9. QUANTUM CHROMODYNAMICS

9.1. The QCD Lagrangian

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Quantum Chromodynamics (QCD), the gauge field theory which describes the strong interactions of colored quarks and gluons, is one of the components of the SU(3)×SU(2)×U(1) Standard Model. A quark of specific flavor (such as a charm quark) comes in 3 colors; gluons come in eight colors; hadrons are color-singlet combinations of quarks, anti-quarks, and gluons. The Lagrangian describing the interactions of quarks and gluons is (up to gauge-fixing terms)

$$\begin{aligned} L_{\rm QCD} = & -\frac{1}{4} F^{(a)}_{\mu\nu}\, F^{(a)\mu\nu} + i \sum_q \overline{\psi}^i_q\, \gamma^\mu\, (D_\mu)_{ij}\, \psi^j_q \\ & - \sum_q m_q\, \overline{\psi}^i_q\, \psi_{qi}\ , \end{aligned} \tag{9.1}$$

$$F^{(a)}_{\mu\nu} = \partial_\mu\, A^a_\nu - \partial_\nu\, A^a_\mu + g_s\, f_{abc}\, A^b_\mu\, A^c_\nu\ , \tag{9.2}$$

$$(D_\mu)_{ij} = \delta_{ij}\, \partial_\mu - i g_s\, \sum_a \frac{\lambda^a_{i,j}}{2} A^a_\mu\ , \tag{9.3}$$

where g_s is the QCD coupling constant, and the f_{abc} are the structure constants of the SU(3) algebra (the λ matrices and values for f_{abc} can be found in "SU(3) Isoscalar Factors and Representation Matrices," Sec. 32 of this *Review*). The $\psi^i_q(x)$ are the 4-component Dirac spinors associated with each quark field of (3) color i and flavor q, and the $A^a_\mu(x)$ are the (8) Yang-Mills (gluon) fields. A complete list of the Feynman rules which derive from this Lagrangian, together with some useful color-algebra identities, can be found in Ref. 1.

The principle of "asymptotic freedom" (see below) determines that the renormalized QCD coupling is small only at high energies, and it is only in this domain that high-precision tests—similar to those in QED—can be performed using perturbation theory. Nonetheless, there has been in recent years much progress in understanding and quantifying the predictions of QCD in the nonperturbative domain, for example, in soft hadronic processes and on the lattice [2]. This short review will concentrate on QCD at short distances (large momentum transfers), where perturbation theory is the standard tool. It will discuss the processes that are used to determine the coupling constant of QCD. Other recent reviews of the coupling constant measurements may be consulted for a different perspective [3,4].

9.2. The QCD coupling and renormalization scheme

The renormalization scale dependence of the effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the β-function:

$$\mu\frac{\partial\alpha_s}{\partial\mu} = -\frac{\beta_0}{2\pi}\,\alpha_s^2 - \frac{\beta_1}{4\pi^2}\,\alpha_s^3 - \frac{\beta_2}{64\pi^3}\,\alpha_s^4 - \cdots \,, \tag{9.4a}$$

$$\beta_0 = 11 - \frac{2}{3}n_f \,, \tag{9.4b}$$

$$\beta_1 = 51 - \frac{19}{3}n_f \,, \tag{9.4c}$$

$$\beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2 \,; \tag{9.4d}$$

where n_f is the number of quarks with mass less than the energy scale μ. The expression for the next term in this series (β_3) can be found in Ref. 5. In solving this differential equation for α_s, a constant of integration is introduced. This constant is the one fundamental constant of QCD that must be determined from experiment. The most sensible choice for this constant is the value of α_s at a fixed-reference scale μ_0. It has become standard to choose $\mu_0 = M_Z$. It is also convenient to introduce the dimensional parameter Λ, since this provides a parameterization of the μ dependence of α_s. The definition of Λ is arbitrary. One way to define it (adopted here) is to write a solution of Eq. (9.4) as an expansion in inverse powers of $\ln(\mu^2)$:

$$\begin{aligned}\alpha_s(\mu) = \frac{4\pi}{\beta_0\ \ln(\mu^2/\Lambda^2)}\Bigg[1 - \frac{2\beta_1}{\beta_0^2}\ \frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4\ln^2(\mu^2/\Lambda^2)} \\ \times\left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\Bigg] \,.\end{aligned} \tag{9.5a}$$

The last term in this expansion is

$$\mathcal{O}\left(\frac{\ln^2\left[\ln(\mu^2/\Lambda^2)\right]}{\ln^3(\mu^2/\Lambda^2)}\right) \,, \tag{9.5b}$$

and is usually neglected in the definition of Λ. We choose to include it. For a fixed value of $\alpha_s(M_Z)$, the inclusion of this term shifts the value of Λ by ~ 15 MeV. This solution illustrates the *asymptotic freedom* property: $\alpha_s \to 0$ as $\mu \to \infty$.

Consider a "typical" QCD cross section which, when calculated perturbatively, starts at $\mathcal{O}(\alpha_s)$:

$$\sigma = A_1\ \alpha_s + A_2\ \alpha_s^2 + \ \cdots \ . \tag{9.6}$$

The coefficients A_1, A_2 come from calculating the appropriate Feynman diagrams. In performing such calculations, various divergences arise, and these must be regulated in a consistent way. This requires a particular renormalization scheme (RS). The most commonly used one is the modified minimal subtraction ($\overline{\text{MS}}$) scheme [6]. This involves continuing momentum integrals from 4 to 4–2ϵ dimensions, and then subtracting off the resulting $1/\epsilon$ poles and also $(\ln 4\pi - \gamma_E)$, which is another artifact of continuing the dimension. (Here γ_E is the Euler-Mascheroni constant.) To preserve the dimensionless nature of the coupling, a mass scale μ must also be introduced: $g \to \mu^\epsilon g$. The finite coefficients A_i $(i \geq 2)$ thus obtained depend implicitly on the renormalization convention used and explicitly on the scale μ.

The first two coefficients (β_0, β_1) in Eq. (9.4) are independent of the choice of RS's. In contrast, the coefficients of terms proportional to α_s^n for $n > 3$ *are* RS-dependent. The form given above for β_2 is in the $\overline{\text{MS}}$ scheme.

The fundamental theorem of RS dependence is straightforward. Physical quantities, in particular the cross section, calculated to all orders in perturbation theory, do not depend on the RS. It follows that a truncated series *does* exhibit RS dependence. In practice, QCD cross sections are known to leading order (LO), or to next-to-leading order (NLO), or in a few cases, to next-to-next-to-leading order (NNLO); and it is only the latter two cases, which have reduced RS dependence, that are useful for precision tests. At NLO the RS dependence is completely given by one condition which can be taken to be the value of the renormalization scale μ. At NNLO this is not sufficient, and μ is no longer equivalent to a choice of scheme; both must now be specified. One, therefore, has to address the question of what is the "best" choice for μ within a given scheme, usually $\overline{\text{MS}}$. There is no definite answer to this question—higher-order corrections do not "fix" the scale, rather they render the theoretical predictions less sensitive to its variation.

One should expect that choosing a scale μ characteristic of the typical energy scale (E) in the process would be most appropriate. In general, a poor choice of scale generates terms of order $\ln(E/\mu)$ in the A_i's. Various methods have been proposed including choosing the scale for which the next-to-leading-order correction vanishes ("Fastest Apparent Convergence [7]"); the scale for which the next-to-leading-order prediction is stationary [8], (*i.e.*, the value of μ where $d\sigma/d\mu = 0$); or the scale dictated by the effective charge scheme [9] or by the BLM scheme [10]. By comparing the values of α_s that different reasonable schemes give, an estimate of theoretical errors can be obtained. It has also been suggested to replace the perturbation series by its Pade approximant [11]. Results obtained using this method have, in certain cases, a reduced scale dependence [12,13]. One can also attempt to determine the scale from data by allowing it to vary and using a fit to determine it. This method can allow a determination of the error due to the scale choice and can give more confidence in the end result [14]. In many of the cases discussed below this scale uncertainty is the dominant error.

An important corollary is that if the higher-order corrections are naturally small, then the additional uncertainties introduced by the μ dependence are likely to be small. There are some processes, however, for which the choice of scheme *can* influence the extracted value of $\alpha_s(M_Z)$. There is no resolution to this problem other than to try to calculate even more terms in the perturbation series. It is important to note that, since the perturbation series is an asymptotic expansion, there is a limit to the precision with which any theoretical quantity can be calculated. In some processes, the highest-order perturbative terms may be comparable in size to nonperturbative corrections (sometimes called higher-twist or renormalon effects, for a discussion see [15]); an estimate of these terms and their uncertainties is required if a value of α_s is to be extracted.

Cases occur where there is more than one large scale, say μ_1 and μ_2. In these cases, terms appear of the form $\log(\mu_1/\mu_2)$. If the ratio μ_1/μ_2 is large, these logarithms can render naive perturbation theory unreliable and a modified perturbation expansion that takes these terms into account must be used. A few examples are discussed below.

In the cases where the higher-order corrections to a process are known and are large, some caution should be exercised when quoting the value of α_s. In what follows, we will attempt to indicate the size of the theoretical uncertainties on the extracted value of α_s. There are two simple ways to determine this error. First, we can estimate it by comparing the value of $\alpha_s(\mu)$ obtained by fitting data using the QCD formula to highest known order in α_s, and then comparing it with the value obtained using the next-to-highest-order formula (μ is chosen as the typical energy scale in the process). The corresponding Λ's are then obtained by evolving $\alpha_s(\mu)$ to $\mu = M_Z$ using Eq. (9.4) to the same order in α_s as the fit. Alternatively, we can vary the value of μ over a reasonable range, extracting a value of Λ for each choice of μ. This method is by its nature imprecise, since "reasonable" involves a subjective judgment. In either case, if the perturbation series is well behaved, the resulting error on $\alpha_s(M_Z)$ will be small.

