized in Table 2, which is an updated version of the Heavy Flavors Averaging Group summary [51]. All endpoint results have QED radiative corrections applied correctly. The listed uncertainties do not include contributions for potentially large theoretical corrections that have been categorized but remain incalculable (see below). The last five results in the table, which we will use below, have been updated to a common framework based on

Table 2: Summary of inclusive $|V_{ub}|$ measurements. The last five measurements are incorporated into the analysis presented below. The errors in the first group are the experimental and theoretical uncertainties. The errors in the second group are from the statistical, experimental systematic, E_γ-based rate fraction, and $\Gamma_{\rm tot}$ uncertainties. The two groups are *not* directly comparable as they have not been evaluated with identical theoretical inputs.

	$\vert V_{ub}\vert(10^{-3})$	
ALEPH [53]	$4.12 \pm 0.67 \pm 0.76$	neural net
L3 [54]	$5.70 \pm 1.00 \pm 1.40$	cut and count
DELPHI	$4.07 \pm 0.65 \pm 0.61$	M_X
OPAL [55]	$4.00 \pm 0.71 \pm 0.71$	neural net
LEP Avg.	$4.09 \pm 0.37 \pm 0.56$	
CLEO [56]	$4.05 \pm 0.61 \pm 0.65$	$d\Gamma/dq^2 dM_X^2 dE_\ell$
Belle	$5.00 \pm 0.64 \pm 0.53$	M_X, $D^{(*)}\ell\nu$ tag
CLEO	$4.11 \pm 0.13 \pm 0.31 \pm 0.46 \pm 0.28$	$2.2 < p < 2.6$
BaBar	$4.31 \pm 0.20 \pm 0.20 \pm 0.49 \pm 0.30$	$2.3 < p < 2.6$
Belle	$3.99 \pm 0.17 \pm 0.16 \pm 0.45 \pm 0.27$	$2.3 < p < 2.6$
Belle	$4.63 \pm 0.28 \pm 0.39 \pm 0.48 \pm 0.32$	$M_X < 1.7$, $q^2 > 8$
BaBar	$4.79 \pm 0.29 \pm 0.28 \pm 0.60 \pm 0.33$	$M_X < 1.55$

the CLEO E_γ-derived shape function. The rate fractions [52] for the BaBar M_X analysis (f_M) and the Belle $M_X - q^2$ analysis f_{qM} are $f_M = 0.55 \pm 0.14$ and $f_{qM} = 0.33 \pm 0.07$. The central values for these and for the endpoint fractions (Table 1) correspond to an exponential shape function ansatz [48] and $(\lambda_1^{SF}, \overline{\Lambda}^{SF}) = (-0.342, 0.545)$, with small corrections related to background subtractions in the $b \to s\gamma$ spectrum. The errors are dominated by the statistical uncertainty in the $f(k_+)$ fit to the E_γ spectrum, but include contributions from experimental systematics, α_s uncertainties and modeling. Incorporation of results beyond those used here will require significant input from the experimental analyses, and is left to the HFAG.

Combining inclusive information:

Evaluation of the total uncertainty on $|V_{ub}|$ remains problematic because of a variety of theoretical complications. A recent review [16] discusses these issues in detail. There are three main contributions. The first arises from subleading (higher twist) contributions to the shape function resummation [57–61]. These involve incorporation of effects such as the variation of $k_\perp$, and are not universal for all B decay processes. Hence with the use of $b \to s\gamma$ to obtain a shape function, there are two contributions, one from subleading contributions to the use of a shape function in $b \to u\ell\overline{\nu}$ process itself, and the second from the different corrections in $b \to s\gamma$ from which the shape function is obtained. These contributions are potentially large, since they are of order $\Lambda_{\rm QCD}/m_b$. Indeed, a partial estimate of these effects [59] for the momentum endpoint region finds corrections that are similar in size to the total uncertainties of those analyses. Recent work indicates that the subleading contributions for $u\ell\nu$ and $s\gamma$ may partially cancel in the low M_X and the low M_X-high q^2 regions [61].

