B^0–$\overline{B}^0$ MIXING

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Formalism in quantum mechanics

There are two neutral B^0–$\overline{B}^0$ meson systems, B_d–$\overline{B}_d$ and B_s–$\overline{B}_s$ (generically denoted B_q–$\overline{B}_q$, $q = s, d$), which exhibit the phenomenon of particle-antiparticle mixing [1]. Such a system is produced in one of its two possible states of well-defined flavor: $|B^0\rangle$ ($\overline{b}q$) or $|\overline{B}^0\rangle$ ($b\overline{q}$). Due to flavor-changing interactions, this initial state evolves into a time-dependent quantum superposition of the two flavor states, $a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle$, satisfying the equation

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{\Gamma}$, known as the mass and decay matrices, describe the dispersive and absorptive parts of B^0–$\overline{B}^0$ mixing. These matrices are hermitian, and CPT invariance requires $M_{11} = M_{22} \equiv M$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$, where M and Γ are the mass and decay width of the B^0 and $\overline{B}^0$ flavor states.

The two eigenstates of the effective hamiltonian matrix $(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma})$ are given by

$$|B_\pm\rangle = p|B^0\rangle \pm q|\overline{B}^0\rangle\,, \tag{2}$$

and correspond to the eigenvalues

$$\lambda_\pm = \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\,, \tag{3}$$

where

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}\,. \tag{4}$$

We choose a convention where $\mathrm{Re}(q/p) > 0$ and $CP|B^0\rangle = |\overline{B}^0\rangle$.

An alternative notation is

$$|B_\pm\rangle = \frac{(1+\epsilon)|B^0\rangle \pm (1-\epsilon)|\overline{B}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}} \qquad \text{with} \qquad \frac{1-\epsilon}{1+\epsilon} = \frac{q}{p}\,. \tag{5}$$

The time dependence of these eigenstates of well-defined masses $M_\pm = \mathrm{Re}(\lambda_\pm)$ and widths $\Gamma_\pm = -2\,\mathrm{Im}(\lambda_\pm)$ is given by the phases $e^{-i\lambda_\pm t} = e^{-iM_\pm t} e^{-\frac{1}{2}\Gamma_\pm t}$: the evolution of a pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ state at $t = 0$ is thus given by

$$|B^0(t)\rangle = g_+(t)\,|B^0\rangle + \frac{q}{p}\,g_-(t)\,|\overline{B}^0\rangle\,, \tag{6}$$

$$|\overline{B}^0(t)\rangle = g_+(t)\,|\overline{B}^0\rangle + \frac{p}{q} g_-(t)\,|B^0\rangle\,, \tag{7}$$

where

$$g_\pm(t) = \frac{1}{2}\left(e^{-i\lambda_+ t} \pm e^{-i\lambda_- t}\right)\,. \tag{8}$$

This means that the flavor states oscillate into each other with time-dependent probabilities proportional to

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma}{2}\,t\right) \pm \cos(\Delta m\,t)\right]\,, \tag{9}$$

where

$$\Delta m = |M_+ - M_-|\,, \qquad \Delta\Gamma = |\Gamma_+ - \Gamma_-|\,. \tag{10}$$

Time-integrated mixing probabilities are only well defined when considering decays to flavor-specific final states, *i.e.* final states f such that the instantaneous decay amplitudes $A_{\overline{f}} = \langle \overline{f}|H|B^0\rangle$ and $\overline{A}_f = \langle f|H|\overline{B}^0\rangle$, where H is the weak interaction hamiltonian, are both zero. Due to mixing, a produced B^0 can decay to the final state $\overline{f}$ (mixed event) in addition to the final state f (unmixed event). Restricting the sample to these two decay channels, the time-integrated mixing probability is given by

$$\begin{aligned}\chi_f^{B^0\to\overline{B}^0} &= \frac{\int_0^\infty |\langle\overline{f}|H|B^0(t)\rangle|^2 dt}{\int_0^\infty |\langle\overline{f}|H|B^0(t)\rangle|^2 dt + \int_0^\infty |\langle f|H|B^0(t)\rangle|^2 dt} \\ &= \frac{|\xi_f|^2(x^2+y^2)}{|\xi_f|^2(x^2+y^2)+2+x^2-y^2}\,,\end{aligned} \tag{11}$$

where we have defined $\xi_f = \frac{q}{p}\frac{\overline{A}_{\overline{f}}}{A_f}$ and

$$x = \frac{\Delta m}{\Gamma}\,, \qquad y = \frac{\Delta\Gamma}{2\Gamma}\,. \tag{12}$$

