## 13. QUARK MODEL

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### 13.1. Quantum numbers of the quarks

Each quark has spin $1 / 2$ and baryon number $1 / 3$. Table 13.1 gives the additive quantum numbers (other than baryon number) of the three generations of quarks. Our convention is that the flavor of a quark ( $\mathrm{I}_{z}, \mathrm{~S}, \mathrm{C}, \mathrm{B}$, or T ) has the same sign as its charge. With this convention, any flavor carried by a charged meson has the same sign as its charge; e.g., the strangeness of the $K^{+}$ is +1 , the bottomness of the $B^{+}$is +1 , and the charm and strangeness of the $D_{s}^{-}$are each -1 .

By convention, each quark is assigned positive parity. Then each antiquark has negative parity.
Table 13.1: Additive quantum numbers of the quarks.

| Property Quark | $d$ | $u$ | $s$ | $c$ | $b$ | $t$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Q - electric charge | $-\frac{1}{3}$ | $+\frac{2}{3}$ | $-\frac{1}{3}$ | $+\frac{2}{3}$ | $-\frac{1}{3}$ | $+\frac{2}{3}$ |
| $\mathrm{I}_{z}$ - isospin $z$-component | $-\frac{1}{2}$ | $+\frac{1}{2}$ | 0 | 0 | 0 | 0 |
| $\mathrm{~S}-$ strangeness | 0 | 0 | -1 | 0 | 0 | 0 |
| $\mathrm{C}-$ charm | 0 | 0 | 0 | +1 | 0 | 0 |
| B - bottomness | 0 | 0 | 0 | 0 | -1 | 0 |
| $\mathrm{~T}-$ topness | 0 | 0 | 0 | 0 | 0 | +1 |

### 13.2. Mesons: $q \bar{q}$ states

Nearly all known mesons are bound states of a quark $q$ and an antiquark $\bar{q}^{\prime}$ (the flavors of $q$ and $q^{\prime}$ may be different). If the orbital angular momentum of the $q \bar{q}^{\prime}$ state is $L$, then the parity $P$ is $(-1)^{L+1}$. A state $q \bar{q}$ of a quark and its own antiquark is also an eigenstate of charge conjugation, with $C=(-1)^{L+S}$, where the spin $S$ is 0 or 1 . The $L=0$ states are the pseudoscalars, $J^{P}=0^{-}$, and the vectors, $J^{P}=1^{-}$. Assignments for many of the known mesons are given in Table 13.2. States in the "normal" spin-parity series, $P=(-1)^{J}$, must, according to the above, have $S=1$ and hence $C P=+1$. Thus mesons with normal spin-parity and $C P=-1$ are forbidden in the $q \bar{q}^{\prime}$ model. The $J^{P C}=0^{--}$state is forbidden as well. Mesons with such $J^{P C}$ may exist, but would lie outside the $q \bar{q}^{\prime}$ model.

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Table 13.2: Suggested $q \bar{q}$ quark-model assignments for most of the known mesons. Some assignments, especially for the $0^{++}$multiplet and for some of the higher multiplets, are controversial. Mesons in bold face are included in the Meson Summary Table. Of the light mesons in the Summary Table, the $f_{0}(1500), f_{1}(1510), f_{2}(1950), f_{2}(2300), f_{2}(2340)$, and one of the two peaks in the $\eta(1440)$ entry are not in this table. Within the $q \bar{q}$ model, it is especially hard to find a place for the first two of these $f$ mesons and for one of the $\eta(1440)$ peaks. See the "Note on Non $-q \bar{q}$ Mesons" at the end of the Meson Listings.