The second contribution, from "weak annihilation" processes, is formally of order $(\Lambda_{\rm QCD}/m_b)^3$ but receives a large multiplicative enhancement of $16\pi^2$ [62,63]. The contribution, which requires factorization violation to be nonzero, is expected to be localized near $q^2 \sim m_b^2$, and this localization can result in a further enhancement of the effect on $|V_{ub}|$. For the endpoint region, which sees about 10% of the total rate, an effect on the total rate of 2-3% (corresponding to factorization violation of about 10%), produces an effect on the measured rate of 20-30%.

Finally, there are unknown contributions from potential violation of local quark hadron duality. The true differential distribution cannot be predicted via the OPE—the resonant substructure is not described. However, spectra integrated over a sufficiently broad range should be better described.

The problems just outlined present a challenge to the averaging of the various inclusive results. Results with a potentially large bias might be included with neither a correction nor an appropriate uncertainty due to these effects. The resulting $|V_{ub}|$ determination would be potentially biased and the attached uncertainty unreliable. This motivated us not to provide an average result in the first edition of this review two years ago.

As an alternative, we here choose measurements in the region of phase space that appears to have the best compromise of the affects discussed to obtain an estimate of $|V_{ub}|$. Measurements from the other regions of phase space, which have increased sensitivity to one or more of the corrections, then provide limits on the uncertainties from these effects and thereby allow as complete as possible an estimation of the theoretical uncertainty, as first proposed by Ref. 52. At this time, the low M_X, high q^2 region appears to be the best motivated choice. It has reduced (though by no means negligible) corrections from the shape function and thus also from the subleading contributions to the shape function. Yet it integrates over a sufficient fraction of the spectrum to dilute weak annihilation contributions and concerns on local quark hadron duality.

While this choice is at present subjective, it offers the advantage of a reduction of the shape function influence coupled with the ability to bound the remaining theoretical uncertainties. In the opinion of the reviewers, this is a reasonable tradeoff for the statistical loss relative to the low M_X region. We expect that each experiment will perform an improved combination of information from the different regions of phase space where the experimental and theoretical correlations can be made manifest more straightforwardly.

We further stress that we view all three regions as equally crucial in this combination of information, as a more complete evaluation of the inclusive uncertainty than has previously existed is necessary for proper use of the inclusive results. The choice of the phase space region should not be misconstrued as a preference of experimental technique. Indeed, we look forward to a similar (or improved) analysis when a sample of clean results based on fully tagged B samples have been obtained for all regions of phase space.

At present only Belle [46] has contributed a result for this region of phase space, so for now we take this result as the "central value":

$$
\begin{aligned}
|V_{ub}|/10^{-3} = 4.63 \pm 0.28_{\mathrm{stat}} \pm 0.39_{\mathrm{sys}} \pm 0.48_{\mathrm{f_{qM}}} \pm 0.32_{\Gamma\mathrm{thy}} \\
\pm \sigma_{\mathrm{WA}} \pm \sigma_{\mathrm{SSF}} \pm \sigma_{\mathrm{LQD}} \ . \qquad (5)
\end{aligned}
$$

Additional measurements by the B factories of the rate in this region of phase space will soon improve the experimental uncertainties.

We must determine the last three uncertainties for weak annihilation (WA), subleading shape function corrections (SSF) and local quark hadron duality (LQD). The measurements from other regions of phase space are crucial for this task.