The mixing probability $\chi_f^{\overline{B}^0\to B^0}$ for the case of a produced $\overline{B}^0$ is obtained by replacing ξ_f with $1/\xi_f$ in Eq. (11). It is different from $\chi_f^{B^0\to\overline{B}^0}$ if $|\xi_f|^2 \neq 1$, a condition reflecting non-invariance under the CP transformation. CP violation in the decay amplitudes is discussed elsewhere [2] and we assume $|\overline{A}_{\overline{f}}| = |A_f|$ from now on. The deviation of $|q/p|^2$ from 1, namely the quantity

$$1 - \left|\frac{q}{p}\right|^2 = \frac{4\,\mathrm{Re}(\epsilon)}{1+|\epsilon|^2} + \mathcal{O}\left(\left(\frac{\mathrm{Re}(\epsilon)}{1+|\epsilon|^2}\right)^2\right), \tag{13}$$

describes CP violation in B^0–$\overline{B}^0$ mixing. As can be seen from Eq. (4), this can occur only if $M_{12} \neq 0$, $\Gamma_{12} \neq 0$ and if the phase difference between M_{12} and Γ_{12} is different from 0 or π.

In the absence of CP violation, $|q/p|^2 = 1$, $\mathrm{Re}(\epsilon) = 0$, the mass eigenstates are also CP eigenstates,

$$CP\,|B_\pm\rangle = \pm|B_\pm\rangle\,, \tag{14}$$

the phases $\varphi_{M_{12}} = \arg(M_{12})$ and $\varphi_{\Gamma_{12}} = \arg(\Gamma_{12})$ satisfy

$$\sin(\varphi_{M_{12}} - \varphi_{\Gamma_{12}}) = 0\,, \tag{15}$$

the mass and decay width differences reduce to

$$\Delta m = 2\,|M_{12}|\,, \qquad \Delta\Gamma = 2\,|\Gamma_{12}|\,, \tag{16}$$

and the time-integrated mixing probabilities $\chi_f^{B^0\to\overline{B}^0}$ and $\chi_f^{\overline{B}^0\to B^0}$ become both equal to

$$\chi = \frac{x^2+y^2}{2(x^2+1)}\,. \tag{17}$$

Standard Model predictions and phenomenology

In the Standard Model, the transitions $B_q^0 \to \overline{B}_q^0$ and $\overline{B}_q^0 \to B_q^0$ are due to the weak interaction. They are described, at the lowest order, by the box diagrams involving two W bosons and two up-type quarks, as is the case for K^0–$\overline{K}^0$ mixing. However, the long range interactions arising from intermediate virtual states are negligible for the neutral B meson systems, because the large B mass is away from the region of hadronic resonances. The calculation of the dispersive and absorptive parts of the box diagrams yields the following predictions for the off-diagonal element of the mass and decay matrices [3],

$$M_{12} = -\frac{G_F^2 m_W^2 \eta_B m_{B_q} B_{B_q} f_{B_q}^2}{12\pi^2} \, S_0(m_t^2/m_W^2)\,(V_{tq}^* V_{tb})^2 \tag{18}$$

$$\begin{aligned}\Gamma_{12} &= \frac{G_F^2 m_b^2 \eta'_B m_{B_q} B_{B_q} f_{B_q}^2}{8\pi} \\ &\quad \times \left[(V_{tq}^* V_{tb})^2 + V_{tq}^* V_{tb} V_{cq}^* V_{cb} \; \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \right. \\ &\qquad \left. + \, (V_{cq}^* V_{cb})^2 \; \mathcal{O}\left(\frac{m_c^4}{m_b^4}\right) \right]\end{aligned} \tag{19}$$