In the above discussion we have ignored quark-mass effects, *i.e.*, we have assumed an idealized situation where quarks of mass greater than μ are neglected completely. In this picture, the β-function coefficients change by discrete amounts as flavor thresholds (a quark of mass M) are crossed when integrating the differential equation for α_s. Now imagine an experiment at energy scale μ; for example, this could be $e^+e^- \to$ hadrons at center-of-mass energy μ. If $\mu \gg M$, the mass M is negligible and the process is well described by QCD with n_f massless flavors and its parameter $\alpha_{(n_f)}$ up to terms of order M^2/μ^2. Conversely if $\mu \ll M$, the heavy quark plays no role and the process is well described by QCD with $n_f - 1$ massless flavors and its parameter $\alpha_{(n_f-1)}$ up to terms of order μ^2/M^2. If $\mu \sim M$, the effects of the quark mass are process-dependent and cannot be absorbed into the running coupling. The values of $\alpha^{(n_f)}$ and $\alpha^{(n_f-1)}$ are related so that a physical quantity calculated in both "theories" gives the same result [16]. This implies

$$\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M) - \frac{7}{72\pi^2}\alpha^2_{(n_f-1)}(M) \tag{9.7}$$

which is almost identical to the naive result $\alpha_{(n_f)}(M) = \alpha_{(n_f-1)}(M)$. Here M is the mass of the value of the running quark mass defined in the $\overline{\text{MS}}$ scheme (see the note on "Quark Masses" in the Particle Listings for more details), *i.e.*, where $M_{\overline{\text{MS}}}(M) = M$.

It also follows that, for a relationship such as Eq. (9.5) to remain valid for all values of μ, Λ must also change as flavor thresholds are crossed, the value corresponds to an effective number of massless quarks: $\Lambda \to \Lambda^{(n_f)}$ [16,17]. The formulae are given in the previous edition of this review.

An alternative matching procedure can be used [18]. This procedure requires the equality $\alpha_s(\mu)^{(n_f)} = \alpha_s(\mu)^{(n_f-1)}$ for $\mu = M$. This matching is somewhat arbitrary; a different relation between $\Lambda^{(n_f)}$ and $\Lambda^{(n_f-1)}$ would result if $\mu = M/2$ were used. In practice, the differences between these procedures are very small. $\Lambda^{(5)} = 200$ MeV corresponds to $\Lambda^{(4)} = 289$ MeV in the scheme of Ref. 18 and $\Lambda^{(4)} = 280$ MeV in the scheme we adopt. Note that the differences between $\Lambda^{(5)}$ and $\Lambda^{(4)}$ are numerically very significant.

Data from deep-inelastic scattering are in a range of energy where the bottom quark is not readily excited, and hence, these experiments quote $\Lambda^{(4)}_{\overline{\text{MS}}}$. Most data from PEP, PETRA, TRISTAN, LEP, and SLC quote a value of $\Lambda^{(5)}_{\overline{\text{MS}}}$ since these data are in an energy range where the bottom quark is light compared to the available energy. We have converted it to $\Lambda^{(4)}_{\overline{\text{MS}}}$ as required. A few measurements, including the lattice gauge theory values from the $J\psi$ system, and from τ decay are at sufficiently low energy that $\Lambda^{(3)}_{\overline{\text{MS}}}$ is appropriate.

In order to compare the values of α_s from various experiments, they must be evolved using the renormalization group to a common scale. For convenience, this is taken to be the mass of the Z boson. This evolution uses third-order perturbation theory and can introduce additional errors particularly if extrapolation from very small scales is used. The variation in the charm and bottom quark masses ($m_b = 4.3 \pm 0.2$ GeV and $m_c = 1.3 \pm 0.3$ GeV are used) can also introduce errors. These result in a fixed value of $\alpha_s(2\ \text{GeV})$ giving an uncertainty in $\alpha_s(M_Z) = \pm 0.001$ if only perturbative evolution is used. There could be additional errors from nonperturbative effects that enter at low energy.

9.3. QCD in deep-inelastic scattering

The original and still one of the most powerful quantitative tests of perturbative QCD is the breaking of Bjorken scaling in deep-inelastic lepton-hadron scattering. In the leading-logarithm approximation, the measured structure functions $F_i(x, Q^2)$ are related to the quark distribution functions $q_i(x, Q^2)$ according to the naive parton model, by the formulae in "Cross-section Formulae for Specific Processes," Sec. 35 of this *Review*. (In that section, q_i is denoted by the notation f_q). In describing the way in which scaling is broken in QCD, it is convenient to define nonsinglet and singlet quark distributions:

$$F^{NS} = q_i - q_j \qquad\qquad F^S = \sum_i (q_i + \overline{q}_i) \ . \tag{9.8}$$

The nonsinglet structure functions have nonzero values of flavor quantum numbers such as isospin or baryon number. The variation with Q^2 of these is described by the so-called DGLAP equations [19,20]:

$$Q^2 \frac{\partial F^{NS}}{\partial Q^2} = \frac{\alpha_s(|Q|)}{2\pi} P^{qq} * F^{NS} \tag{9.9a}$$

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} F^S \\ G \end{pmatrix} = \frac{\alpha_s(|Q|)}{2\pi} \begin{pmatrix} P^{qq} & 2n_f P^{qg} \\ P^{gq} & P^{gg} \end{pmatrix} * \begin{pmatrix} F^S \\ G \end{pmatrix} \tag{9.9b}$$

where $*$ denotes a convolution integral:

$$f * g = \int_x^1 \frac{dy}{y} \, f(y) \, g\left(\frac{x}{y}\right) \; . \tag{9.10}$$

The leading-order Altarelli-Parisi [20] splitting functions are

$$P^{qq} = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+}\right] + 2\delta(1-x) \; , \tag{9.11a}$$

$$P^{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right] \; , \tag{9.11b}$$

$$P^{gq} = \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right] \; , \tag{9.11c}$$

$$\begin{aligned} P^{gg} = 6 & \left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+} + \frac{11}{12}\delta(1-x)\right] \\ & - \frac{n_f}{3}\delta(1-x) \; . \end{aligned} \tag{9.11d}$$

Here the gluon distribution $G(x,Q^2)$ has been introduced and $1/(1-x)_+$ means

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} = \int_0^1 dx \, \frac{f(x) - f(1)}{(1-x)} \; . \tag{9.12}$$

The precision of contemporary experimental data demands that higher-order corrections also be included [21]. The above results are for massless quarks. At low Q^2 values, there are also important "higher-twist" (HT) contributions of the form:

$$F_i(x,Q^2) = F_i^{(LT)}(x,Q^2) + \frac{F_i^{(HT)}(x,Q^2)}{Q^2} + \cdots \; . \tag{9.13}$$

Leading twist (LT) indicates a term whose behavior is predicted by perturbative QCD. These corrections are numerically important only for $Q^2 < \mathcal{O}(\text{few GeV}^2)$ except for x very close to 1. At very large values of x perturbative corrections proportional to $\log(1-x)$ can become important [22].

A detailed review of the current status of the experimental data can be found, for example, in Refs. [23–26], and only a brief summary will be presented here. We shall only include determinations of Λ from the recently published results; the earlier editions of this *Review* should be consulted for the earlier data. Data now exist from HERA at much smaller values of x than the fixed-target data. They provide valuable information about the shape of the antiquark and gluon distribution functions at $x \sim 10^{-4}$ [27].

From Eq. (9.9), it is clear that a nonsinglet structure function offers in principle the most precise test of the theory, since the Q^2 evolution is independent of the unmeasured gluon distribution. The CCFR collaboration fit to the Gross-Llewellyn Smith sum rule [28] which is known to order α_s^3 [29,30](Estimates of the order α_s^4 term are available [31])

$$\int_0^1 dx \left(F_3^{\overline{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2) \right) =$$
$$3\left[1 - \frac{\alpha_s}{\pi}(1 + 3.58\frac{\alpha_s}{\pi} + 19.0\left(\frac{\alpha_s}{\pi})^2\right) - \Delta HT\right] \quad , \tag{9.14}$$

where the higher-twist contribution ΔHT is estimated to be $(0.09 \pm 0.045)/Q^2$ in Refs. [29,32] and to be somewhat smaller by Ref. 33. The CCFR collaboration [35], combines their data with that from other experiments [36] and gives α_s ($\sqrt{3}$ GeV) = 0.28 ± 0.035 (expt.) $\pm\, 0.05$ (sys)$^{+0.035}_{-0.03}$ (theory). The error from higher-twist terms (assumed to be $\Delta HT = 0.05 \pm 0.05$) dominates the theoretical error. If the higher twist result of Ref. 33 is used, the central value increases to 0.31 in agreement with the fit of [37]. This value corresponds to $\alpha_s(M_Z) = 0.118 \pm 0.011$.

Measurements involving singlet-dominated structure functions, such as F_2, result in measurements of α_s and the gluon structure function. A full next-to-leading-order fit combining data from SLAC [38], BCDMS [39], E665 [40] and HERA [27] has been performed by Ref. 41. These authors extend the analysis to next-to-next-to-leading order (NNLO). In this case the full theoretical calculation is not available as not all the three loop anomalous dimensions are known; their analysis uses moments of structure functions and is restricted to those moments where the full calculation is available [21,42,37]. The NNLO result is $\alpha_s(M_Z) = 0.1172 \pm 0.0017$ (expt.) $\pm\, 0.0017$ (sys). Here the first error is a combination of statistical and systematic experimental errors, and the second error is due to the uncertainties in the quark masses, higher twist and target mass corrections, and errors from the gluon distribution. If only a next-to-leading-order fit is performed then the value decreases to $\alpha_s(M_Z) = 0.116$ indicating that the theoretical results are stable. Scale uncertainties are not included. This result is consistent with earlier determinations [43,44,45]. The second of these authors estimated the scale uncertainty at ± 0.004 when a NLO fit was used. The error of Ref. 41 should be increased to take account of the possible scale error. We will therefore use $\alpha_s(M_Z) = 0.1172 \pm 0.0045$ in the final average.

The spin-dependent structure functions, measured in polarized lepton-nucleon scattering, can also be used to test QCD and to determine α_s. Here the values of $Q^2 \sim 2.5$ GeV^2 are small, particularly for the E143 data [49], and higher-twist corrections are important. A fit [46] using the measured spin dependent structure functions for several experiments themselves from Refs. [48,49] gives $\alpha_s(M_Z) = 0.121 \pm 0.002(\text{expt.}) \pm 0.006(\text{theory and syst.})$. Data from HERMES [50] are not included in this fit; they are consistent with the older data. α_s can also be determined from the Bjorken sum rule [51]; a fit gives [47] $\alpha_s(M_Z) = 0.118^{+0.010}_{-0.024}$; consistent with an earlier determination [52], the larger error being due to the extrapolation into the (unmeasured) small x region. Theoretically, the sum rule is preferable as the perturbative QCD result is known to higher order and these terms are important at the low Q^2 involved. It has been shown that the theoretical errors associated with the choice of scale are considerably reduced by the use of Pade approximants [12] which results in $\alpha_s(1.7 \text{ GeV}) = 0.328 \pm 0.03(\text{expt.}) \pm 0.025(\text{theory})$ corresponding to $\alpha_s(M_Z) = 0.116^{+0.003}_{-0.005}(\text{expt.}) \pm 0.003(\text{theory})$. No error is included from the extrapolation into the region of x that is unmeasured. Should data become available at smaller values of x so that this extrapolation could be more tightly constrained, the sum rule method could provide the best determination of α_s; the result from the structure functions themselves is used in the average.

