

## $K_{\ell 3}^{\pm}$ AND $K_{\ell 3}^0$ FORM FACTORS

Revised May 2002 by T.G. Trippe (LBNL).

Assuming that only the vector current contributes to  $K \rightarrow \pi \ell \nu$  decays, we write the matrix element as

$$M \propto f_+(t) [(P_K + P_\pi)_\mu \bar{\ell} \gamma_\mu (1 + \gamma_5) \nu] + f_-(t) [m_\ell \bar{\ell} (1 + \gamma_5) \nu] , \quad (1)$$

where  $P_K$  and  $P_\pi$  are the four-momenta of the  $K$  and  $\pi$  mesons,  $m_\ell$  is the lepton mass, and  $f_+$  and  $f_-$  are dimensionless form factors which can depend only on  $t = (P_K - P_\pi)^2$ , the square of the four-momentum transfer to the leptons. If time-reversal invariance holds,  $f_+$  and  $f_-$  are relatively real.  $K_{\mu 3}$  experiments, discussed immediately below, measure  $f_+$  and  $f_-$ , while  $K_{e 3}$  experiments, discussed further below, are sensitive only to  $f_+$  because the small electron mass makes the  $f_-$  term negligible.

For this edition, fits to the form factor data have been made with and without the assumption of  $\mu$ - $e$  universality, as described near the end of the note.

**$K_{\mu 3}$  Experiments.** Analyses of  $K_{\mu 3}$  data frequently assume a linear dependence of  $f_+$  and  $f_-$  on  $t$ , *i.e.*,

$$f_\pm(t) = f_\pm(0) [1 + \lambda_\pm(t/m_\pi^2)] \quad (2)$$

Most  $K_{\mu 3}$  data are adequately described by Eq. (2) for  $f_+$  and a constant  $f_-$  (*i.e.*,  $\lambda_- = 0$ ). There are two equivalent parametrizations commonly used in these analyses:

**(1)  $\lambda_+$ ,  $\xi(0)$  parametrization.** Analyses of  $K_{\mu 3}$  data often introduce the ratio of the two form factors

$$\xi(t) = f_-(t)/f_+(t) . \quad (3)$$

The  $K_{\mu 3}$  decay distribution is then described by the two parameters  $\lambda_+$  and  $\xi(0)$  (assuming time reversal invariance and  $\lambda_- = 0$ ). These parameters can be determined by three different methods:

**Method A.** By studying the Dalitz plot or the pion spectrum of  $K_{\mu 3}$  decay. The Dalitz plot density is (see, *e.g.*, Chounet *et al.* [1]):

$$\rho(E_\pi, E_\mu) \propto f_+^2(t) [A + B\xi(t) + C\xi(t)^2] ,$$

where

$$A = m_K (2E_\mu E_\nu - m_K E'_\pi) + m_\mu^2 \left( \frac{1}{4} E'_\pi - E_\nu \right) ,$$

$$B = m_\mu^2 \left( E_\nu - \frac{1}{2} E'_\pi \right) ,$$

$$C = \frac{1}{4} m_\mu^2 E'_\pi ,$$

$$E'_\pi = E_\pi^{\max} - E_\pi = (m_K^2 + m_\pi^2 - m_\mu^2) / 2m_K - E_\pi . \quad (4)$$

Here  $E_\pi$ ,  $E_\mu$ , and  $E_\nu$  are, respectively, the pion, muon, and neutrino energies in the kaon center of mass. The density  $\rho$  is fit to the data to determine the values of  $\lambda_+$ ,  $\xi(0)$ , and their correlation.

**Method B.** By measuring the  $K_{\mu 3}/K_{e 3}$  branching ratio and comparing it with the theoretical ratio (see, *e.g.*, Fearing *et al.* [2]) as given in terms of  $\lambda_+$  and  $\xi(0)$ , assuming  $\mu$ - $e$  universality:

$$\begin{aligned} \Gamma(K_{\mu 3}^\pm) / \Gamma(K_{e 3}^\pm) &= 0.6457 + 1.4115\lambda_+ + 0.1264\xi(0) \\ &\quad + 0.0192\xi(0)^2 + 0.0080\lambda_+\xi(0) , \\ \Gamma(K_{\mu 3}^0) / \Gamma(K_{e 3}^0) &= 0.6452 + 1.3162\lambda_+ + 0.1264\xi(0) \\ &\quad + 0.0186\xi(0)^2 + 0.0064\lambda_+\xi(0) . \end{aligned} \quad (5)$$

This cannot determine  $\lambda_+$  and  $\xi(0)$  simultaneously but simply fixes a relationship between them.