where G_F is the Fermi constant, m_W the W mass, m_i the mass of quark i, and where $m_{B_q} = M$, f_{B_q} and B_{B_q} are the B_q^0 mass, decay constant and bag parameter. The known function $S_0(x_t)$ can be approximated very well with $0.784\, x_t^{0.76}$ [4] and V_{ij} are the elements of the CKM matrix [5]. The QCD corrections η_B and η'_B are of order unity. The only non negligible contributions to M_{12} are from top-top diagrams. The phases of M_{12} and Γ_{12} satisfy

$$\varphi_{M_{12}} - \varphi_{\Gamma_{12}} = \pi + \mathcal{O}\left(\frac{m_c^2}{m_b^2}\right) \tag{20}$$

implying that the mass eigenstates have mass and width differences of opposite signs. This means that, like in the K^0–$\overline{K}^0$ system, the "heavy" state with mass $M_{\rm heavy} = \max(M_+, M_-)$ has a smaller decay width than that of the "light" state with mass $M_{\rm light} = \min(M_+, M_-)$. We thus redefine

$$\Delta m = M_{\rm heavy} - M_{\rm light}\,, \qquad \Delta\Gamma = \Gamma_{\rm light} - \Gamma_{\rm heavy}\,, \tag{21}$$

where Δm is positive by definition and $\Delta\Gamma$ is expected to be positive in the Standard Model.

Furthermore, since Γ_{12} is, like M_{12}, dominated by the top-top diagrams, the quantity

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq \frac{3\pi}{2} \frac{m_b^2}{m_W^2} \frac{1}{S_0(m_t^2/m_W^2)} \sim \mathcal{O}\left(\frac{m_b^2}{m_t^2}\right) \tag{22}$$

is small, and a power expansion of $|q/p|^2$ yields

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin(\varphi_{M_{12}} - \varphi_{\Gamma_{12}}) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right) . \tag{23}$$

Therefore, considering both Eqs. (20) and (22), the CP-violating parameter

$$1 - \left|\frac{q}{p}\right|^2 \simeq \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) \tag{24}$$

is expected to be tiny: $\sim \mathcal{O}(10^{-3})$ for the B_d–$\overline{B}_d$ system and $\lesssim \mathcal{O}(10^{-4})$ for the B_s–$\overline{B}_s$ system [6].

In the approximation of negligible CP violation in the mixing, the ratio $\Delta\Gamma/\Delta m$ is equal to the small quantity $|\Gamma_{12}/M_{12}|$ of Eq. (22); it is hence independent of CKM matrix elements, *i.e.* the same for the B_d–$\overline{B}_d$ and B_s–$\overline{B}_s$ systems. It can be calculated with lattice QCD techniques; typical results are $\sim 5 \times 10^{-3}$ with quoted uncertainties of 30% at least. Given the current experimental knowledge (discussed below) on the mixing parameter x,

$$\begin{cases} x_d = 0.73 \pm 0.03 & (B_d\text{–}\overline{B}_d \text{ system}) \\ x_s \gtrsim 20 \ \text{ at } 95\% \text{ CL} & (B_s\text{–}\overline{B}_s \text{ system}) \end{cases}, \tag{25}$$

the Standard Model thus predicts that $\Delta\Gamma/\Gamma$ is very small for the B_d–$\overline{B}_d$ system (below 1%), but may be quite large for the B_s–$\overline{B}_s$ system (up to $\sim$ 20%). This width difference is caused by the existence of final states to which both the B^0_q and $\overline{B}^0_q$ mesons can decay. Such decays involve $b \to c\bar{c}q$ quark-level transitions, which are Cabibbo-suppressed if $q = d$ and Cabibbo-allowed if $q = s$. If the final states common to B^0_s and $\overline{B}^0_s$ are predominantly CP-even as discussed in Ref. 7, then the B_s–$\overline{B}_s$ mass eigenstate with the largest decay width corresponds to the CP-even eigenstate. Taking Eq. (21) into account, one thus expects $\Gamma_{\text{light}} = \Gamma_+$ and

$$\Delta m_s = M_- - M_+ > 0\,, \qquad \Delta\Gamma_s = \Gamma_+ - \Gamma_- > 0\,. \tag{26}$$

Experimental issues and methods for oscillation analyses

Time-integrated measurements of B^0–$\overline{B}^0$ mixing were published for the first time in 1987 by UA1 [8] and ARGUS [9], and since then by many different experiments. These are typically based on counting same-sign and opposite-sign lepton pairs from the semileptonic decay of the produced $b\bar{b}$ pairs. At high energy colliders, such analyses cannot easily separate the B_d and B_s contributions, therefore experiments at $\Upsilon(4S)$ machines are best suited to measure χ_d.