At very small values of x and Q^2, the x and Q^2 dependence of the structure functions is predicted by perturbative QCD [53]. Here terms to all orders in $\alpha_s \ln(1/x)$ are summed. The data from HERA [27] on $F_2^{ep}(x, Q^2)$ can be fitted to this form [54], including the NLO terms which are required to fix the Q^2 scale. The data are dominated by $4 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2$. The fit [55] using H1 data [56] gives $\alpha_s(M_Z) = 0.122 \pm 0.004 \text{ (expt.)} \pm 0.009 \text{ (theory)}$. (The theoretical error is taken from Ref. 54.) The dominant part of the theoretical error is from the scale dependence; errors from terms that are suppressed by $1/\log(1/x)$ in the quark sector are included [57] while those from the gluon sector are not.

Typically, Λ is extracted from the deep inelastic scattering data by parameterizing the parton densities in a simple analytic way at some Q_0^2, evolving to higher Q^2 using the next-to-leading-order evolution equations, and fitting globally to the measured structure functions to obtain $\Lambda^{(4)}_{\overline{\text{MS}}}$. Thus, an important by-product of such studies is the extraction of parton densities at a fixed-reference value of Q_0^2. These can then be evolved in Q^2 and used as input for phenomenological studies in hadron-hadron collisions (see below). To avoid having to evolve from the starting Q_0^2 value each time, a parton density is required; it is useful to have available a simple analytic approximation to the densities valid over a range of x and Q^2 values. A package is available from the CERN computer library that includes an exhaustive set of fits [58]. Most of these fits are obsolete. In using a parameterization to predict event rates, a next-to-leading order fit must be used if the process being calculated is known to next-to-leading order in QCD perturbation theory. In such a case, there is an additional scheme dependence; this scheme dependence is reflected in the $\mathcal{O}(\alpha_s)$ corrections that appear in the relations between the structure functions and the quark distribution functions. There are two common schemes: a deep-inelastic scheme where there are no order α_s corrections in the formula for $F_2(x, Q^2)$ and the minimal subtraction scheme. It is important when these next-to-leading order fits are used in other processes (see below), that the same scheme is used in the calculation of the partonic rates. Most current sets of parton distributions are obtained using fits to all relevant event data [59]. In particular, data from purely hadronic initial states are used as they can provide important constraints on the gluon distributions.

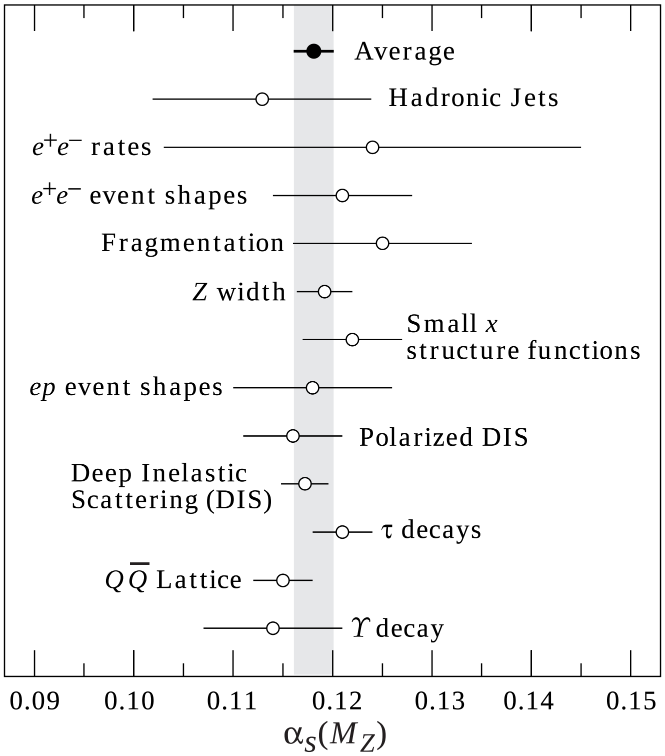

Figure 9.1: Summary of the value of $\alpha_s(M_Z)$ from various processes. The values shown indicate the process and the measured value of α_s extrapolated up to $\mu = M_Z$. The error shown is the *total* error including theoretical uncertainties.

9.4. QCD in decays of the τ lepton

The semi-leptonic branching ratio of the tau ($\tau \rightarrow \nu_\tau +$ hadrons, R_τ) is an inclusive quantity. It is related to the contribution of hadrons to the imaginary part of the W self energy $\big(\Pi(s)\big)$. It is sensitive to a range of energies since it involves an integral

$$R_\tau \sim \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}(1-\frac{s}{m_\tau^2})^2 \ \mathrm{Im}\,(\Pi(s)) \ \ .$$

Since the scale involved is low, one must take into account nonperturbative (higher-twist) contributions which are suppressed by powers of the τ mass.

$$\begin{aligned} R_\tau =& 3.058\Big[1+\frac{\alpha_s(m_\tau)}{\pi}+5.2\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^2+26.4\Big(\frac{\alpha_s(m_\tau)}{\pi}\Big)^3 \\ &+ a\frac{m^2}{m_\tau^2}+b\frac{m\psi\overline{\psi}}{m_\tau^4}+c\frac{\psi\overline{\psi}\psi\overline{\psi}}{m_\tau^6}+\cdots\Big] \ . \end{aligned} \tag{9.15}$$

Here a, b, and c are dimensionless constants and m is a light quark mass. The term of order $1/m_\tau^2$ is a kinematical effect due to the light quark masses and is consequently very small. The nonperturbative terms are estimated using sum rules [60]. In total, they are estimated to be -0.014 ± 0.005 [61,62]. This estimate relies on there being no term of order Λ^2/m_τ^2 $\left(\text{note that } \dfrac{\alpha_s(m_\tau)}{\pi} \sim (\dfrac{0.5 \text{ GeV}}{m_\tau})^2\right)$. The a, b, and c can be determined from the data [63] by fitting to moments of the $\Pi(s)$ and separately to the final states accessed by the vector and axial parts of the W coupling. The values so extracted [64,65] are consistent with the theoretical estimates. If the nonperturbative terms are omitted from the fit, the extracted value of $\alpha_s(m_\tau)$ decreases by ~ 0.02.

For $\alpha_s(m_\tau) = 0.35$ the perturbative series for R_τ is $R_\tau \sim 3.058(1{+}0.112{+}0.064{+}0.036)$. The size (estimated error) of the nonperturbative term is 20% (7%) of the size of the order α_s^3 term. The perturbation series is not very well convergent; if the order α_s^3 term is omitted, the extracted value of $\alpha_s(m_\tau)$ increases by 0.05. The order α_s^4 term has been estimated [111] and attempts made to resum the entire series [67,68]. These estimates can be used to obtain an estimate of the errors due to these unknown terms [69,70]. We assign an uncertainty of ± 0.02 to $\alpha_s(m_\tau)$ from these sources.

R_τ can be extracted from the semi-leptonic branching ratio from the relation $R_\tau = 1/(\mathrm{B}(\tau \to e\nu\overline{\nu}) - 1.97256$; where $\mathrm{B}(\tau \to e\nu\overline{\nu})$ is measured directly or extracted from the lifetime, the muon mass, and the muon lifetime assuming universality of lepton couplings. Using the average lifetime of 290.0 ± 1.2 fs and a τ mass of 1777.05 ± 0.29 MeV from the PDG fit gives $R_\tau = 3.655 \pm 0.023$. The direct measurement of $\mathrm{B}(\tau \to e\nu\overline{\nu})$ can be combined with $\mathrm{B}(\tau \to \mu\nu\overline{\nu})$ to give $\mathrm{B}(\tau \to e\nu\overline{\nu}) = 0.1783 \pm 0.0007$ which gives $R_\tau = 3.636 \pm 0.021$. Averaging these yields $\alpha_s(m_\tau) = 0.351 \pm 0.008$ using the experimental error alone. We assign a theoretical error equal to 40% of the contribution from the order α^3 term and all of the nonperturbative contributions. This then gives $\alpha_s(m_\tau) = 0.35 \pm 0.03$ for the final result. This corresponds to $\alpha_s(M_Z) = 0.121 \pm 0.003$.

9.5. QCD in high-energy hadron collisions

There are many ways in which perturbative QCD can be tested in high-energy hadron colliders. The quantitative tests are only useful if the process in question has been calculated beyond leading order in QCD perturbation theory. The production of hadrons with large transverse momentum in hadron-hadron collisions provides a direct probe of the scattering of quarks and gluons: $qq \to qq$, $qg \to qg$, $gg \to gg$, *etc.* Higher–order QCD calculations of the jet rates [71] and shapes are in impressive agreement with data [72]. This agreement has led to the proposal that these data could be used to provide a determination of α_s [73]. A set of structure functions is assumed and Tevatron collider data are fitted over a very large range of transverse momenta, to the QCD prediction for the underlying scattering process that depends on α_s. The evolution of the coupling over this energy range (40 to 250 GeV) is therefore tested in the analysis. CDF obtains $\alpha_s(M_Z) = 0.1129 \pm 0.0001$ (stat.) ± 0.0085 (syst.) [74]. Estimation of the theoretical errors is not straightforward. The structure functions used depend implicitly on α_s and an iteration procedure must be used to obtain a consistent result; different sets of structure functions yield different correlations between the two values of α_s. I estimate an uncertainty of ± 0.005 from examining the fits. Ref. 73 estimates the error from unknown higher order QCD corrections to be ± 0.005. Combining these then gives. $\alpha_s(M_Z) = 0.113 \pm 0.011$. Data are also available on the angular distribution of jets; these are also in agreement with QCD expectations [75,76].