We assume that the WA contribution is largely contained within each of the $p_\ell > 2.2$ GeV/c, the $M_X < 1.55$ GeV and the combined $M_X < 1.7$ GeV, $q^2 > 8$ GeV^2 regions. The contribution will be most diluted in the low M_X region, with the rate fraction $f_M = 0.55 \pm 0.14$, and most concentrated in the endpoint region, with the rate fraction $f_e = 0.14 \pm 0.03$

(without radiative corrections). It is simple to show that for a neglected WA contribution, a comparison of $|V_{ub}|$ from these two regions would predict the bias in the M_X, q^2 region (with rate fraction $f_{qM} = 0.33 \pm 0.07$) to be

$$[(1 - f_{qM})/f_{qM}][f_e f_M/(f_M - f_e)] \approx 0.39 \tag{6}$$

of the observed difference. Comparison of the endpoint result from CLEO and the low M_X result from BaBar, taking into consideration the almost total correlation in the shape function and Γ_{tot} uncertainties, yields $\Delta|V_{ub}|/10^{-3} = 0.69 \pm 0.53$. There is not sufficient sensitivity to draw conclusions regarding the presence of a WA component, but we can place a bound. We take the larger of the error and central value and scale according to Eq. (6) to obtain

$$\sigma_{\mathrm{WA}} \approx 0.27 \ . \tag{7}$$

To estimate the uncertainty from the subleading corrections to the shape function, we assume that subleading corrections will scale like the fractional change in the predicted rate $(\Delta\Gamma/\Gamma)$ with and without convolution of the parton-level expression with the shape function. As the base comparison, we take the low M_X region, with $(\Delta\Gamma/\Gamma)_M = 0.15$, and compare to the combined M_X, q^2 region, with $(\Delta\Gamma/\Gamma)_{qM} = -0.075$). The shifts again depend on the shape function modeling, and the quoted values correspond to the $f(k_+)$ from the best fit to the CLEO E_γ spectrum . The theory uncertainties are again correlated, and we find $\Delta|V_{ub}|/10^{-3} = 0.16 \pm 0.63$. Scaling the uncertainty of the comparison by $|(\Delta\Gamma/\Gamma)_{qM}/(\Delta\Gamma/\Gamma)_M| = 0.49$, we have

$$\sigma_{\mathrm{SSF}} \approx 0.31 \ . \tag{8}$$

Finally, we must make an estimate of the local duality uncertainty. We assume that a potential violation will scale with the fraction of rate f in a given region as $(1-f)/f$. This form ranges from no "local" violation for integration of the full phase space ($f=1$), to large uncertainty for use of a very localized region of phase space ($f \to 0$). The estimate derives from comparison of the CLEO $p_\ell > 2.2$ GeV/c analysis ($f \sim 0.14 \pm 0.03$) to the average of the BaBar and Belle $p_\ell > 2.3$ GeV/c analyses ($f \sim 0.07 \pm 0.02$). The subleading correction estimates of Ref. 59 are applied to minimize potential cancelation between duality violation and subleading corrections. This yields $(|V_{ub}|^{2.3} - |V_{ub}|^{2.2} + 0.27)/10^{-3} = 0.29 \pm 0.38$, where the 0.27 is the estimate of the relative subleading correction. With our scaling assumption, we then apply a scale factor s of

$$s = \frac{(1-f_{qM})/f_{qM}}{(1-f_{2.3})/f_{2.3} - (1-f_{2.2})/f_{2.2}} \approx 0.29 \tag{9}$$

to the uncertainty in this difference estimate. Our local duality estimate therefore is

$$\sigma_{\mathrm{LQD}} \sim 0.11.$$

From this analysis, we finally obtain

$$\begin{aligned}|V_{ub}|/10^{-3} = 4.63 &\pm 0.28_{\mathrm{stat}} \pm 0.39_{\mathrm{sys}} \pm 0.48_{f_{qM}} \pm 0.32_{\Gamma\mathrm{thy}} \\ &\pm 0.27_{\mathrm{WA}} \pm 0.31_{\mathrm{SSF}} \pm 0.11_{\mathrm{LQD}} \ ,\end{aligned} \tag{10}$$