However, better sensitivity is obtained from time-dependent analyses aimed at the direct measurement of the oscillation frequencies Δm_d and Δm_s, from the proper time distributions of B_d or B_s candidates identified through their decay in (mostly) flavor-specific modes and suitably tagged as mixed or unmixed. This is particularly true for the B_s–$\overline{B}_s$ system where the large value of x_s implies maximal mixing, *i.e.* $\chi_s \simeq 1/2$. In such analyses, performed at high-energy colliders, the neutral B mesons are either partially reconstructed from a charm meson, or selected from a lepton with high transverse momentum with respect to the b jet, or selected from a reconstructed displaced vertex. The proper time $t = \dfrac{m_B}{p}L$ is measured from the distance L between the production vertex and the B decay vertex, as measured with a silicon vertex detector, and from an estimate of the B momentum p.

The statistical significance $\mathcal{S}$ of an oscillation signal can be approximated as [10]

$$\mathcal{S} \approx \sqrt{N/2}\, f_{\rm sig}\,(1-2\eta)\, e^{-(\Delta m\, \sigma_t)^2/2}\,, \qquad (27)$$

where N and $f_{\rm sig}$ are the number of candidates and the fraction of signal in the selected sample, η is the mistag probability, and σ_t is the proper time resolution. The quantity $\mathcal{S}$ decreases very quickly as Δm increases; this dependence is controlled by σ_t, which is therefore a critical parameter for Δm_s analyses. The proper time resolution $\sigma_t \sim \dfrac{m_B}{\langle p\rangle}\sigma_L \oplus t\dfrac{\sigma_p}{p}$ includes a constant contribution due to the decay length resolution σ_L (typically 0.1–0.3 ps), and a term due to the relative momentum resolution $\dfrac{\sigma_p}{p}$ (typically 10–20% for partially reconstructed decays), which increases with proper time.

In order to tag a B candidate as mixed or unmixed, it is necessary to determine its flavor state both at production (initial state) and at decay (final state). The initial and final state mistag probabilities, η_i and η_f, degrade $\mathcal{S}$ by a total factor $(1-2\eta) = (1-2\eta_i)(1-2\eta_f)$. In inclusive lepton analyses, the final state is tagged by the charge of the lepton from $b \to \ell^-$ decays; the biggest contribution to η_f is then due to $\overline{b} \to \overline{c} \to \ell^-$ decays. Alternatively, the charge of a reconstructed charm meson (D^{*-} from B_d^0 or D_s^- from B_s^0), or that of a kaon thought to come from a $b \to c \to s$ decay [11], can be used. For fully inclusive analyses based on topological vertexing, final state tagging techniques include jet charge [12] and charge dipole methods [11].

The initial state tags are somewhat less dependent on the procedure used to select B candidates. They can be divided in two groups: the ones that tag the initial charge of the $\overline{b}$ quark contained in the B candidate itself (same-side tag), and the ones that tag the initial charge of the other b quark produced in the event (opposite-side tag). On the same side, the charge of a track from the primary vertex is correlated with the production state of the B if that track is a decay product of a B^{**} state or the first particle in the fragmentation chain [13,14]. Jet charge techniques work on both sides. Finally, the charge of a lepton from $b \to \ell^-$ or of a kaon from $b \to c \to s$ can be used as opposite side tags, keeping in mind that their performance depends on integrated mixing. At SLC, the beam polarization produced a sizeable forward-backward asymmetry in the $Z \to b\overline{b}$ decays and provided another very interesting and effective initial state tag based on the polar angle of the B candidate [11]. Initial state tags have also been combined to reach $\eta_i \sim 26\%$ at LEP [14,15] or even 16% at SLD [11] with full efficiency. The equivalent figure at CDF is currently $\sim 40\%$ [16].