QCD corrections to Drell-Yan type cross sections (*i.e.*, the production in hadron collisions by quark-antiquark annihilation of lepton pairs of invariant mass Q from virtual photons, or of real W or Z bosons), are known [77]. These $\mathcal{O}(\alpha_s)$ QCD corrections are sizable at small values of Q. The correction to W and Z production, as measured in $p\overline{p}$ collisions at $\sqrt{s} = 0.63$ TeV and $\sqrt{s} = 1.8$ TeV, is of order 30%. The NNLO corrections to this process are known [78].

The production of W and Z bosons and photons at large transverse momentum can also be used to test QCD. The leading-order QCD subprocesses are $q\overline{q} \to \gamma g$ and $qg \to \gamma q$. If the parton distributions are taken from other processes and a value of α_s assumed, then an absolute prediction is obtained. Conversely, the data can be used to extract information on quark and gluon distributions and on the value of α_s. The next-to-leading-order QCD corrections are known [79,80] (for photons), and for W/Z production [81], and so a precision test is possible. Data exist on photon production from the CDF and DØ collaborations [82,83] and from fixed target experiments [84]. Detailed comparisons with QCD predictions [85] may indicate an excess of the data over the theoretical prediction at low value of transverse momenta. although other authors [86] find smaller excesses.

The UA2 collaboration [87] extracted a value of $\alpha_s(M_W) = 0.123 \pm 0.018(\text{stat.}) \pm 0.017(\text{syst.})$ from the measured ratio $R_W = \dfrac{\sigma(W + 1\text{jet})}{\sigma(W + 0\text{jet})}$. The result depends on the algorithm used to define a jet, and the dominant systematic errors due to fragmentation and corrections for underlying events (the former causes jet energy to be lost, the latter causes it to be increased) are connected to the algorithm. There is also dependence on the parton distribution functions, and hence, α_s appears explicitly in the formula for R_W, and implicitly in the distribution functions. This result is not used in the final average. Data from CDF and D0 on the W p_t distribution [89] are in agreement with QCD but are not able to determine α_s with sufficient precision to have any weight in a global average.

In the region of low p_t, fixed order perturbation theory is not applicable; one must sum terms of order $\alpha_s^n \ln^n(p_t/M_W)$ [88]. Data from D0 [90] on the p_t distribution of Z bosons agree well with these predictions.

The production rates of b quarks in $p\overline{p}$ have been used to determine α_s [91]. The next-to-leading-order QCD production processes [92] have been used. By selecting events where the b quarks are back-to-back in azimuth, the next-to-leading-order calculation can be used to compare rates to the measured value and a value of α_s extracted. The errors are dominated by the measurement errors, the choice of μ and M, and uncertainties in the choice of structure functions. The last were estimated by varying the structure functions used. The result is $\alpha_s(M_Z) = 0.113^{+0.009}_{-0.013}$.

9.6. QCD in heavy-quarkonium decay

Under the assumption that the hadronic and leptonic decay widths of heavy $Q\overline{Q}$ resonances can be factorized into a nonperturbative part—dependent on the confining potential—and a calculable perturbative part, the ratios of partial decay widths allow measurements of α_s at the heavy-quark mass scale. The most precise data come from the decay widths of the 1^{--} $J/\psi(1S)$ and Υ resonances. The total decay width of the Υ is predicted by perturbative QCD [93]

$$\begin{aligned} R_\mu(\Upsilon) &= \frac{\Gamma(\Upsilon \to \text{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)} \\ &= \frac{10(\pi^2-9)\alpha_s^3(M)}{9\pi\alpha_{\text{em}}^2} \\ &\quad \times \left[1 + \frac{\alpha_s}{\pi}\left(-19.4 + \frac{3\beta_0}{2}\left(1.162 + \ln\left(\frac{2M}{M_\Upsilon}\right)\right)\right)\right] . \end{aligned} \tag{9.16}$$

Data are available for the Υ, Υ', Υ'', and J/ψ. The result is very sensitive to α_s and the data are sufficiently precise ($R_\mu(\Upsilon) = 32.5 \pm 0.9$) [94] that the theoretical errors will dominate. There are theoretical corrections to this simple formula due to the relativistic nature of the $Q\overline{Q}$ system; $v^2/c^2 \sim 0.1$ for the Υ. They are more severe for the J/ψ. There are also nonperturbative corrections of the form Λ^2/M_Υ^2; again these are more severe for the J/ψ. A fit to Υ, Υ', and Υ'' [95] gives $\alpha_s(M_Z) = 0.113 \pm 0.001$ (expt.). The results from each state separately and also from the J/ψ are consistent with each other. There is an uncertainty of order ± 0.005 from the choice of scale; the error from v^2/c^2 corrections is a little larger. The ratio of widths $\dfrac{\Upsilon \to \gamma gg}{\Upsilon \to ggg}$ has been measured by the CLEO collaboration who use it to determine $\alpha_s(9.45\ \text{GeV}) = 0.163 \pm 0.002 \pm 0.014$ [97] which corresponds to $\alpha_s(M_Z) = 0.110 \pm 0.001 \pm 0.007$. The error is dominated by theoretical uncertainties associated with the scale choice. The theoretical uncertainties due to the production of photons in fragmentation [96] are small [97]. Higher order QCD calculations of the photon energy distribution are available [98]; this distribution could now be used to further test the theory. The width $\Gamma(\Upsilon \to e^+e^-)$ can also be used to determine α_s by using moments of the quantity $R_b(s) = \dfrac{\sigma(e^+e^- \to b\overline{b})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ defined by $M_n = \int_0^\infty \dfrac{R_b(s)}{s^{n+1}}$ [99]. At large values of n, M_n is dominated by $\Gamma(\Upsilon \to e^+e^-)$. Higher order corrections are available and the method gives [100] $\alpha_s(m_b) = 0.220 \pm 0.027$. The dominant error is theoretical and is dominated by the choice of scale and by uncertainties in Coulomb corrections. It corresponds to $\alpha_s(M_Z) = 0.119 \pm 0.008$. These various Υ measurements can be combined and give $\alpha_s(M_Z) = 0.114 \pm 0.008$.

9.7. Perturbative QCD in e^+e^- collisions

The total cross section for $e^+e^- \to$ hadrons is obtained (at low values of $\sqrt{s}$) by multiplying the muon-pair cross section by the factor $R = 3\Sigma_q e_q^2$. The higher-order QCD corrections to this quantity have been calculated, and the results can be expressed in terms of the factor:

$$R = R^{(0)} \left[1 + \frac{\alpha_s}{\pi} + C_2 \left(\frac{\alpha_s}{\pi}\right)^2 + C_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \cdots \right] , \tag{9.17}$$

where $C_2 = 1.411$ and $C_3 = -12.8$ [101].

$R^{(0)}$ can be obtained from the formula for $d\sigma/d\Omega$ for $e^+e^- \to f\overline{f}$ by integrating over Ω. The formula is given in Sec. 35.2 of this *Review*. This result is only correct in the zero-quark-mass limit. The $\mathcal{O}(\alpha_s)$ corrections are also known for massive quarks [102]. The principal advantage of determining α_s from R in e^+e^- annihilation is that there is no dependence on fragmentation models, jet algorithms, *etc.*

A measurement by CLEO [103] at $\sqrt{s}$ = 10.52 GeV yields α_s(10.52 GeV) = $0.20 \pm 0.01 \pm 0.06$, which corresponds to $\alpha_s(M_Z) = 0.13 \pm 0.005 \pm 0.03$. A comparison of the theoretical prediction of Eq. (9.17) (corrected for the b-quark mass), with all the available data at values of $\sqrt{s}$ between 20 and 65 GeV, gives [104] α_s(35 GeV) $= 0.146 \pm 0.030$. The size of the order α_s^3 term is of order 40% of that of the order α_s^2 and 3% of the order α_s. If the order α_s^3 term is not included, a fit to the data yields α_s (34 GeV) $= 0.142 \pm 0.03$, indicating that the theoretical uncertainty is smaller than the experimental error.

Measurements of the ratio of hadronic to leptonic width of the Z at LEP and SLC, Γ_h/Γ_μ probe, the same quantity as R. Using the average of $\Gamma_h/\Gamma_\mu = 20.783 \pm 0.029$ gives $\alpha_s(M_Z) = 0.123 \pm 0.004$ [105]. There are theoretical errors arising from the values of top-quark and Higgs masses which enter due to electroweak corrections to the Z width and from the choice of scale.

While this method has small theoretical uncertainties from QCD itself, it relies sensitively on the electroweak couplings of the Z to quarks [106]. The presence of new physics which changes these couplings via electroweak radiative corrections would invalidate the value of $\alpha_s(M_Z)$. However, given the excellent agreement [107] of the many measurements at the Z, there is no reason not to use the value of $\alpha_s(M_Z) = 0.1192 \pm 0.0028 \pm 0.002(\mathit{scale})$ from the global fits of the various precision measurements at LEP/SLC and the W and top masses in the world average (see the section on "Electroweak model and constraints on new physics," Sec. 10 of this *Review*).

An alternative method of determining α_s in e^+e^- annihilation is from measuring quantities that are sensitive to the relative rates of two-, three-, and four-jet events. A review should be consulted for more details [108] of the issues mentioned briefly here. In addition to simply counting jets, there are many possible choices of such "shape variables": thrust [109], energy-energy correlations [110], average jet mass, *etc.* All of these are infrared safe, which means they can be reliably calculated in perturbation theory. The starting point for all these quantities is the multijet cross section. For example, at order α_s, for the process $e^+e^- \to qqg$: [111]

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} , \tag{9.18}$$

$$x_i = \frac{2E_i}{\sqrt{s}} \tag{9.19}$$

where x_i are the center-of-mass energy fractions of the final-state (massless) quarks. A distribution in a "three-jet" variable, such as those listed above, is obtained by integrating this differential cross section over an appropriate phase space region for a fixed value of the variable. The order α_s^2 corrections to this process have been computed, as well as the 4-jet final states such as $e^+e^- \to qqgg$ [112].

There are many methods used by the e^+e^- experimental groups to determine α_s from the event topology. The jet-counting algorithm, originally introduced by the JADE collaboration [113], has been used by many other groups. Here, particles of momenta p_i and p_j are combined into a pseudo-particle of momentum $p_i + p_j$ if the invariant mass of the pair is less than $y_0\sqrt{s}$. The process is then iterated until no more pairs of particles or pseudo-particles remain. The remaining number is then defined to be the number of jets in the event, and can be compared to the QCD prediction. The Durham algorithm is slightly different: in computing the mass of a pair of partons, it uses $M^2 = 2\min(E_1^2, E_2^2)(1-\cos\theta_{ij})$ for partons of energies E_i and E_j separated by angle θ_{ij} [114].