for a total theory error of 15% and total precision of 18%. Given that the uncertainties are dominated by experimental limits, addition in quadrature seems appropriate. Note that these estimates apply *only* in the combined low M_X, high q^2 region of phase space. The limits presented here can be improved both in robustness, through more sophisticated scaling estimates, and in magnitude, through additional and improved $|V_{ub}|$ measurements and through inputs from other sources. The consistency of the values of $|V_{ub}|$ extracted with different inclusive methods and the stability of the results over changes in the selected region of phase space will provide increasing confidence in the reliability of the extracted results and of their estimated uncertainties. Improvement of the $b \to s\gamma$ photon energy spectrum is key until a self-consistent extraction of the shape function from $b \to u\ell\overline{\nu}$ transition becomes available. Comparisons of the D^0 versus D_s semileptonic widths and of the rates for charged versus neutral B mesons can provide estimates of the weak annihilation contributions [63]. Finally, improved theoretical guidance concerning the scaling of the effects over phase space would allow development of a simultaneous extraction of $|V_{ub}|$ and the corrections, with all experimental information contributing directly to $|V_{ub}|$.

Exclusive measurements of $b \to u\ell\overline{\nu}$:

Reconstruction of exclusive $b \to u\ell\overline{\nu}$ channels provides powerful kinematic constraints for suppression of the $b \to c\ell\overline{\nu}$ background. For this suppression to be effective, an estimate of the four momentum of the undetected neutrino must be provided. The measurements to date have made use of detector hermeticity and the well-determined beam parameters to define a missing momentum that is used as the neutrino momentum. Signal-to-background ratios (S/B) of order two have been achieved in these channels.

To extract $|V_{ub}|$ from an exclusive channel, the form factors for that channel must be known. The form factor normalization dominates the uncertainty on $|V_{ub}|$. The q^2-dependence of the form factors, which is needed to determine the experimental efficiency, also contributes to the uncertainty, but at a much reduced level. For example, the requirement of a stiff lepton for background reduction in these analyses introduces a q^2-dependence to the efficiency. In the limit of a massless charged lepton (a reasonable limit for the electron and muon decay channels), the $B \to \pi \ell \nu$ decay depends on one form factor $f_1(q^2)$:

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dy\, d\cos\theta_\ell} = |V_{ub}|^2 \frac{G_F^2 p_\pi^3 M_B^2}{32\pi^3} \sin^2\theta_\ell |f_1(q^2)|^2 \,, \qquad (11)$$

where $y = q^2/M_B^2$ and θ_ℓ is the angle between the charged lepton direction in the virtual W $(\ell + \nu)$ rest frame and the direction of the virtual W. For the vector meson final states ρ and ω, three form factors A_1, A_2, and V are necessary (see *e.g.* reference [64]).

Calculation of these form factors constitutes a considerable theoretical industry, with a variety of techniques now being employed. Form factors based on lattice QCD calculations [65–77] and on light cone sum rules [78–87] currently have uncertainties in the 15% to 20% range. A variety of quark model calculations exists [88–102]. Finally, a number of other approaches [103–109], such as dispersive bounds and experimentally-constrained models based on Heavy Quark Symmetry, seek to improve the q^2 range where the form factors can be estimated, without introducing significant model dependence.

Of particular interest are the light cone sum rules (LCSR) and lattice QCD (LQCD) calculations, which minimize modeling assumptions as they are QCD-based calculations and provide a much firmer basis compared to the quark model calculations for systematic evaluation of the uncertainties. The calculations used in the current results have been summarized nicely in Ref. 11. The LCSR are expected to be valid in the region $q^2 \lesssim 16$ GeV^2. The light cone sum rules calculations use quark-hadron duality to estimate some spectral densities, and offer a "canonical" contribution to the related uncertainty of 10% with no known means of rigorously limiting that uncertainty. The theory community is currently debating the size of potential contributions to the form factors missing from the LCSR approach [110–114] that have been revealed using the newly-developed soft collinear effective theory (SCET) [115–118]. The

$B \to \rho\ell\nu$ form factors, in particular, could be appreciably overestimated, biasing $|V_{ub}|$ low. Two exclusive results will therefore be presented in this review, one based on the full set of exclusive results, and the second based only on results in the $q^2 > 16$ GeV^2 region for $B \to \rho\ell\nu$.