In the absence of experimental evidence for a width difference, and since $\Delta\Gamma/\Delta m$ is predicted to be very small, oscillation analyses typically neglect $\Delta\Gamma$ and describe the data with the physics functions $\Gamma e^{-\Gamma t}(1 \pm \cos \Delta m t)/2$. As can be seen from Eq. (9), a non zero value of $\Delta\Gamma$ would effectively reduce the oscillation amplitude with a small time-dependent factor that would be very difficult to distinguish from time resolution effects. Whereas measurements of Δm_d are usually extracted from the data using a maximum likelihood fit, no significant B_s–$\overline{B}_s$ oscillations have been seen so far, and all B_s

analyses set lower limits on Δm_s. The original technique used to set such limits was to study the likelihood as a function of Δm_s. However, these limits turned out to be difficult to combine. A method was therefore developed [10], in which a B_s oscillation amplitude $\mathcal{A}$ is measured at each fixed value of Δm_s, using a maximum likelihood fit based on the functions $\Gamma_s e^{-\Gamma_s t}(1 \pm \mathcal{A} \cos \Delta m_s t)/2$. To a very good approximation, the statistical uncertainty on $\mathcal{A}$ is Gaussian and equal to $1/\mathcal{S}$ [10]. Measurements of $\mathcal{A}$ performed at a given value of Δm_s can be averaged easily. If $\Delta m_s = \Delta m_s^{\rm true}$, one expects $\mathcal{A} = 1$ within the total uncertainty $\sigma_{\mathcal{A}}$; however, if Δm_s is far from its true value, a measurement consistent with $\mathcal{A} = 0$ is expected. A value of Δm_s can be excluded at 95% CL if $\mathcal{A} + 1.645\,\sigma_{\mathcal{A}} \leq 1$. If $\Delta m_s^{\rm true}$ is very large, one expects $\mathcal{A} = 0$, and all values of Δm_s such that $1.645\,\sigma_{\mathcal{A}}(\Delta m_s) < 1$ are expected to be excluded at 95% CL. Because of the proper time resolution, the quantity $\sigma_{\mathcal{A}}(\Delta m_s)$ is an increasing function of Δm_s and one therefore expects to be able to exclude individual Δm_s values up to $\Delta m_s^{\rm sens}$, where $\Delta m_s^{\rm sens}$, called here the sensitivity of the analysis, is defined by $1.645\,\sigma_{\mathcal{A}}(\Delta m_s^{\rm sens}) = 1$.

B_d mixing studies

Many B_d–$\overline{B}_d$ oscillations analyses have been performed by the ALEPH [17,12], CDF [13,18], DELPHI [19], L3 [20], OPAL [21] and SLD [11] collaborations. Although a variety of different techniques have been used, the Δm_d results have remarkably similar precision. The systematic uncertainties are not negligible; they are often dominated by sample composition, mistag probability, or b-hadron lifetime contributions. Before being combined, the measurements are adjusted on the basis of a common set of input values, including the b-hadron lifetimes and fractions published in this *Review*. Some measurements are statistically correlated. Systematic correlations arise both from common physics sources (fragmentation fractions, lifetimes, branching ratios of b hadrons), and from purely experimental or algorithmic effects (efficiency, resolution, tagging, background description). Combining all published measurements [17,13,19,20,21] and accounting for all identified correlations as described in Ref. 22 yields $\Delta m_d = 0.478 \pm 0.012(\mathrm{stat}) \pm 0.013(\mathrm{syst})\ \ \mathrm{ps}^{-1}$.

On the other hand, ARGUS and CLEO have published time-integrated measurements based on semileptonic decays [23,24], which average to $\chi_d^{\Upsilon(4S)} = 0.156 \pm 0.024$. The width difference $\Delta\Gamma_d$ could in principle be extracted from the measured value of Γ_d, and the above averages for Δm_d and χ_d (see Eqs. (12) and (17)). The results are however compatible with $\Delta\Gamma_d = 0$, and their precision is still insufficient to provide an interesting constraint. Neglecting $\Delta\Gamma_d$ and using the measured B_d lifetime, the Δm_d and χ_d results are combined to yield the world average