There are theoretical ambiguities in the way this process is carried out. Quarks and gluons are massless, whereas the observed hadrons are not, so that the massive jets that result from this scheme cannot be compared directly to the massless jets of perturbative QCD. Different recombination schemes have been tried, for example combining 3-momenta and then rescaling the energy of the cluster so that it remains massless. These schemes result in the same data giving a slightly different values [115,116] of α_s. These differences can be used to determine a systematic error. In addition, since what is observed are hadrons rather than quarks and gluons, a model is needed to describe the evolution of a partonic final state into one involving hadrons, so that detector corrections can be applied. The QCD matrix elements are combined with a parton-fragmentation model. This model can then be used to correct the data for a direct comparison with the parton calculation. The different hadronization models that are used [117–120] model the dynamics that are controlled by nonperturbative QCD effects which we cannot yet calculate. The fragmentation parameters of these Monte Carlos are tuned to get agreement with the observed data. The differences between these models contribute to the systematic errors. The systematic errors from recombination schemes and fragmentation effects dominate over the statistical and other errors of the LEP/SLD experiments.

The scale M at which $\alpha_s(M)$ is to be evaluated is not clear. The invariant mass of a typical jet (or $\sqrt{sy_0}$) is probably a more appropriate choice than the e^+e^- center-of-mass energy. While there is no justification for doing so, if the value is allowed to float in the fit to the data, the fit improves and the data tend to prefer values of order $\sqrt{s}/10$ GeV for some variables [116,121]; the exact value depends on the variable that is fitted.

The perturbative QCD formulae can break down in special kinematical configurations. For example, the thrust (T) distribution contains terms of the type $\alpha_s \ln^2(1-T)$. The higher orders in the perturbation expansion contain terms of order $\alpha_s^n \ln^m(1-T)$. For $T \sim 1$ (the region populated by 2-jet events), the perturbation expansion is unreliable. The terms with $n \leq m$ can be summed to all orders in α_s [122]. If the jet recombination methods are used higher-order terms involve $\alpha_s^n \ln^m(y_0)$, these too can be resummed [123]. The resummed results give better agreement with the data at large values of T. Some caution should be exercised in using these resummed results because of the possibility of overcounting; the showering Monte Carlos that are used for the fragmentation corrections also generate some of these leading-log corrections. Different schemes for combining the order α_s^2 and the resummations are available [124]. These different schemes result in shifts in α_s of order ± 0.002. The use of the resummed results improves the agreement between the data and the theory. An average of the recent results at the Z resonance from SLD [116], OPAL [125], L3 [126], ALEPH [127], and DELPHI [128], using the combined α_s^2 and resummation fitting to a large set of shape variables, gives $\alpha_s(M_Z) = 0.122 \pm 0.007$. The errors in the values of $\alpha_s(M_Z)$ from these shape variables are totally dominated by the theoretical uncertainties associated with the choice of scale, and the effects of hadronization Monte Carlos on the different quantities fitted.

Similar studies on event shapes have been undertaken at lower energies at TRISTAN, PEP/PETRA, and CLEO. A combined result from various shape parameters by the TOPAZ collaboration gives $\alpha_s(58\ \text{GeV}) = 0.125 \pm 0.009$, using the fixed order QCD result, and $\alpha_s(58\ \text{GeV}) = 0.132 \pm 0.008$ (corresponding to $\alpha_s(M_Z) = 0.123 \pm 0.007$), using the same method as in the SLD and LEP average [129]. The measurements of event shapes at PEP/PETRA are summarized in earlier editions of this note. A recent reevaluation of the JADE data [130] obtained using resummed QCD results and by averaging over several shape variables gives $\alpha_s(35\ \text{GeV}) = 0.145^{+0.012}_{-0.007}$. An analysis by the TPC group [131] gives $\alpha_s(29\ \text{GeV}) = 0.160 \pm 0.012$, using the same method as TOPAZ. This value corresponds to $\alpha_s(M_Z) = 0.131 \pm 0.010$

The CLEO collaboration fits to the order α_s^2 results for the two jet fraction at $\sqrt{s} =$ 10.53 GeV, and obtains $\alpha_s(10.93\ \text{GeV}) = 0.164 \pm 0.004\ (\text{expt.}) \pm 0.014\ (\text{theory})$ [132]. The dominant systematic error arises from the choice of scale (μ), and is determined from the range of α_s that results from fit with $\mu = 10.53$ GeV, and a fit where μ is allowed to vary to get the lowest χ^2. The latter results in $\mu = 1.2$ GeV. Since the quoted result corresponds to $\alpha_s(1.2\ \text{GeV}) = 0.35$, it is by no means clear that the perturbative QCD expression is reliable and the resulting error should, therefore, be treated with caution. A fit to many different variables as is done in the LEP/SLC analyses would give added confidence to the quoted error.

Recently studies have been carried out at energies between ~130 GeV [133] and ~189 GeV [134]. These can be combined to give $\alpha_s(130\ \text{GeV}) = 0.114 \pm 0.008$ and $\alpha_s(189\ \text{GeV}) = 0.1104 \pm 0.005$. The dominant errors are theoretical and systematic and, as most of these are in common at the two energies. These data and those at the Z

resonance provide clear confirmation of the expected decrease in α_s as the energy is increased.

Since the errors in the event shape measurements are dominantly systematic, and are common to the experiments, the results from PEP/PETRA, TRISTAN, LEP, SLC, and CLEO are combined to give $\alpha_s(M_Z) = 0.121 \pm 0.007$. All of the experiments are consistent with this average and, taken together, provide verification of the running of the coupling constant with energy.

Estimates are available for the nonperturbative corrections to the mean value of $1 - T$ [136]. These are of order $1/E$ and involve a single parameter to be determined from experiment. These corrections can then be used as an alternative to those modeled by the fragmentation Monte-Carlos. The DELPHI collaboration [135] uses data up to the Z mass from many experiments and determines $\alpha_s(M_Z) = 0.119 \pm 0.006$, the error being dominated by the choice of scale. The value is also determined by a fit to a second variable (the mean jet mass); while the extracted values of $\alpha_s(M_Z)$ are consistent with each other, the values of the non perturbative parameter are not. The analysis is useful as one can directly determine the size of the $1/E$ corrections; they are approximately 20% (50%) of the perturbative result at $\sqrt{s} = 91(11)$ GeV.

9.8. Scaling violations in fragmentation functions

Measurements of the fragmentation function $d_i(z, E)$, (the probability that a hadron of type i be produced with energy zE in e^+e^- collisions at $\sqrt{s} = 2E$) can be used to determine α_s. As in the case of scaling violations in structure functions, QCD predicts only the E dependence. Hence, measurements at different energies are needed to extract a value of α_s. Because the QCD evolution mixes the fragmentation functions for each quark flavor with the gluon fragmentation function, it is necessary to determine each of these before α_s can be extracted. The ALEPH collaboration has used data from energies ranging from $\sqrt{s} = 22$ GeV to $\sqrt{s} = 91$ GeV. A flavor tag is used to discriminate between different quark species, and the longitudinal and transverse cross sections are used to extract the gluon fragmentation function [137]. The result obtained is $\alpha_s(M_Z) = 0.126 \pm 0.007$ (expt.) ± 0.006 (theory) [138]. The theory error is due mainly to the choice of scale. The OPAL collaboration [139] has also extracted the separate fragmentation functions. DELPHI [140] has also performed a similar analysis using data from other experiments at lower energy with the result $\alpha_s(M_Z) = 0.124 \pm 0.007 \pm 0.009$ (theory). The larger theoretical error is due to the larger range of scales that were used in the fit. These results can be combined to give $\alpha_s(M_Z) = 0.125 \pm 0.005 \pm 0.008$ (theory).

9.9. Photon structure functions

e^+e^- can also be used to study photon-photon interactions, which can be used to measure the structure function of a photon [141], by selecting events of the type $e^+e^- \to e^+e^- + hadrons$ which proceeds via two photon scattering. If events are selected where one of the photons is almost on mass shell and the other has a large invariant mass Q, then the latter probes the photon structure function at scale Q; the process is analogous to deep inelastic scattering where a highly virtual photon is used to probe the proton structure. This process was included in earlier versions of this *Review* which can be consulted for details on older measurements [142–145]. A recent review of the data can be found in Ref. 146. Data have become available from LEP [147–150] and from TRISTAN [151,152] which extend the range of Q^2 to of order 300 GeV^2 and x as low as $2 \times 10^{-3}$and show Q^2 dependence of the structure function that is consistent with QCD expectations. Experiments at HERA can also probe the photon structure function by looking at jet production in γp collisions; this is analogous to the jet production in hadron-hadron collisions which is sensitive to hadron structure functions. The data [153] are consistent with theoretical models [154].

9.10. Jet rates in *ep* collisions

At lowest order in α_s, the ep scattering process produces a final state of (1+1) jets, one from the proton fragment and the other from the quark knocked out by the process $e + quark \to e + quark$. At next order in α_s, a gluon can be radiated, and hence a (2+1) jet final state produced. By comparing the rates for these (1+1) and (2+1) jet processes, a value of α_s can be obtained. A NLO QCD calculation is available [155]. The basic methodology is similar to that used in the jet counting experiments in e^+e^- annihilation discussed above. Unlike those measurements, the ones in ep scattering are not at a fixed value of Q^2. In addition to the systematic errors associated with the jet definitions, there are additional ones since the structure functions enter into the rate calculations. Results from H1 [156] and ZEUS [157] can be combined to give $\alpha_s(M_Z) = 0.118 \pm 0.0015$ (stat.) $\pm\, 0.009$ (syst.). The contributions to the systematic errors from experimental effects (mainly the hadronic energy scale) are comparable to the theoretical ones arising from scale choice, structure functions, and jet definitions. The theoretical errors are common to the two measurements; therefore, we have not reduced the systematic error after forming the average.

9.11. QCD in diffractive events

In approximately 10% of the deep-inelastic scattering events at HERA a rapidity gap is observed [158]; that is events are seen where there are almost no hadrons produced in the direction of the incident proton. This was unexpected; QCD based models of the final state predicted that the rapidity interval between the quark that is hit by the electron and the proton remnant should be populated approximately evenly by the hadrons. Similar phenomena have been observed at the Tevatron in W and jet production. For a review see Ref. 159.