| $N{ }^{2 S+1} L_{J}$ | $J^{P C}$ | $\begin{gathered} u \bar{d}, u \bar{u}, d \bar{d} \\ I=1 \end{gathered}$ | $\begin{gathered} u \bar{u}, d \bar{d}, s \bar{s} \\ I=0 \end{gathered}$ | $\begin{gathered} c \bar{c} \\ I=0 \end{gathered}$ | $\begin{gathered} b \bar{b} \\ I=0 \end{gathered}$ | $\begin{gathered} \bar{s} u, \bar{s} d \\ I=1 / 2 \end{gathered}$ | $\begin{gathered} c \bar{u}, c \bar{d} \\ I=1 / 2 \end{gathered}$ | $\begin{gathered} c \bar{s} \\ I=0 \end{gathered}$ | $\begin{gathered} \bar{b} u, \bar{b} d \\ I=1 / 2 \end{gathered}$ | $\begin{gathered} \bar{b} s \\ I=0 \end{gathered}$ | $\begin{gathered} \bar{b} c \\ I=0 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1{ }^{1} S_{0}$ | $0^{-+}$ | $\pi$ | $\eta, \eta^{\prime}$ | $\boldsymbol{\eta}_{c}$ |  | K | D | $D_{s}$ | B | $B_{s}$ | $B_{c}$ |
| $1{ }^{3} S_{1}$ | $1^{--}$ | $\rho$ | $\omega, \phi$ | $J / \psi(1 S)$ | $\Upsilon(1 S)$ | $K^{*}(892)$ | $D^{*}(2010)$ | $D_{s}^{*}$ | $B^{*}$ | $B_{s}^{*}$ |  |
| $1{ }^{1} P_{1}$ | $1^{+-}$ | $b_{1}(1235)$ | $\boldsymbol{h}_{\mathbf{1}}(\mathbf{1 1 7 0}), h_{1}(1380)$ | $h_{c}(1 P)$ |  | $K_{1 B}{ }^{\dagger}$ | $D_{1}(2420)$ | $D_{s 1}(2536)$ |  |  |  |
| $1{ }^{3} P_{0}$ | $0^{++}$ | $a_{0}(1450)^{*}$ | $f_{0}(1370)^{*}, f_{0}(1710)^{*}$ | $\chi_{c 0}(1 P)$ | $\chi_{b 0}(1 P)$ | $K_{0}^{*}(1430)$ |  |  |  |  |  |
| $1{ }^{3} P_{1}$ | $1^{++}$ | $a_{1}(1260)$ | $f_{1}(1285), f_{1}(1420)$ | $\chi_{c 1}(1 P)$ | $\chi_{b 1}(1 P)$ | $K_{1 A}{ }^{\dagger}$ |  |  |  |  |  |
| $1{ }^{3} P_{2}$ | $2^{++}$ | $a_{2}(1320)$ | $f_{2}(1270), f_{2}^{\prime}(1525)$ | $\chi_{c 2}(1 P)$ | $\chi_{b 2}(1 P)$ | $K_{2}^{*}(1430)$ | $D_{2}^{*}(2460)$ |  |  |  |  |
| $1{ }^{1} D_{2}$ | $2^{-+}$ | $\pi_{2}(1670)$ | $\eta_{2}(1645), \eta_{2}(1870)$ |  |  | $K_{2}(1770)$ |  |  |  |  |  |
| $1{ }^{3} D_{1}$ | $1^{--}$ | $\rho(1700)$ | $\omega(1650)$ | $\psi(3770)$ |  | $K^{*}(1680)^{\ddagger}$ |  |  |  |  |  |
| $1{ }^{3} D_{2}$ | $2^{--}$ |  |  |  |  | $K_{2}(1820)$ |  |  |  |  |  |
| $1{ }^{3} D_{3}$ | $3^{--}$ | $\rho_{3}(1690)$ | $\omega_{3}(1670), \phi_{3}(1850)$ |  |  | $K_{3}^{*}(1780)$ |  |  |  |  |  |
| $1{ }^{3} F_{4}$ | $4^{++}$ | $a_{4}(2040)$ | $\boldsymbol{f}_{4}(\mathbf{2 0 5 0}), f_{4}(2220)$ |  |  | $K_{4}^{*}(2045)$ |  |  |  |  |  |
| $2{ }^{1} S_{0}$ | $0^{-+}$ | $\pi(1300)$ | $\eta(1295), \eta(1440)$ | $\eta_{c}(2 S)$ |  | $K(1460)$ |  |  |  |  |  |
| $2{ }^{3} S_{1}$ | $1^{--}$ | $\rho(1450)$ | $\omega(1420), \phi(1680)$ | $\psi(2 S)$ | $\Upsilon(2 S)$ | $K^{*}(1410)^{\ddagger}$ |  |  |  |  |  |
| $2^{3} P_{2}$ | $2^{++}$ |  | $f_{2}(1810), \boldsymbol{f}_{\mathbf{2}}(\mathbf{2 0 1 0})$ |  | $\chi_{b 2}(2 P)$ | $K_{2}^{*}(1980)$ |  |  |  |  |  |
| $3{ }^{1} S_{0}$ | $0^{-+}$ | $\pi(1800)$ | $\eta(1760)$ |  |  | $K(1830)$ |  |  |  |  |  |