$$\Delta m_d = 0.472 \pm 0.017\ \ \mathrm{ps}^{-1} \tag{28}$$

or, equivalently,

$$\chi_d = 0.174 \pm 0.009\,. \tag{29}$$

Evidence for CP violation in B_d mixing has been searched for, both with semileptonic and inclusive B_d decays, in samples where the initial flavor state is tagged. In the semileptonic case, where the final state tag is also available, the following asymmetry

$$\begin{aligned} &\frac{N(\overline{B}{}^0_d(t) \to \ell^+\nu_\ell X) - N(B^0_d(t) \to \ell^-\overline{\nu}_\ell X)}{N(\overline{B}{}^0_d(t) \to \ell^+\nu_\ell X) + N(B^0_d(t) \to \ell^-\overline{\nu}_\ell X)} \\ &= a_{CP} \simeq 1 - |q/p|^2_d \simeq \frac{4\mathrm{Re}(\epsilon_d)}{1+|\epsilon_d|^2} \end{aligned} \tag{30}$$

has been measured, either in time-integrated analyses at CLEO [24] and CDF [25], or in more recent and sensitive time-dependent analyses at LEP [26,27,28]. In the inclusive case, also investigated at LEP [29,27,30], no final state tag is used, and the asymmetry [31]

$$\begin{aligned} &\frac{N(B^0_d(t) \to \mathrm{all}) - N(\overline{B}{}^0_d(t) \to \mathrm{all})}{N(B^0_d(t) \to \mathrm{all}) + N(\overline{B}{}^0_d(t) \to \mathrm{all})} \\ &\simeq a_{CP}\left[\frac{x_d}{2}\sin(\Delta m_d\, t) - \sin^2\left(\frac{\Delta m_d\, t}{2}\right)\right] \end{aligned} \tag{31}$$

must be measured as a function of the proper time to extract information on CP violation. In all cases asymmetries compatible with zero have been found, with a precision limited by the available statistics. A simple average of all published and preliminary results [24–30] neglecting small possible statistical correlations and assuming half of the systematics to be correlated, is $a_{CP} = -0.017 \pm 0.016$, a result which does not yet constrain the Standard Model.

The Δm_d result of Eq. (28) provides an estimate of $|M_{12}|$ and can be used, together with Eqs. (16) and (18), to extract the modulus of the CKM matrix element V_{td} within the Standard Model [32]. The main experimental uncertainties on the resulting estimate of $|V_{td}|$ come from m_t and Δm_d; however, these are at present completely dominated by the 15–20% uncertainty usually quoted on the hadronic matrix element $f_{B_d}\sqrt{B_{B_d}} \sim 200$ MeV obtained from lattice QCD calculations [33].

B**$_s$* ***mixing studies

B_s–$\overline{B}_s$ oscillation has been the subject of many recent studies from ALEPH [14], CDF [34], DELPHI [35,15], OPAL [36] and SLD [37]. No oscillation signal has been found so far. The most sensitive analyses appear to be the ones based on inclusive lepton samples, and on samples where a lepton and a D_s meson have been reconstructed in the same jet. All results are limited by the available statistics. These are combined to yield the amplitudes $\mathcal{A}$ shown in Fig. 1 as a function of Δm_s [22].

As before, the individual results have been adjusted to common physics inputs, and all known correlations have been accounted for; furthermore, the sensitivities of the inclusive analyses, which depend directly through Eq. (27) on the assumed fraction f_s of B_s mesons in an unbiased sample of weakly-decaying b hadrons, have been rescaled to a common value of $f_s = 0.100 \pm 0.012$ [22]. The combined sensitivity for 95% CL exclusion of Δm_s values is found to be 14.5 ps^{-1}. All values of Δm_s below 14.3 ps^{-1} are excluded at 95% CL, and no deviation from $\mathcal{A} = 0$ is seen in Fig. 1 that would indicate the observation of a signal.