The LQCD calculations that can be applied to experimental $B \to X_u\ell\nu$ decay remain, to date, in the "quenched" approximation (no light quark loops in the propagators), which limits the ultimate precision to the 15% to 20% range. The q^2 range accessible to these calculations has been $q^2 \gtrsim 16$ GeV^2. Significant progress has been made towards unquenched lattice QCD calculations, and a recent comparison [119] of a range unquenched results to experiment shows much better agreement (few percent) than the corresponding quenched results. Work has begun on the unquenched form factors needed for $|V_{ub}|$, though the initial results have been limited to valence quarks closer to the strange quark mass. Nevertheless, initial results [120,121] are compatible with the 15% to 20% uncertainties used for the quenching uncertainty, lending them some validity.

The exclusive $|V_{ub}|$ results are summarized in Table 3. These include a simultaneous measurement of the $B \to \pi\ell\overline{\nu}$ and the $B \to \rho\ell\overline{\nu}$ transitions by CLEO [122], and measurement of the $B \to \rho\ell\overline{\nu}$ rate by CLEO [123] and BaBar [124]. All measurements employ the missing energy and momentum to estimate the neutrino momentum. With that technique, the major background results from $b \to c\ell\overline{\nu}$ decays in events that cannot be properly reconstructed (for example, because of additional neutrinos in the event) and hence which overestimate the neutrino energy. All measurements also employ the isospin relations

$$\Gamma(B^0 \to \pi^- \ell^+ \nu) = 2\Gamma(B^+ \to \pi^0 \ell^+ \nu)$$

and

$$\Gamma(B^0 \to \rho^- \ell^+ \nu) = 2\Gamma(B^+ \to \rho^0 \ell^+ \nu) \tag{12}$$

to combine the charged and neutral decays. These relationships can be distorted by $\rho - \omega$ mixing [125], and all results discussed here allow for this possibility in their systematic evaluation.

In the combined π and ρ measurement, strict event quality requirements were made that resulted in a low efficiency, but a relatively low background to signal ratio over a fairly broad lepton momentum range. The ρ-only analyses employ looser event cleanliness requirements, resulting in a much higher efficiency. The efficiency gain comes at the price of an increased background, and the analyses are primarily sensitive to signal with lepton momenta above 2.3 GeV/c, which is near (and beyond) the kinematic endpoint for $b \to c\ell\overline{\nu}$ decays which are therefore highly suppressed.

The combined π and ρ analysis of CLEO employs relatively loose lepton selection criteria and extracts rates independently in three separate q^2 intervals. Form factor dependence of the rates is then evaluated using models and calculations that exhibit a broad variation in $d\Gamma/dq^2$, which shows that this approach has eliminated model dependence of the rates in $\pi\ell\nu$, and significantly reduced it in $\rho\ell\nu$. To further reduce modeling uncertainties, CLEO then extracts $|V_{ub}|$ using only the LQCD and LCSR QCD-based calculations restricted to their respective valid q^2 ranges, thereby eliminating modeling used for extrapolation. Averages of the CLEO results, with and without the low q^2 region for $\rho\ell\nu$ are listed in Table 3.

A more complete review of recent $B \to X_u \ell\nu$ branching fractions, including analyses too incomplete for inclusion in this $|V_{ub}|$ summary, can be found in Reference [126]. Of note is the recent evidence presented for $B \to \omega\ell\nu$ by Belle [127].