* See our scalar minireview in the Particle Listings. The candidates for the $I=1$ states are $a_{0}(980)$ and $a_{0}(1450)$, while for $I=0$ they are: $f_{0}(400-1200), f_{0}(980), f_{0}(1370)$, and $f_{0}(1710)$.
The light scalars are problematic, since there may be two poles for one $q \bar{q}$ state and $a_{0}(980), f_{0}(980)$ may be $K \bar{K}$ bound states.
$\dagger$ The $K_{1 A}$ and $K_{1 B}$ are nearly equal $\left(45^{\circ}\right)$ mixes of the $K_{1}(1270)$ and $K_{1}(1400)$.
${ }^{\ddagger}$ The $K^{*}(1410)$ could be replaced by the $K^{*}(1680)$ as the $2^{3} S_{1}$ state.

The nine possible $q \bar{q}^{\prime}$ combinations containing $u$, $d$, and $s$ quarks group themselves into an octet and a singlet:

$$
\begin{equation*}
\mathbf{3} \otimes \overline{\mathbf{3}}=\mathbf{8} \oplus \mathbf{1} \tag{13.1}
\end{equation*}
$$

States with the same $I J^{P}$ and additive quantum numbers can mix. (If they are eigenstates of charge conjugation, they must also have the same value of $C$.) Thus the $I=0$ member of the ground-state pseudoscalar octet mixes with the corresponding pseudoscalar singlet to produce the $\eta$ and $\eta^{\prime}$. These appear as members of a nonet, which is shown as the middle plane in Fig. 13.1(a). Similarly, the ground-state vector nonet appears as the middle plane in Fig. 13.1(b).

A fourth quark such as charm can be included in this scheme by extending the symmetry to $\mathrm{SU}(4)$, as shown in Fig. 13.1. Bottom extends the symmetry to $\mathrm{SU}(5)$; to draw the multiplets would require four dimensions.

For the pseudoscalar mesons, the Gell-Mann-Okubo formula is

$$
\begin{equation*}
m_{\eta}^{2}=\frac{1}{3}\left(4 m_{K}^{2}-m_{\pi}^{2}\right), \tag{13.2}
\end{equation*}
$$

assuming no octet-singlet mixing. However, the octet $\eta_{8}$ and singlet $\eta_{1}$ mix because of $\mathrm{SU}(3)$ breaking. In general, the mixing angle is mass dependent and becomes complex for resonances of finite width. Neglecting this, the physical states $\eta$ and $\eta^{\prime}$ are given in terms of a mixing angle $\theta_{P}$ by

$$
\begin{align*}
\eta & =\eta_{8} \cos \theta_{P}-\eta_{1} \sin \theta_{P}  \tag{13.3a}\\
\eta^{\prime} & =\eta_{8} \sin \theta_{P}+\eta_{1} \cos \theta_{P} \tag{13.3b}
\end{align*}
$$

These combinations diagonalize the mass-squared matrix

$$
M^{2}=\left(\begin{array}{cc}
M_{11}^{2} & M_{18}^{2}  \tag{13.4}\\
M_{18}^{2} & M_{88}^{2}
\end{array}\right)
$$

where $M_{88}^{2}=\frac{1}{3}\left(4 m_{K}^{2}-m_{\pi}^{2}\right)$. It follows that

$$
\begin{equation*}
\tan ^{2} \theta_{P}=\frac{M_{88}^{2}-m_{\eta}^{2}}{m_{\eta^{\prime}}^{2}-M_{88}^{2}} \tag{13.5}
\end{equation*}
$$