9.12. Lattice QCD

Lattice gauge theory calculations can be used to calculate, using non-perturbative methods, a physical quantity that can be measured experimentally. The value of this quantity can then be used to determine the QCD coupling that enters in the calculation. For a review of the methodology see Ref. 160. For example, the energy levels of a $Q\overline{Q}$ system can be determined and then used to extract α_s. The masses of the $Q\overline{Q}$ states depend only on the quark mass and on α_s. A limitation is that calculations cannot be performed for three light quark flavors. Results are available for zero ($n_f = 0$, quenched approximation) and two light flavors, which allow extrapolation to three. The coupling constant so extracted is in a lattice renormalization scheme, and must be converted to the $\overline{\text{MS}}$ scheme for comparison with other results. Using the mass differences of Υ and Υ' and Υ'' and χ_b, Davies *et al.* [161] extract a value of $\alpha_s(M_Z) = 0.1174 \pm 0.0024$. A similar result with larger errors is reported by [162], where results are consistent with $\alpha_s(M_Z) = 0.111 \pm 0.006$. The SESAM collaboration [163] uses the Υ and Υ' and χ_b masses to obtain $\alpha_s(M_Z) = 0.1118 \pm 0.0017$ using Wilson fermions. These authors point out that their result is more than 3σ from that of Davies *et al.* which uses Kogut-Susskind fermions. A combination of the results from quenched [164] and ($n_f = 2$) [165] gives $\alpha_s(M_Z) = 0.116 \pm 0.003$ [166]. Calculations [167] using the strength of the force between two heavy quarks computed in the quenched approximation obtains a value of $\alpha_s(5\ \text{GeV})$ that is consistent with these results. There have also been investigations of the running of α_s [168]. These show remarkable agreement with the two loop perturbative result of Eq. (9.5).

There are several sources of error in these estimates of $\alpha_s(M_Z)$. The experimental error associated with the measurements of the particle masses is negligible. The conversion from the lattice coupling constant to the $\overline{\text{MS}}$ constant is obtained using a perturbative expansion where one coupling expanded as a power series in the other. This series is only known to second order. A third order calculation exists only from the $n_f = 0$ case [169]. Its inclusion leads to a shift in the extracted value of $\alpha_s(M_Z)$ of $+0.002$. Other theoretical errors arising from the limited statistics of the Monte-Carlo calculation, extrapolation in n_f, and corrections for light quark masses are smaller than this.

The result of averaging [163,161,164] gives with a more conservative error $\alpha_s(M_Z) = 0.115 \pm 0.003$. This will be used in the average.

9.13. Conclusions

The need for brevity has meant that many other important topics in QCD phenomenology have had to be omitted from this review. One should mention in particular the study of exclusive processes (form factors, elastic scattering, ...), the behavior of quarks and gluons in nuclei, the spin properties of the theory, and QCD effects in hadron spectroscopy.

We have focused on those high-energy processes which currently offer the most quantitative tests of perturbative QCD. Figure 9.1 shows the values of $\alpha_s(M_Z)$ deduced from the various experiments. Figure 9.2 shows the values and the values of Q where they are measured. This figure clearly shows the experimental evidence for the variation of $\alpha_s(Q)$ with Q.

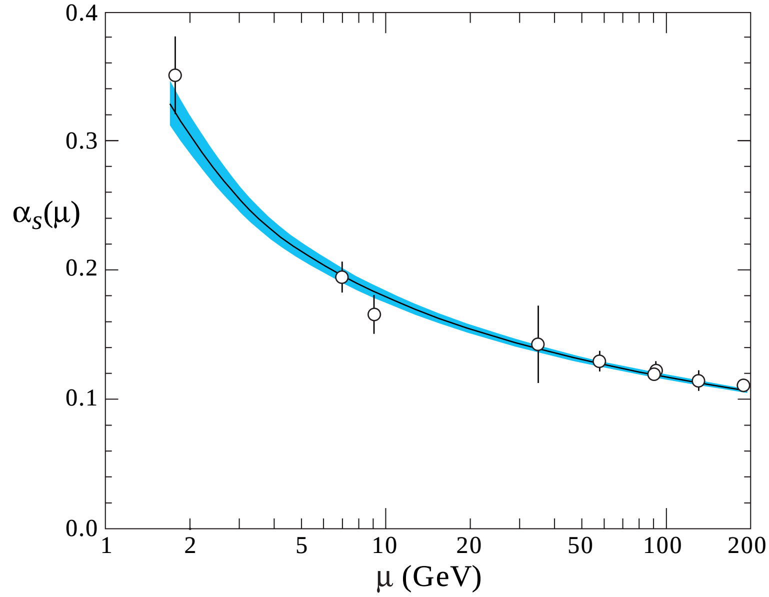

Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ. The data are, in increasing order of μ, τ width, deep inelastic scattering, Υ decays, e^+e^- event rate at 25 GeV, event shapes at TRISTAN, Z width, e^+e^- event shapes of M_Z, 135, and 189 GeV.

An average of the values in Fig. 9.1 gives $\alpha_s(M_z) = 0.1181$, with a total χ^2 of 3.8 for twelve fitted points, showing good consistency among the data. The error on the average, assuming that all of the errors in the contributing results are uncorrelated, is ± 0.0014, and may be an underestimate. Almost all of the values used in the average are dominated by systematic, usually theoretical, errors. Only some of these, notably from the choice of scale, are correlated. The average is not dominated by a single measurement; there are several results with comparable small errors: these are the ones from τ decay, lattice gauge theory, deep inelastic scattering, and the Z^0 width. We quote our average value as $\alpha_s(M_Z) = 0.1181 \pm 0.002$, which corresponds to $\Lambda^{(5)} = 208^{+25}_{-23}$ MeV using Eq. (9.5*a*). Future experiments can be expected to improve the measurements of α_s somewhat. Precision at the 1% level may be achievable if the systematic and theoretical errors can be reduced [170].

The value of α_s at any scale corresponding to our average can be obtained from `http://www-theory.lbl.gov/~ianh/alpha/alpha.html`

References:

1. R.K. Ellis, W.J. Stirling, and B.R. Webber, "QCD and Collider Physics" (Cambridge 1996).
2. For reviews see, for example, A.S. Kronfeld and P.B. Mackenzie, Ann. Rev. Nucl. and Part. Sci. **43**, 793 (1993);

   H. Wittig, Int. J. Mod. Phys. **A12**, 4477 (1997).
3. For example see, S. Bethke, in *Proceedings of the IVth Int. Symposium on Radiative Corrections*, Barcelona, Spain (Sept. 1998), `hep-ex/9812026`;

   M Davier, *33rd Rencontres de Moriond: Electroweak Interactions and Unified Theories*, Les Arcs, France (14–21 Mar. 1998);

   P.N. Burrows, Acta. Phys. Pol. **28**, 701 (1997).
4. J. Womersley, *International Conference on Lepton Photon Interactions*, Stanford, USA (Aug. 1999).
5. S.A. Larin, T. van Ritbergen, and J.A.M. Vermaseren, Phys. Lett. **B400**, 379 (1997).
6. W.A. Bardeen *et al.*, Phys. Rev. **D18**, 3998 (1978).
7. G. Grunberg, Phys. Lett. **95B**, 70 (1980); Phys. Rev. **D29**, 2315 (1984).
8. P.M. Stevenson, Phys. Rev. **D23**, 2916 (1981); and Nucl. Phys. **B203**, 472 (1982).
9. S. Brodsky and H.J. Lu, SLAC-PUB-6389 (Nov. 1993).
10. S. Brodsky, G.P. Lepage, and P.B. Mackenzie, Phys. Rev. **D28**, 228 (1983).
11. M.A. Samuel, G. Li, and E. Steinfelds, Phys. Lett. **B323**, 188 (1994);

    M.A. Samuel, J. Ellis, and M. Karliner, Phys. Rev. Lett. **74**, 4380 (1995).
12. J. Ellis *et al.*, Phys. Rev. **D54**, 6986 (1996).
13. P.N. Burrows *et al.*, Phys. Lett. **B382**, 157 (1996).
14. P. Abreu *et al.*, Z. Phys. **C54**, 55 (1992).
15. A.H. Mueller, Phys. Lett. **B308**, 355, (1993).
16. W. Bernreuther, Annals of Physics **151**, 127 (1983); Erratum Nucl. Phys. **B513**, 758 (1998);

    S.A. Larin, T. van Ritbergen, and J.A.M. Vermaseren, Nucl. Phys. **B438**, 278 (1995).
17. K.G. Chetyrkin, B.A. Kniehl, and M. Steinhauser, Phys. Rev. Lett. **79**, 2184 (1977).
18. W. Marciano, Phys. Rev. **D29**, 580 (1984).
19. V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972);

    Yu.L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).
20. G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
21. G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980);

    W. Furmanski and R. Petronzio, Phys. Lett. **97B**, 437 (1980); and Z. Phys. **C11**, 293 (1982);

    E.G. Floratos, C. Kounnas, and R. Lacaze, Phys. Lett. **98B**, 89 (1981); Phys. Lett. **98B**, 285 (1981); and Nucl. Phys. **B192**, 417 (1981);

R.T. Herrod and S. Wada, Phys. Lett. **96B**, 195 (1981); and Z. Phys. **C9**, 351 (1981).