Some Δm_s analyses are still preliminary [15,37]. Using only published results, the combined Δm_s result is

$$\Delta m_s > 10.6 \ \text{ps}^{-1} \quad \text{at 95\% CL}\,, \tag{32}$$

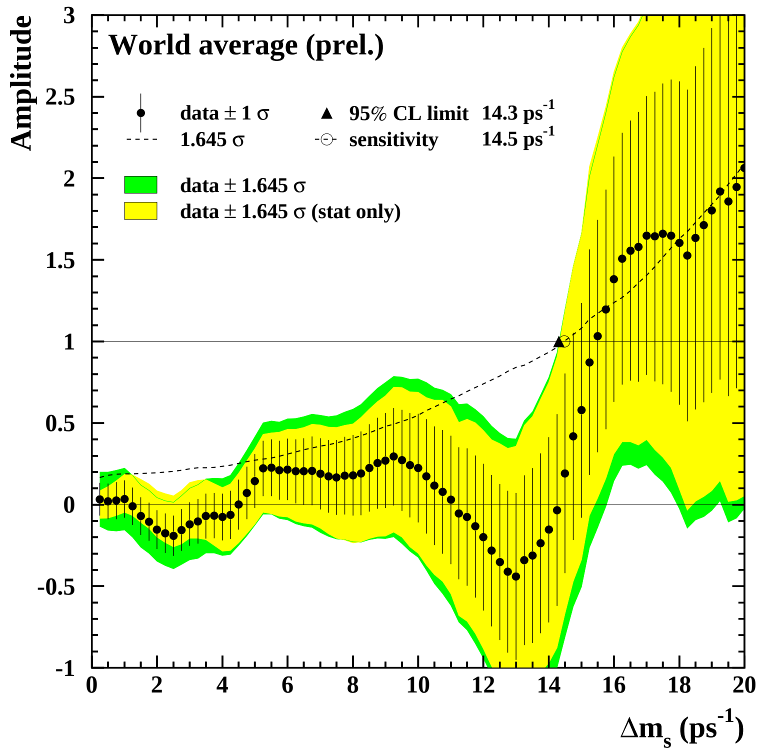

Figure 1: Combined measurements of the B_s oscillation amplitude as a function of Δm_s [22], including all preliminary results available at the end of 1999. The measurements are dominated by statistical uncertainties. Neighboring points are statistically correlated.

with a sensitivity of 12.1 ps^{-1}.

The information on $|V_{ts}|$ obtained, in the framework of the Standard Model, from the combined limit is hampered by the hadronic uncertainty, as in the B_d case. However, many uncertainties cancel in the frequency ratio

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}} \xi^2 \left| \frac{V_{ts}}{V_{td}} \right|^2 , \tag{33}$$

where $\xi = (f_{B_s}\sqrt{B_{B_s}})/(f_{B_d}\sqrt{B_{B_d}})$, of order unity, is currently estimated from lattice QCD with a 5–6% uncertainty [33]. The CKM matrix can be constrained using the experimental results on Δm_d, Δm_s, $|V_{ub}/V_{cb}|$ and ϵ_K, together with theoretical inputs and unitarity conditions [32]. Given the information available from $|V_{ub}/V_{cb}|$ and ϵ_K measurements, the constraint from our knowledge on the ratio $\Delta m_d/\Delta m_s$ is presently more effective in limiting the position of the apex of the CKM unitarity triangle than the one obtained from the Δm_d measurements alone, due to the reduced hadronic uncertainty in Eq. (33). We note also that the Standard Model would not easily accommodate values of Δm_s above $\sim 25\ \mathrm{ps}^{-1}$.

Information on $\Delta\Gamma_s$ can be obtained by studying the proper time distribution of untagged data samples enriched in B_s mesons [38]. In the case of an inclusive B_s selection [39] or a semileptonic B_s decay selection [40,41], both the short- and long-lived components are present, and the proper time distribution is a superposition of two exponentials with decay constants $\Gamma_s \pm \Delta\Gamma_s/2$. In principle, this provides sensitivity to both Γ_s and $(\Delta\Gamma_s/\Gamma_s)^2$. Ignoring $\Delta\Gamma_s$ and fitting for a single exponential leads to an estimate of Γ_s with a relative bias proportional to $(\Delta\Gamma_s/\Gamma_s)^2$. An alternative approach, which is directly sensitive to first order in $\Delta\Gamma_s/\Gamma_s$, is to determine the lifetime of B_s candidates decaying to CP eigenstates; measurements already exist for $B_s^0 \to J/\psi\phi$ [42] and $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ [43], which are mostly CP-even states [7]. An estimate of $\Delta\Gamma_s/\Gamma_s$ has also been obtained directly from a measurement of the $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ branching ratio [43], under the assumption that these decays practically account for all the CP-even final states.