The sign of $\theta_{P}$ is meaningful in the quark model. If

$$
\begin{align*}
& \eta_{1}=(u \bar{u}+d \bar{d}+s \bar{s}) / \sqrt{3}  \tag{13.6a}\\
& \eta_{8}=(u \bar{u}+d \bar{d}-2 s \bar{s}) / \sqrt{6}, \tag{13.6b}
\end{align*}
$$

then the matrix element $M_{18}^{2}$, which is due mostly to the strange quark mass, is negative. From the relation

$$
\begin{equation*}
\tan \theta_{P}=\frac{M_{88}^{2}-m_{\eta}^{2}}{M_{18}^{2}} \tag{13.7}
\end{equation*}
$$

we find that $\theta_{P}<0$. However, caution is suggested in the use of the $\eta-\eta^{\prime}$ mixing-angle formulas, as they are extremely sensitive to $\operatorname{SU}(3)$ breaking. If we allow $M_{88}^{2}=\frac{1}{3}\left(4 m_{K}^{2}-m_{\pi}^{2}\right)(1+\Delta)$, the mixing angle is determined by

$$
\begin{equation*}
\tan ^{2} \theta_{P}=0.0319(1+17 \Delta) \tag{13.8}
\end{equation*}
$$



Figure 13.1: $S U(4) 16$-plets for the (a) pseudoscalar and (b) vector mesons made of $u$, $d$, $s$, and $c$ quarks. The nonets of light mesons occupy the central planes, to which the $c \bar{c}$ states have been added. The neutral mesons at the centers of these planes are mixtures of $u \bar{u}, d \bar{d}$, $s \bar{s}$, and $c \bar{c}$ states.

$$
\begin{equation*}
\theta_{P}=-10.1^{\circ}(1+8.5 \Delta) \tag{13.9}
\end{equation*}
$$

to first order in $\Delta$. A small breaking of the Gell-Mann-Okubo relation can produce a major modification of $\theta_{P}$.

For the vector mesons, $\pi \rightarrow \rho, K \rightarrow K^{*}, \eta \rightarrow \phi$, and $\eta^{\prime} \rightarrow \omega$, so that

$$
\begin{gather*}
\phi=\omega_{8} \cos \theta_{V}-\omega_{1} \sin \theta_{V}  \tag{13.10}\\
\omega=\omega_{8} \sin \theta_{V}+\omega_{1} \cos \theta_{V} \tag{13.11}
\end{gather*}
$$

For "ideal" mixing, $\phi=s \bar{s}$, so $\tan \theta_{V}=1 / \sqrt{2}$ and $\theta_{V}=35.3^{\circ}$. Experimentally, $\theta_{V}$ is near $35^{\circ}$, the sign being determined by a formula like that for $\tan \theta_{P}$. Following this procedure we find the mixing angles given in Table 13.3.

Table 13.3: Singlet-octet mixing angles for several nonets, neglecting possible mass dependence and imaginary parts. The sign conventions are given in the text. The values of $\theta_{\text {quad }}$ are obtained from the equations in the text, while those for $\theta_{\text {lin }}$ are obtained by replacing $m^{2}$ by $m$ throughout. Of the two isosinglets in a nonet, the mostly octet one is listed first.

| $J^{P C}$ | Nonet members | $\theta_{\text {quad }}$ | $\theta_{\text {lin }}$ |
| :--- | :--- | :---: | :---: |
| $0^{-+}$ | $\pi, K, \eta, \eta^{\prime}$ | $-10^{\circ}$ | $-23^{\circ}$ |
| $1^{--}$ | $\rho, K^{*}(892), \phi, \omega$ | $39^{\circ}$ | $36^{\circ}$ |
| $2^{++}$ | $a_{2}(1320), K_{2}^{*}(1430), f_{2}^{\prime}(1525), f_{2}(1270)$ | $28^{\circ}$ | $26^{\circ}$ |
| $3^{--}$ | $\rho_{3}(1690), K_{3}^{*}(1780), \phi_{3}(1850), \omega_{3}(1670)$ | $29^{\circ}$ | $28^{\circ}$ |