22. G. Sterman, Nucl. Phys. **B281**, 310 (1987);

    S. Catani and L. Trentadue, Nucl. Phys. **B327**, 323 (1989); Nucl. Phys. **B353**, 183 (1991).

23. R.D. Ball and A. DeRoeck, `hep-ph/9609309`.
24. J. Feltesse, in *Proceedings of the XXVII International Conference on High Energy Physics*, Glasgow, Scotland, (July 1994).
25. M Klein at *International Conference on Lepton Photon Interactions* , Stanford, USA (August 1999).
26. F. Eisele, at the European Physical Society meeting, Brussels, (July 1995).
27. M. Derrick *et al.*, Phys. Lett. **B345**, 576 (1995); Z. Phys. **C62**, 399 (1999);

    S. Aid *et al.*, Phys. Lett. **B354**, 494 (1995); Nucl. Phys. **B470**, 3 (1996).

28. D. Gross and C.H. Llewellyn Smith, Nucl. Phys. **B14**, 337 (1969).
29. J. Chyla and A.L. Kataev, Phys. Lett. **B297**, 385 (1992).
30. S.A. Larin and J.A.M. Vermaseren, Phys. Lett. **B259**, 345 (1991).
31. A.L. Kataev and V.V. Starchenko, Mod. Phys. Lett. **A10**, 235 (1995).
32. V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. **B283**, 723 (1987).
33. M. Dasgupta and B. Webber, Phys. Lett. **B382**, 273 (1993).
34. M. Maul *et al.*, Phys. Lett. **401**, 100 (1997).
35. J. Kim *et al.*, Phys. Rev. Lett. **81**, 3595 (1998).
36. D. Allasia *et al.*, Z. Phys. **C28**, 321 (1985);

    K. Varvell *et al.*, Z. Phys. **C36**, 1 (1997);

    V.V. Ammosov *et al.*, Z. Phys. **C30**, 175 (1986);

    P.C. Bosetti *et al.*, Nucl. Phys. **B142**, 1 (1978).

37. A.L. Kataev *et al.*, Nucl. Phys. **A666 & 667**, 184 (2000) `hep-ph/9907310`.
38. L.W. Whitlow *et al.*, Phys. Lett. **B282**, 475 (1992).
39. A.C. Benvenuti *et al.*, Phys. Lett. **B223**, 490 (1989); Phys. Lett. **B223**, 485, (1989); Phys. Lett. **B237**, 592 (1990); and Phys. Lett. **B237**, 599 (1990).
40. M.R. Adams *et al.*, Phys. Rev. **D54**, 3006 (1996).
41. J. Santiago and F.J. Yndurain, Nucl. Phys. **B563**, 45 (1999) `hep-ph/9904344`.
42. K. Adel, F. Barriero, and F.J. Yndurain, Nucl. Phys. **B495**, 221 (1997);

    W.L. Van Neerven, and E.B. Zijlstra, Phys. Lett. **B272**, 127 (1991); Nucl. Phys. **B383**, 525 (1992);

    S. A. Larin *et al.*, Nucl. Phys. **B427**, 41 (1994); Nucl. Phys. **B492**, 338 (1997).

43. M. Arneodo *et al.*, Phys. Lett. **B309**, 222 (1993).
44. M. Virchaux and A. Milsztajn, Phys. Lett. **B274**, 221 (1992).

45. P.Z. Quintas, Phys. Rev. Lett. **71**, 1307 (1993).
46. B. Adeva *et al.*, Phys. Rev. **D58**, 112002 (1998).
47. G. Altarelli *et al.*, Nucl. Phys. **B496**, 337 (1997), hep-ph/9803237.
48. D. Adams *et al.*, Phys. Lett. **B329**, 399 (1995); Phys. Rev. **D56**, 5330 (1998); Phys. Rev. **D58**, 1112001 (1998).
49. K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995); Phys. Lett. **B364**, 61 (1995); Phys. Rev. Lett. **75**, 25 (1995);
    P.L. Anthony *et al.*, Phys. Rev. **D54**, 6620 (1996).
50. A. Airapetian *et al.*, Phys. Lett. **B442**, 484 (1998), hep-ex/99-06035.
51. J.D. Bjorken, Phys. Rev. **148**, 1467 (1966).
52. J. Ellis and M. Karliner, Phys. Lett. **B341**, 397 (1995).
53. A. DeRujula *et al.*, Phys. Rev. **D10**, 1669 (1974);
    E.A. Kurayev, L.N. Lipatov, and V.S. Fadin, Sov. Phys. JETP **45**, 119 (1977);
    Ya.Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28**, 882 (1978).
54. R.D. Ball and S. Forte, Phys. Lett. **B335**, 77 (1994); Phys. Lett. **B336**, 77 (1994);
    H1Collaboration: S. Aid *et al.*, Nucl. Phys. **B470**, 3 (1996).
55. R.D. Ball and S. Forte, hep-ph/9607289.
56. **H1** Collaboration: T. Ahmed *et al.*, Nucl. Phys. **B439**, 471 (1995).
57. S. Catani and F. Hautmann, Nucl. Phys. **B427**, 475 (1994).
58. H. Plothow-Besch, Comp. Phys. Comm. **75**, 396 (1993).
59. H.L. Lai *et al.*, Eur. Phys. J. **C12**, 375 (2000) hep-ph/9903282;
    A.D. Martin *et al.*, Nucl. Phys. B Proc. Suppl. **79**, 105 (1999) hep-ph/9906231.
60. M.A. Shifman, A.I Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
61. S. Narison and A. Pich, Phys. Lett. **B211**, 183 (1988);
    E. Braaten, S. Narison, and A. Pich, Nucl. Phys. **B373**, 581 (1992).
62. M. Neubert, Nucl. Phys. **B463**, 511 (1996).
63. F. Le Diberder and A. Pich, Phys. Lett. **B289**, 165 (1992).
64. R. Barate *et al.*, Z. Phys. **C76**, 1 (1997); Z. Phys. **C76**, 15 (1997);
    K. Ackerstaff *et al.*, Eur. Phys. J. **C7**, 571 (1999).
65. T. Coan *et al.*, Phys. Lett. **B356**, 580 (1995).
66. A.L. Kataev and V.V. Starshenko, Mod. Phys. Lett. **A10**, 235 (1995).
67. F. Le Diberder and A. Pich, Phys. Lett. **B286**, 147 (1992).
68. C.J. Maxwell and D.J. Tong, Nucl. Phys. **B481**, 681 (1996).
69. G. Altarelli, Nucl. Phys. **B40**, 59 (1995);
    G. Altarelli, P. Nason, and G. Ridolfi, Z. Phys. **C68**, 257 (1995).
70. S. Narison, Nucl. Phys. **B40**, 47 (1995).

71. S.D. Ellis, Z. Kunszt, and D.E. Soper, Phys. Rev. Lett. **64**, 2121 (1990);
F. Aversa *et al.*, Phys. Rev. Lett. **65**, 401 (1990);
W.T. Giele, E.W.N. Glover, and D. Kosower, Phys. Rev. Lett. **73**, 2019 (1994);
S. Frixione, Z. Kunszt, and A. Signer, Nucl. Phys. **B467**, 399 (1996).
72. F. Abe *et al.*, Phys. Rev. Lett. **77**, 438 (1996);
B. Abbott *et al.*, Phys. Rev. Lett. **82**, 2451 (1999).
73. W.T. Giele, E.W.N. Glover, and J. Yu, Phys. Rev. **D53**, 120 (1996).
74. **CDF** Collaboration reported in [4].
75. **UA1** Collaboration: G. Arnison *et al.*, Phys. Lett. **B177**, 244 (1986).
76. F. Abe *et al.*, Phys. Rev. Lett. **77**, 533 (1996);
ibid., erratum Phys. Rev. Lett. **78**, 4307 (1997);
B. Abbott, Phys. Rev. Lett. **80**, 666 (1998);
S. Abache *et al.*, Phys. Rev. **D53**, 6000 (1996).
77. G. Altarelli, R.K. Ellis, and G. Martinelli, Nucl. Phys. **B143**, 521 (1978).
78. R. Hamberg, W.L. Van Neerven, and T. Matsuura, Nucl. Phys. **B359**, 343 (1991).
79. P. Aurenche, R. Baier, and M. Fontannaz, Phys. Rev. **D42**, 1440 (1990);
P. Aurenche *et al.*, Phys. Lett. **140B**, 87 (1984);
P. Aurenche *et al.*, Nucl. Phys. **B297**, 661 (1988).
80. H. Baer, J. Ohnemus, and J.F. Owens, Phys. Lett. **B234**, 127 (1990).
81. H. Baer and M.H. Reno, Phys. Rev. **D43**, 2892 (1991);
P.B. Arnold and M.H. Reno, Nucl. Phys. **B319**, 37 (1989).
82. F. Abe *et al.*, Phys. Rev. Lett. **73**, 2662 (1994).
83. S. Abachi *et al.*, Phys. Rev. Lett. **77**, 5011 (1996).
84. G. Alverson *et al.*, Phys. Rev. **D48**, 5 (1993).
85. L. Apanasevich *et al.*, Phys. Rev. **D59**, 074007 (1999); Phys. Rev. Lett. **81**, 2642 (1998).
86. W. Vogelsang and A. Vogt, Nucl. Phys. **B453**, 334 (1995);
P. Aurenche *et al.*, Eur. Phys. J. **C9**, 107 (1999).
87. J. Alitti *et al.*, Phys. Lett. **B263**, 563 (1991).
88. R. K . Ellis and S. Veseli, Nucl. Phys. **B511**, 649 (1998);
C.T. Davies, B.R. Webber, and W.J. Stirling, Nucl. Phys. **B256**, 413 (1985).
89. S. Abache *et al.*, Phys. Rev. Lett. **75**, 3226 (1995);
J. Womersley, private communication;
J. Huston, in the *Proceedings to the 29th International Conference on High-Energy Physics (ICHEP98)*, Vancouver, Canada (23–29 Jul 1998) `hep-ph/9901352`.
90. **DØ** Collaboration: B. Abbott *et al.*, Phys. Rev. **D61**, 032004 (2000) `hep-ex/9907009`;