**Method C.** By measuring the muon polarization in  $K_{\mu 3}$  decay. In the rest frame of the  $K$ , the  $\mu$  is expected to be polarized in the direction  $\mathbf{A}$  with  $\mathbf{P} = \mathbf{A} / |\mathbf{A}|$ , where  $\mathbf{A}$  is given (Cabined

and Maksymowicz [3]) by

$$\begin{aligned} \mathbf{A} = & a_1(\xi)\mathbf{p}_\mu \\ & - a_2(\xi) \left[ \frac{\mathbf{p}_\mu}{m_\mu} \left( m_K - E_\pi + \frac{\mathbf{p}_\pi \cdot \mathbf{p}_\mu}{|\mathbf{p}_\mu|^2} (E_\mu - m_\mu) \right) + \mathbf{p}_\pi \right] \\ & + m_K \text{Im}\xi(t)(\mathbf{p}_\pi \times \mathbf{p}_\mu) . \end{aligned} \quad (6)$$

If time-reversal invariance holds,  $\xi$  is real, and thus there is no polarization perpendicular to the  $K$ -decay plane. Polarization experiments measure the weighted average of  $\xi(t)$  over the  $t$  range of the experiment, where the weighting accounts for the variation with  $t$  of the sensitivity to  $\xi(t)$ .

**(2)  $\lambda_+$ ,  $\lambda_0$  parametrization.** Most of the more recent  $K_{\mu 3}$  analyses have parameterized in terms of the form factors  $f_+$  and  $f_0$  which are associated with vector and scalar exchange, respectively, to the lepton pair.  $f_0$  is related to  $f_+$  and  $f_-$  by

$$f_0(t) = f_+(t) + [t/(m_K^2 - m_\pi^2)] f_-(t) . \quad (7)$$

Here  $f_0(0)$  must equal  $f_+(0)$  unless  $f_-(t)$  diverges at  $t = 0$ . The earlier assumption that  $f_+$  is linear in  $t$  and  $f_-$  is constant leads to  $f_0$  linear in  $t$ :

$$f_0(t) = f_0(0) [1 + \lambda_0(t/m_\pi^2)] . \quad (8)$$

With the assumption that  $f_0(0) = f_+(0)$ , the two parametrizations,  $(\lambda_+, \xi(0))$  and  $(\lambda_+, \lambda_0)$  are equivalent as long as correlation information is retained.  $(\lambda_+, \lambda_0)$  correlations tend to be less strong than  $(\lambda_+, \xi(0))$  correlations.

The experimental results for  $\xi(0)$  and its correlation with  $\lambda_+$  are listed in the  $K^\pm$  and  $K_L^0$  sections of the Particle Listings in section  $\xi_A$ ,  $\xi_B$ , or  $\xi_C$  depending on whether method A, B, or C discussed above was used. The corresponding values of  $\lambda_+$  are also listed.

Because recent experiments tend to use the  $(\lambda_+, \lambda_0)$  parametrization, we include a subsection for  $\lambda_0$  results. Whenever possible we have converted  $\xi(0)$  results into  $\lambda_0$  results and vice versa.

See the 1982 version of this note [4] for additional discussion of the  $K_{\mu 3}^0$  parameters, correlations, and conversion between parametrizations.

**$K_{e3}$  Experiments.** Analysis of  $K_{e3}$  data is simpler than that of  $K_{\mu 3}$  because the second term of the matrix element assuming a pure vector current [Eq. (1) above] can be neglected. Here  $f_+$  is usually assumed to be linear in  $t$ , and the linear coefficient  $\lambda_+$  of Eq. (2) is determined.

If we remove the assumption of a pure vector current, then the matrix element for the decay, in addition to the terms in Eq. (1), would contain

$$\begin{aligned}
 &+2m_K f_S \bar{\ell}(1 + \gamma_5)\nu \\
 &+(2f_T/m_K)(P_K)_\lambda(P_\pi)_\mu \bar{\ell} \sigma_{\lambda\mu}(1 + \gamma_5)\nu , \quad (9)
 \end{aligned}$$

where  $f_S$  is the scalar form factor, and  $f_T$  is the tensor form factor. In the case of the  $K_{e3}$  decays where the  $f_-$  term can be neglected, experiments have yielded limits on  $|f_S/f_+|$  and  $|f_T/f_+|$ .

**Fits for  $K_{\ell 3}$  Form Factors.** Fits are made for the two parameters  $\lambda_+$  and  $\lambda_0$  and their correlation, or equivalently for  $\lambda_+$  and  $\xi(0)$  and their correlation. The fits are done both with and without assuming  $\mu$ - $e$  universality. This assumption sets  $\lambda_+(K_{\mu 3}) = \lambda_+(K_{e3})$  so that the more precise  $\lambda_+$  values from  $K_{e3}$  can be included with the  $K_{\mu 3}$  data in the fit. In addition it allows the  $K_{\mu 3}/K_{e3}$  branching ratio to be used in the determination of  $\xi(0)$  or  $\lambda_0$  as a function of  $\lambda_+$ . Without this assumption,  $\lambda_+(K_{e3})$  values are averaged separately and excluded from the fit to  $K_{\mu 3}$  data, and branching ratio data are not used. The Kaon Particle Listings show the results with and without assuming  $\mu$ - $e$  universality.

## References

1. L.M. Chounet, J.M. Gaillard, and M.K. Gaillard, Phys. Reports **4C**, 199 (1972).
2. H.W. Fearing, E. Fischbach, and J. Smith, Phys. Rev. **D2**, 542 (1970).
3. N. Cabined and A. Mathematica, Phys. Lett. **9**, 352 (1964).
4. Particle Data Group, Phys. Lett. **111B**, 73 (1982).