In the quark model, the coupling of neutral mesons to two photons is proportional to $\sum_{i} Q_{i}^{2}$, where $Q_{i}$ is the charge of the $i$-th quark. This provides an alternative characterization of mixing. For example, defining

$$
\begin{equation*}
\operatorname{Amp}\left[P \rightarrow \gamma\left(k_{1}\right) \gamma\left(k_{2}\right)\right]=M \epsilon^{\mu \nu \alpha \beta} \epsilon_{1 \mu}^{*} k_{1 \nu} \epsilon_{2 \alpha}^{*} k_{2 \beta}, \tag{13.12}
\end{equation*}
$$

where $\epsilon_{i \lambda}$ is the $\lambda$ component of the polarization vector of the $i^{t h}$ photon, one finds

$$
\begin{align*}
\frac{M(\eta \rightarrow \gamma \gamma)}{M\left(\pi^{0} \rightarrow \gamma \gamma\right)} & =\frac{1}{\sqrt{3}}\left(\cos \theta_{P}-2 \sqrt{2} \sin \theta_{P}\right) \\
& =\frac{1.73 \pm 0.18}{\sqrt{3}}  \tag{13.13a}\\
\frac{M\left(\eta^{\prime} \rightarrow \gamma \gamma\right)}{M\left(\pi^{0} \rightarrow \gamma \gamma\right)} & =2 \sqrt{2 / 3}\left(\cos \theta_{P}+\frac{\sin \theta_{P}}{2 \sqrt{2}}\right) \\
& =2 \sqrt{2 / 3}(0.78 \pm 0.04), \tag{13.13b}
\end{align*}
$$

where the numbers with errors are experimental. These data favor $\theta_{P} \approx-20^{\circ}$, which is compatible with the quadratic mass mixing formula with about $12 \% \mathrm{SU}(3)$ breaking in $M_{88}^{2}$.

### 13.3. Baryons: $q q q$ states

All the established baryons are apparently 3-quark ( $q q q$ ) states, and each such state is an $\mathrm{SU}(3)$ color singlet, a completely antisymmetric state of the three possible colors. Since the quarks are fermions, the state function for any baryon must be antisymmetric under interchange of any two equal-mass quarks (up and down quarks in the limit of isospin symmetry). Thus the state function may be written as

$$
\begin{equation*}
\left.\left.|q q q\rangle_{A}=\mid \text { color }\right\rangle_{A} \times \mid \text { space }, \text { spin, flavor }\right\rangle_{S}, \tag{13.14}
\end{equation*}
$$

where the subscripts $S$ and $A$ indicate symmetry or antisymmetry under interchange of any two of the equal-mass quarks. Note the contrast with the state function for the three nucleons in ${ }^{3} \mathrm{H}$ or ${ }^{3} \mathrm{He}$ :

$$
\begin{equation*}
\left.|N N N\rangle_{A}=\mid \text { space, spin, isospin }\right\rangle_{A} . \tag{13.15}
\end{equation*}
$$

This difference has major implications for internal structure, magnetic moments, etc. (For a nice discussion, see Ref. 1.)

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The "ordinary" baryons are made up of $u, d$, and $s$ quarks. The three flavors imply an approximate flavor $\mathrm{SU}(3)$, which requires that baryons made of these quarks belong to the multiplets on the right side of

$$
\begin{equation*}
\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}=\mathbf{1 0}_{S} \oplus \mathbf{8}_{M} \oplus \mathbf{8}_{M} \oplus \mathbf{1}_{A} \tag{13.16}
\end{equation*}
$$