T. Affolder *et al.*, FERMILAB-PUB-99/220.
91. C. Albajar *et al.*, Phys. Lett. **B369**, 46 (1996).
92. M.L. Mangano, P. Nason, and G. Ridolfi, Nucl. Phys. **B373**, 295 (1992).
93. R. Barbieri *et al.*, Phys. Lett. **95B**, 93 (1980);
B.P. Mackenzie and G.P. Lepage, Phys. Rev. Lett. **47**, 1244 (1981).
94. M. Kobel *et al.*, Z. Phys. **C53**, 193 (1992).
95. M. Kobel, DESY-F31-91-03 (thesis).
96. S. Catani and F. Hautmann, Nucl. Phys. **B** (Proc. Supp.), vol. **39BC**, 359 (1995).
97. B. Nemati *et al.*, Phys. Rev. **D55**, 5273 (1997).
98. M. Kramer, Phys. Rev. **D60**, 111503 (1999) `hep-ph/9904416`.
99. M. Voloshin, Int. J. Mod. Phys. **A10**, 2865 (1999).
100. M. Jamin and A. Pich, Nucl. Phys. **B507**, 334 (1997).
101. S.G. Gorishny, A. Kataev, and S.A. Larin, Phys. Lett. **B259**, 114 (1991);
L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett. **66**, 560 (1991).
102. K.G. Chetyrkin and J.H. Kuhn, Phys. Lett. **B308**, 127 (1993).
103. R. Ammar *et al.*, Phys. Rev. **D57**, 1350 (1998).
104. D. Haidt, in *Directions in High Energy Physics,* vol. 14, p. 201, ed. P. Langacker (World Scientific, 1995).
105. G. Quast, presented at the *International Europhysics Conference on High Energy Physics, EPS-HEP99*, Tampere, Finland (July 1999).
106. A. Blondel and C. Verzegrassi, Phys. Lett. **B311**, 346 (1993);
G. Altarelli *et al.*, Nucl. Phys. **B405**, 3 (1993).
107. See the section on “Standard Model of Electroweak Interactions” (Sec. 10) in this *Review.*
108. S. Bethke and J. Pilcher, Ann. Rev. Nucl. and Part. Sci. **42**, 251 (1992).
109. E. Farhi, Phys. Rev. Lett. **39**, 1587 (1977).
110. C.L. Basham *et al.*, Phys. Rev. **D17**, 2298 (1978).
111. J. Ellis, M.K. Gaillard, and G. Ross, Nucl. Phys. **B111**, 253 (1976);
ibid., erratum Nucl. Phys. **B130**, 516 (1977);
P. Hoyer *et al.*, Nucl. Phys. **B161**, 349 (1979).
112. R.K. Ellis, D.A. Ross, T. Terrano, Phys. Rev. Lett. **45**, 1226 (1980);
Z. Kunszt and P. Nason, ETH-89-0836 (1989).
113. S. Bethke *et al.*, Phys. Lett. **B213**, 235 (1988).
114. S. Bethke *et al.*, Nucl. Phys. **B370**, 310 (1992).
115. M.Z. Akrawy *et al.*, Z. Phys. **C49**, 375 (1991).
116. K. Abe *et al.*, Phys. Rev. Lett. **71**, 2578 (1993); Phys. Rev. **D51**, 962 (1995).
117. B. Andersson *et al.*, Phys. Reports **97**, 33 (1983).

118. A. Ali *et al.*, Nucl. Phys. **B168**, 409 (1980);
A. Ali and R. Barreiro, Phys. Lett. **118B**, 155 (1982).
119. B.R. Webber, Nucl. Phys. **B238**, 492 (1984);
G. Marchesini *et al.*, Phys. Comm. **67**, 465 (1992).
120. T. Sjostrand and M. Bengtsson, Comp. Phys. Comm. **43**, 367 (1987);
T. Sjostrand, CERN-TH-7112/93 (1993).
121. O. Adriani *et al.*, Phys. Lett. **B284**, 471 (1992);
M. Akrawy *et al.*, Z. Phys. **C47**, 505 (1990);
B. Adeva *et al.*, Phys. Lett. **B248**, 473 (1990);
D. Decamp *et al.*, Phys. Lett. **B255**, 623 (1991).
122. S. Catani *et al.*, Phys. Lett. **B263**, 491 (1991).
123. S. Catani *et al.*, Phys. Lett. **B269**, 432 (1991);
S. Catani, B.R. Webber, and G. Turnock, Phys. Lett. **B272**, 368 (1991);
N. Brown and J. Stirling, Z. Phys. **C53**, 629 (1992).
124. G. Catani *et al.*, Phys. Lett. **B269**, 632 (1991); Phys. Lett. **B295**, 269 (1992); Nucl. Phys. **B607**, 3 (1993); Phys. Lett. **B269**, 432 (1991).
125. P.D. Acton *et al.*, Z. Phys. **C55**, 1 (1992); Z. Phys. **C58**, 386 (1993).
126. O. Adriani *et al.*, Phys. Lett. **B284**, 471 (1992).
127. D. Decamp *et al.*, Phys. Lett. **B255**, 623 (1992); Phys. Lett. **B257**, 479 (1992).
128. P. Abreu *et al.*, Z. Phys. **C59**, 21 (1993);
M. Acciarri *et al.*, Phys. Lett. **B404**, 390 (1997).
129. Y. Ohnishi *et al.*, Phys. Lett. **B313**, 475 (1993).
130. P.A. Movilla Fernandez *et al.*, Eur. Phys. J. **C1**, 461 (1998);
O. Biebel *et al.*, Phys. Lett. **B459**, 326 (1999).
131. D.A. Bauer *et al.*, SLAC-PUB-6518.
132. L. Gibbons *et al.*, CLNS 95-1323 (1995).
133. **DELPHI** Collaboration: D. Buskulic *et al.*, Z. Phys. **C73**, 409 (1997); Z. Phys. **C73**, 229 (1997).
134. **ALEPH** Collaboration: 99-023 (1999); **DELPHI** Collaboration: 99-114 (1999); **L3** Collaboration: L3-2414 (1999); **OPAL** Collaboration, PN-403 (1999); all submitted to *International Conference on Lepton Photon Interactions*,Stanford, USA (Aug. 1999);
OPAL Collaboration: M. Acciarri *et al.*, Phys. Lett. **B371**, 137 (1996); Z. Phys. **C72**, 191 (1996);
K. Ackerstaff *et al.*, Z. Phys. **C75**, 193 (1997);
ALEPH Collaboration: ALEPH 98-025 (1998).
135. **DELPHI** Collaboration: Phys. Lett. **B456**, 322 (1999).

136. Y.L. Dokshitzer and B.R. Webber Phys. Lett. **B352**, 451 (1995);
Y.L. Dokshitzer *et. al.* Nucl. Phys. **B511**, 396 (1997);
Y.L. Dokshitzer *et. al.* JHEP 9801,011 (1998).
137. P. Nason and B.R. Webber, Nucl. Phys. **B421**, 473 (1994).
138. D. Buskulic *et al.*, Phys. Lett. **B357**, 487 (1995);
ibid., erratum Phys. Lett. **B364**, 247 (1995).
139. **OPAL** Collaboration: R. Akers *et al.*, Z. Phys. **C68**, 203 (1995).
140. **DELPHI** Collaboration: P. Abreu *et al.*, Phys. Lett. **B398**, 194 (1997).
141. E. Witten, Nucl. Phys. **B120**, 189 (1977).
142. C. Berger *et al.*, Nucl. Phys. **B281**, 365 (1987).
143. H. Aihara *et al.*, Z. Phys. **C34**, 1 (1987).
144. M. Althoff *et al.*, Z. Phys. **C31**, 527 (1986).
145. W. Bartel *et al.*, Z. Phys. **C24**, 231 (1984).
146. J. Butterworth, *International Conference on Lepton Photon Interactions*, Stanford, USA (Aug. 1999).
147. K. Ackerstaff *et al.*, Phys. Lett. **B412**, 225 (1997); Phys. Lett. **B411**, 387 (1997).
148. R. Barate *et al.*, Phys. Lett. **B458**, 152 (1999).
149. M. Acciarri *et al.*, Phys. Lett. **B436**, 403 (1998).
150. P. Abreu *et al.*, Z. Phys. **C69**, 223 (1996).
151. K. Muramatsu *et al.*, Phys. Lett. **B332**, 477 (1994).
152. S.K. Sahu *et al.*, Phys. Lett. **B346**, 208 (1995).
153. **H1** Collaboration: C. Adloff *et al.*, Eur. Phys. J. **C13**, 397 (2000) DESY-98-205;
J.Breitweg *et al.*, Eur. Phys. J. **C11**, 35 (1999) DESY 99-057.
154. S. Frixone, Nucl. Phys. **B507**, 295 (1997);
B.W. Harris and J.F. Owens, Phys. Rev. **D56**, 4007 (1997);
M. Klasen and G. Kramr, Z. Phys. **C72**, 107 (1996).
155. D. Graudenz, Phys. Rev. **D49**, 3921 (1994) `hep-ph/9708362`;
J.G. Korner, E. Mirkes, and G.A. Schuler, Int. J. Mod. Phys. **A4**, 1781, (1989);
S. Catani and M. Seymour, Nucl. Phys. **B485**, 291 (1997);
M. Dasgupta and B.R. Webber, Eur. Phys. J. **C1**, 539 (1998) `hep-ph/9704297`;
E. Mirkes and D. Zeppenfeld, Phys. Lett. **B380**, 205 (1996).
156. **H1** Collaboration: T. Ahmed *et al.*, Phys. Lett. **B346**, 415 (1995); Eur. Phys. J. **C5**, 575 (1998).
157. **ZEUS** Collaboration: M. Derrick *et al.*, Phys. Lett. **B363**, 201 (1995).
158. M.Derrick *et al.*, Phys. Lett. **B**, 369 (1996);
H1 Collaboration: T. Ahmed *et al.*, Nucl. Phys. **B435**, 3 (1995).

159. D.M. Janson, M. Albrow, and R. Brugnera `hep-ex/9905537`.
160. P Weisz, Nucl. Phys. **B** (Proc. Supp.) **47**, 71 (1996).
161. C.T.H. Davies *et al.*, Phys. Rev. **D56**, 2755 (1997).
162. S. Aoki *et al.*, Phys. Rev. Lett. **74**, 222 (1995).
163. A. Spitz *et al.*, Phys. Rev. **D60**, 074502 (1999) `hep-lat/9906009`.
164. A.X. El-Khadra *et al.*, Phys. Rev. Lett. **69**, 729 (1992);
A.X. El-Khadra *et al.*, FNAL 94-091/T (1994);
A.X. El-Khadra *et al.*, `hep-ph/9608220`.
165. S. Collins *et al.*, cited by [166].
166. J. Shigemitsu, Nucl. Phys. **B** (Proc. Supp.) **53**, 16 (1997).
167. G.S Bali and K. Schilling, Phys. Rev. **D47**, 661 (1973);
S.P. Booth *et al.*, Phys. Lett. **B294**, 38 (1992).
168. G. de Divitiis *et al.*, Nucl. Phys. **B437**, 447 (1995);
M. Luscher *et al.*, Nucl. Phys. **B413**, 481 (1994).
169. M. Luscher and P. Weisz, Nucl. Phys. **B452**, 234 (1995).
170. P.N. Burrows *et al.*, in *Proceedings of 1996 DPF/DPB Snowmass Summer Study*, ed. D. Cassel *et al.*, (1997).