With all results resting on use of detector hermeticity, the potential for significant correlation among the dominant experimental systematics exists [126]. Results from the three measurements have been averaged here assuming full correlation in these systematics. The $\rho\ell\nu$-only results [123,124], which depend more heavily on modeling even for the LCSR and LQCD calculations, are deweighted by 5% in the average. This yields

$$|V_{ub}| = (3.27 \pm 0.13 \pm 0.19^{+0.51}_{-0.45}) \times 10^{-3} \qquad (13)$$

where the errors arise from statistical, experimental systematic and form factor uncertainties, respectively. While similar in precision to the exclusive result in the previous $|V_{ub}|$ minireview, this result relies much less heavily on modeling. Should the LCSR form factors prove to be overestimated, we also provide an average excluding any result using information for $q^2 < 16$ GeV^2 in the $\rho\ell\nu$ modes, with the result

$$|V_{ub}| = (3.26 \pm 0.19 \pm 0.15 \pm 0.04^{+0.54}_{-0.39}) \times 10^{-3}\ , \qquad (14)$$

where the errors arise from statistical, experimental systematic $\rho\ell\nu$ form factor uncertainties, and LQCD and LCSR (treated as correlated), respectively.

The future for exclusive determinations of $|V_{ub}|$ appears promising. Unquenched lattice calculations are appearing, with very encouraging results. These calculations will eliminate the primary source of uncontrolled uncertainty in these calculations, and have already provided some validity to the quenching

Table 3: Summary of all exclusive $|V_{ub}|$ measurements. For the CLEO 00 and BaBar 01 measurements, the errors arise from statistical, experimental systematic and form factor modeling uncertainties, respectively. For the CLEO 03 measurements, the errors arise from statistical, experimental systematic, $\rho\ell\nu$ form factor, and LQCD and LCSR calculation uncertainties, respectively. In the CLEO 03 averages, the LQCD and LCSR uncertainties have been treated as correlated.

mode	$\lvert V_{ub}\rvert(10^{-3})$	q^2 range	FF
CLEO 00 $\rho\ell\nu$	$3.23 \pm 0.24^{+0.23}_{-0.26} \pm 0.58$	all	model survey
BaBar 01 $\rho\ell\nu$	$3.64 \pm 0.22 \pm 0.25^{+0.39}_{-0.56}$	all	model survey
CLEO 03 $\pi\ell\nu$	$3.33 \pm 0.24 \pm 0.15 \pm 0.06\ ^{+0.57}_{-0.40}$	$q^2 < 16\ \mathrm{GeV}^2$	LCSR
CLEO 03 $\pi\ell\nu$	$2.88 \pm 0.55 \pm 0.30 \pm 0.18\ ^{+0.45}_{-0.35}$	$q^2 > 16\ \mathrm{GeV}^2$	LQCD
CLEO 03 $\pi\ell\nu$	$3.24 \pm 0.22 \pm 0.13 \pm 0.09^{+0.55}_{-0.39}$	average	
CLEO 03 $\rho\ell\nu$	$2.67 \pm 0.27\ ^{+0.38}_{-0.42} \pm 0.17\ ^{+0.47}_{-0.35}$	$q^2 < 16\ \mathrm{GeV}^2$	LCSR
CLEO 03 $\rho\ell\nu$	$3.34 \pm 0.32\ ^{+0.27}_{-0.36} \pm 0.47\ ^{+0.50}_{-0.40}$	$q^2 > 16\ \mathrm{GeV}^2$	LQCD
CLEO 03 $\rho\ell\nu$	$3.00 \pm 0.21^{+0.29}_{-0.35} \pm 0.28^{+0.49}_{-0.38}$	average	
CLEO 03 $\pi + \rho$	$3.17 \pm 0.17^{+0.16}_{-0.17} \pm 0.03^{+0.53}_{-0.39}$	average	
CLEO 03 $\pi + \rho$	$3.26 \pm 0.19 \pm 0.15 \pm 0.04^{+0.54}_{-0.39}$	average	no $\rho\ell\nu$ LCSR

uncertainty estimate used in the results presented here. Simultaneously, the B factories are performing very well, and very large samples of events in which one B meson has been fully reconstructed are already being used. This will allow a more robust determination of the neutrino momentum, and should allow a significant reduction of backgrounds and experimental systematic uncertainties. The high statistics should also allow more detailed measurements of $d\Gamma/dq^2$, which have already provided a sorely-needed litmus test for the form factor calculations and reduced the form factor shape contribution to the uncertainty on $|V_{ub}|$. Should theory allow use of the full range of q^2 in the extraction of $|V_{ub}|$ [128], the B factories have already logged data sufficient for a 5% statistical determination of $|V_{ub}|$.