(see Sec. 33, on "SU( $n$ ) Multiplets and Young Diagrams"). Here the subscripts indicate symmetric, mixed-symmetry, or antisymmetric states under interchange of any two quarks. The $\mathbf{1}$ is a $u d s$ state $\left(\Lambda_{1}\right)$ and the octet contains a similar state $\left(\Lambda_{8}\right)$. If these have the same spin and parity they can mix. An example is the mainly octet $D_{03} \Lambda(1690)$ and mainly singlet $D_{03} \Lambda(1520)$. In the ground state multiplet, the $\mathrm{SU}(3)$ flavor singlet $\Lambda$ is forbidden by Fermi statistics. The mixing formalism is the same as for $\eta-\eta^{\prime}$ or $\phi-\omega$ (see above), except that for baryons the mass $M$ instead of $M^{2}$ is used. Section 32, on " $\mathrm{SU}(3)$ Isoscalar Factors and Representation Matrices", shows how relative decay rates in, say, $\mathbf{1 0} \rightarrow \mathbf{8} \otimes \mathbf{8}$ decays may be calculated. A summary of results of fits to the observed baryon masses and decay rates for the best-known $\mathrm{SU}(3)$ multiplets is given in Appendix II of our 1982 edition [2].

The addition of the $c$ quark to the light quarks extends the flavor symmetry to $\mathrm{SU}(4)$. Figures $13.2(\mathrm{a})$ and $13.2(\mathrm{~b})$ show the (badly broken) $\mathrm{SU}(4)$ baryon multiplets that have as their bottom levels an $\mathrm{SU}(3)$ octet, such as the octet that includes the nucleon, or an $\mathrm{SU}(3)$ decuplet, such as the decuplet that includes the $\Delta(1232)$. All the particles in a given $\mathrm{SU}(4)$ multiplet have the same spin and parity. The charmed baryons are discussed in more detail in the "Note on Charmed Baryons" in the Particle Listings. The addition of a $b$ quark extends the flavor symmetry to $\operatorname{SU}(5)$; it would require four dimensions to draw the multiplets.

For the "ordinary" baryons (no $c$ or $b$ quark), flavor and spin may be combined in an approximate flavor-spin $\mathrm{SU}(6)$ in which the six basic states are $d \uparrow, d \downarrow, \cdots, s \downarrow$ ( $\uparrow, \downarrow=$ spin up, down). Then the baryons belong to the multiplets on the right side of

$$
\begin{equation*}
\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6}=\mathbf{5 6}_{S} \oplus \mathbf{7 0}_{M} \oplus \mathbf{7 0}_{M} \oplus \mathbf{2 0}_{A} \tag{13.17}
\end{equation*}
$$

These $\mathrm{SU}(6)$ multiplets decompose into flavor $\mathrm{SU}(3)$ multiplets as follows:

$$
\begin{gather*}
\mathbf{5 6}={ }^{4} \mathbf{1 0} \oplus^{2} \mathbf{8}  \tag{13.18a}\\
\mathbf{7 0}={ }^{2} \mathbf{1 0} \oplus{ }^{4} \mathbf{8} \oplus{ }^{2} \mathbf{8} \oplus{ }^{2} \mathbf{1}  \tag{13.18b}\\
\mathbf{2 0}={ }^{2} \mathbf{8} \oplus{ }^{4} \mathbf{1} \tag{13.18c}
\end{gather*}
$$

where the superscript $(2 S+1)$ gives the net spin $S$ of the quarks for each particle in the $\mathrm{SU}(3)$ multiplet. The $J^{P}=1 / 2^{+}$octet containing the nucleon and the $J^{P}=3 / 2^{+}$decuplet containing the $\Delta(1232)$ together make up the "ground-state" 56 -plet in which the orbital angular momenta between the quark pairs are zero (so that the spatial part of the state function is trivially symmetric). The $\mathbf{7 0}$ and $\mathbf{2 0}$ require some excitation of the spatial part of the state function in order to make the overall state function symmetric. States with nonzero orbital angular momenta are classified in $\mathrm{SU}(6) \otimes \mathrm{O}(3)$ supermultiplets. Physical baryons with the same quantum numbers do not belong to a single supermultiplet, since $\mathrm{SU}(6)$ is broken by spin-dependent interactions, differences in quark masses, etc. Nevertheless, the $\mathrm{SU}(6) \otimes \mathrm{O}(3)$ basis provides a suitable framework for describing baryon state functions.

It is useful to classify the baryons into bands that have the same number $N$ of quanta of excitation. Each band consists of a number of supermultiplets, specified by $\left(D, L_{N}^{P}\right)$, where $D$ is the dimensionality of the $\mathrm{SU}(6)$ representation, $L$ is the total quark orbital angular


Figure 13.2: $\mathrm{SU}(4)$ multiplets of baryons made of $u, d, s$, and $c$ quarks. (a) The 20-plet with an $\operatorname{SU}(3)$ octet. (b) The 20-plet with an $\mathrm{SU}(3)$ decuplet.
momentum, and $P$ is the total parity. Supermultiplets contained in bands up to $N=12$ are given in Ref. 3. The $N=0$ band, which contains the nucleon and $\Delta(1232)$, consists only of the $\left(56,0_{0}^{+}\right)$supermultiplet. The $N=1$ band consists only of the $\left(70,1_{1}^{-}\right)$multiplet and contains the negative-parity baryons with masses below about 1.9 GeV . The $N=2$ band contains five supermultiplets: $\left(56,0_{2}^{+}\right),\left(70,0_{2}^{+}\right),\left(56,2_{2}^{+}\right),\left(70,2_{2}^{+}\right)$, and $\left(20,1_{2}^{+}\right)$. Baryons belonging to the $\left(20,1_{2}^{+}\right)$supermultiplet are not ever likely to be observed, since a coupling from the ground-state baryons requires a two-quark excitation. Selection rules are similarly responsible for the fact that many other baryon resonances have not been observed [4].

In Table 13.4, quark-model assignments are given for many of the established baryons whose $\mathrm{SU}(6) \otimes \mathrm{O}(3)$ compositions are relatively unmixed. We note that the unestablished resonances $\Sigma(1480), \Sigma(1560), \Sigma(1580), \Sigma(1770)$, and $\Xi(1620)$ in our Baryon Particle Listings are too low in mass to be accommodated in most quark models $[4,5]$.

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Table 13.4: Quark-model assignments for many of the known baryons in terms of a flavor-spin $\operatorname{SU}(6)$ basis. Only the dominant representation is listed. Assignments for some states, especially for the $\Lambda(1810), \Lambda(2350), \Xi(1820)$, and $\Xi(2030)$, are merely educated guesses. For assignments of the charmed baryons, see the "Note on Charmed Baryons" in the Particle Listings.