For both lattice and the B factories, $\pi\ell\nu$ appears to be a golden mode for future precise determination of $|V_{ub}|$. The one caveat is management of contributions from the B^* pole, but recent work [76] suggests that this problem can be successfully overcome. $B \to \eta\ell\nu$ will provide a valuable cross check. The $\rho\ell\nu$ mode will be more problematic for high precision: the broad width introduces both experimental and theoretical difficulties. Experiments must, for example, assess potential nonresonant $\pi\pi$ contributions, but only crude arguments based on isospin and quark-popping have been brought to bear to date. Theoretically, no calculation, including lattice, has dealt with the width of the ρ. When the lattice calculations become unquenched, the ρ will become unstable and the $\pi\pi$ final state must be faced by the calculations. The methodology for accommodation of high-energy two particle final states on the lattice has yet to be developed. The $\omega\ell\nu$ mode may provide a more tractable alternative to the ρ mode because of the relative narrowness of the ω resonance. Agreement between accurate $|V_{ub}|$ determinations from $\pi\ell\nu$ and from $\omega\ell\nu$ will provide added confidence in both.

Combined results:

The experimental bounds provided for the outstanding uncertainties in the inclusive $|V_{ub}|$ measurements in the low M_X, high q^2 region and theoretical work which clarifies the reliability of the LCSR and quenched LQCD form factors make possible comparison of the inclusive and exclusive determinations of

$|V_{ub}|$. Results agree to better than 1.5 times the quadratic combination of the quoted uncertainties. Therefore it becomes feasible to propose an average the inclusive and exclusive results, which have comparable accuracies.

The uncertainties have been combined in quadrature, using the larger (upward) error for the exclusive numbers. The proposed average of the inclusive and exclusive results, with all the exclusive data considered, is

$$|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3} \ . \tag{15}$$

Including in the average only the exclusive analyses based on data with $q^2 > 16$ GeV^2 in the $\rho\ell\nu$ mode, the average becomes $|V_{ub}| = (3.70 \pm 0.49) \times 10^{-3}$, so exclusion of this region, if appropriate, has only a minor effect.

The procedure proposed here results in a value of $|V_{ub}|$ with a 13% uncertainty. With the experimental and theoretical progress expected over the next few years an improvement of the accuracy at the 10% level, and possibly below, appears now realizable.

References

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3. See the "Determination of $|V_{cb}|$" review by M. Artuso and E. Barberio in this *Review*.
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V_{ub} MEASUREMENTS

For the discussion of V_{ub} measurements, which is not repeated here, see the review on "Determination of $|V_{ub}|$."

The CKM matrix element $|V_{ub}|$ can be determined by studying the rate of the charmless semileptonic decay $b \rightarrow u \ell \nu$. Measurements based on exclusive decay channels and on inclusive techniques can be found in the previous B Listings, which will not repeat here.

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BRIERE	02	PRL 89 081803	R. Briere *et al.*	(CLEO Collab.)
ABREU	01H	PL B510 55	P. Abreu *et al.*	(DELPHI Collab.)
ABBIENDI	00Q	PL B482 15	G. Abbiendi *et al.*	(OPAL Collab.)
BARTELT	99	PRL 82 3746	J. Bartelt *et al.*	(CLEO Collab.)
ACKERSTAFF	97G	PL B395 128	K. Ackerstaff *et al.*	(OPAL Collab.)
ATHANAS	97	PRL 79 2208	M. Athanas *et al.*	(CLEO Collab.)
BUSKULIC	97	PL B395 373	D. Buskulic *et al.*	(ALEPH Collab.)
ABREU	96P	ZPHY C71 539	P. Abreu *et al.*	(DELPHI Collab.)
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