| $J^{P}$ | $\left(D, L_{N}^{P}\right)$ | $S$ | Octet m | embers |  | Singlets |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1 / 2^{+}$ | $\left(56,0_{0}^{+}\right)$ | $1 / 2 N(939)$ | $\Lambda(1116)$ | $\Sigma(1193)$ | $\Xi(1318)$ |  |
| $1 / 2^{+}$ | $\left(56,0_{2}^{+}\right)$ | $1 / 2 N(1440)$ | $\Lambda(1600)$ | $\Sigma(1660)$ | $\Xi(?)$ |  |
| $1 / 2^{-}$ | ( $70,1_{1}^{-}$) | $1 / 2 N(1535)$ | $\Lambda(1670)$ | $\Sigma(1620)$ | $\Xi(?)$ | $\Lambda(1405)$ |
| $3 / 2^{-}$ | ( $70,1_{1}^{-}$) | $1 / 2 N(1520)$ | $\Lambda(1690)$ | $\Sigma(1670)$ | $\Xi(1820)$ | $\Lambda(1520)$ |
| $1 / 2^{-}$ | ( $70,1_{1}^{-}$) | $3 / 2 N(1650)$ | $\Lambda(1800)$ | $\Sigma(1750)$ | $\Xi(?)$ |  |
| $3 / 2^{-}$ | ( $70,11_{1}^{-}$) | $3 / 2 N(1700)$ | $\Lambda(?)$ | $\Sigma(?)$ | $\Xi(?)$ |  |
| $5 / 2^{-}$ | ( $70,11_{1}^{-}$) | $3 / 2 N(1675)$ | ) $\Lambda$ (1830) | $\Sigma(1775)$ | $\Xi(?)$ |  |
| $1 / 2^{+}$ | $\left(70,0_{2}^{+}\right)$ | $1 / 2 N(1710)$ | $\Lambda(1810)$ | $\Sigma(1880)$ | $\Xi(?)$ | $\Lambda(?)$ |
| $3 / 2^{+}$ | $\left(56,2_{2}^{+}\right)$ | $1 / 2 N(1720)$ | ) $\Lambda$ (1890) | $\Sigma(?)$ | $\Xi(?)$ |  |
| $5 / 2^{+}$ | $\left(56,2_{2}^{+}\right)$ | $1 / 2 N(1680)$ | ) $\Lambda$ (1820) | $\Sigma(1915)$ | $\Xi(2030)$ |  |
| 7/2 ${ }^{-}$ | $\left(70,3_{3}^{-}\right)$ | $1 / 2 N(2190)$ |  | $\Sigma(?)$ | $\Xi(?)$ | $\Lambda(2100)$ |
| $9 / 2^{-}$ | ( $70,3_{3}^{-}$) | $3 / 2 N(2250)$ | $\Lambda(?)$ | $\Sigma(?)$ | $\Xi(?)$ |  |
| $9 / 2^{+}$ | $\left(56,4_{4}^{+}\right)$ | $1 / 2 N(2220)$ | $\Lambda(2350)$ | $\Sigma(?)$ | $\Xi(?)$ |  |
|  |  | Decuplet members |  |  |  |  |
| $3 / 2^{+}$ | $\left(56,0_{0}^{+}\right)$ | $3 / 2 \Delta(1232)$ | $\Sigma(1385)$ | $\Xi(1530)$ | $\Omega(1672)$ |  |
| $1 / 2^{-}$ | ( $70,1_{1}^{-}$) | $1 / 2 \Delta(1620)$ | $\Sigma(?)$ | $\Xi(?)$ | $\Omega(?)$ |  |
| $3 / 2^{-}$ | (70,1-) | $1 / 2 \Delta(1700)$ | $\Sigma(?)$ | $\Xi(?)$ | $\Omega(?)$ |  |
| $5 / 2^{+}$ | $\left(56,2_{2}^{+}\right)$ | $3 / 2 \Delta(1905)$ | $\Sigma(?)$ | $\Xi(?)$ | $\Omega(?)$ |  |
| 7/2 ${ }^{+}$ | $\left(56,2_{2}^{+}\right)$ | $3 / 2 \Delta(1950)$ | $\Sigma(2030)$ | $\Xi(?)$ | $\Omega(?)$ |  |
| $11 / 2^{+}$ | $\left(56,4_{4}^{+}\right)$ | $3 / 2 \Delta(2420)$ | $\Sigma(?)$ | $\Xi(?)$ | $\Omega(?)$ |  |

The quark model for baryons is extensively reviewed in Ref. 6 and 7.

### 13.4. Dynamics

Many specific quark models exist, but most contain the same basic set of dynamical ingredients. These include:
i) A confining interaction, which is generally spin-independent.
ii) A spin-dependent interaction, modeled after the effects of gluon exchange in QCD. For example, in the $S$-wave states, there is a spin-spin hyperfine interaction of the form

$$
\begin{equation*}
H_{H F}=-\alpha_{S} M \sum_{i>j}\left(\vec{\sigma} \lambda_{a}\right)_{i}\left(\vec{\sigma} \lambda_{a}\right)_{j} \tag{13.19}
\end{equation*}
$$

where $M$ is a constant with units of energy, $\lambda_{a}(a=1, \cdots, 8$,$) is the set of \mathrm{SU}(3)$ unitary spin matrices, defined in Sec. 32, on "SU(3) Isoscalar Factors and Representation Matrices," and the sum runs over constituent quarks or antiquarks. Spin-orbit interactions, although allowed, seem to be small.
iii) A strange quark mass somewhat larger than the up and down quark masses, in order to split the $\mathrm{SU}(3)$ multiplets.
iv) In the case of isoscalar mesons, an interaction for mixing $q \bar{q}$ configurations of different flavors (e.g., $u \bar{u} \leftrightarrow d \bar{d} \leftrightarrow s \bar{s}$ ), in a manner which is generally chosen to be flavor independent.

These four ingredients provide the basic mechanisms that determine the hadron spectrum.

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