Present data is not precise enough to efficiently constrain both Γ_s and $\Delta\Gamma_s/\Gamma_s$; since the B_s and B_d lifetimes are predicted to be equal within less than a percent [44], an expectation compatible with the current experimental data [45], the constraint $\Gamma_s = \Gamma_d$ can also be used to extract $\Delta\Gamma_s/\Gamma_s$. Applying the combination procedure described in Ref. 22 on the published B_s lifetime results [40,42,46] yields

$$\Delta\Gamma_s/\Gamma_s < 0.65 \quad \text{at 95\% CL} \tag{34}$$

without external constraint, or

$$\Delta\Gamma_s/\Gamma_s < 0.33 \quad \text{at 95\% CL} \tag{35}$$

when constraining $1/\Gamma_s$ to the measured B_d lifetime. These results are not yet precise enough to test Standard Model predictions.

Average b-hadron mixing and b-hadron production fractions

Let f_u, f_d, f_s and $f_{\rm baryon}$ be the B_u, B_d, B_s and b-baryon fractions composing an unbiased sample of weakly-decaying b hadrons produced in high energy colliders. LEP experiments have measured $f_s \times {\rm BR}(B_s^0 \to D_s^- \ell^+ \nu_\ell X)$ [47], ${\rm BR}(b \to \Lambda_b^0) \times {\rm BR}(\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu}_\ell X)$ [48] and ${\rm BR}(b \to \Xi_b^-) \times {\rm BR}(\Xi_b^- \to \Xi^- \ell^- \overline{\nu}_\ell X)$ [49] from partially reconstructed final states including a lepton, $f_{\rm baryon}$ from protons identified in b events [50], and the production rate of charged b hadrons [51]. The various b hadron fractions have also been measured at CDF from electron-charm final states [52]. All the published results have been combined following the procedure and assumptions described in Ref. 22, to yield $f_u = f_d = (38.4 \pm 1.8)\%$, $f_s = (11.7 \pm 3.0)\%$ and $f_{\rm baryon} = (11.5 \pm 2.0)\%$ under the constraints

$$f_u = f_d \qquad \text{and} \qquad f_u + f_d + f_s + f_{\rm baryon} = 1\,. \tag{36}$$

Time-integrated mixing analyses performed with lepton pairs from $b\overline{b}$ events produced at high energy colliders measure the quantity

$$\overline{\chi} = f_d'\,\chi_d + f_s'\,\chi_s\,, \tag{37}$$

where f_d' and f_s' are the fractions of B_d and B_s hadrons in a sample of semileptonic b-hadron decays. Assuming that all b hadrons have the same semileptonic decay width implies $f_q' = f_q/(\Gamma_q \tau_b)$ $(q = s, d)$, where τ_b is the average b-hadron lifetime. Hence $\overline{\chi}$ measurements can be used to improve our knowledge on the fractions f_u, f_d, f_s and $f_{\rm baryon}$.

Combining the above estimates of these fractions with the average $\overline{\chi} = 0.118 \pm 0.005$ (published in this *Review*), χ_d from Eq. (29) and $\chi_s = \frac{1}{2}$ yields, under the constraints of Eq. (36),

$$f_u = f_d = (38.9 \pm 1.3)\%\,, \tag{38}$$

$$f_s = (10.7 \pm 1.4)\%\,, \tag{39}$$

$$f_{\rm baryon} = (11.6 \pm 2.0)\%\,, \tag{40}$$

showing that mixing information substantially reduces the uncertainty on f_s. These results and the averages quoted in Eqs. (28) and (29) for χ_d and Δm_d have been obtained in a consistent way by the B oscillations working group [22], taking into account the fact that many individual measurements of Δm_d depend on the assumed values for the b-hadron fractions.

Summary and prospects

B^0–$\overline{B}^0$ mixing has been a field of intense study in the last few years. The mass difference in the B_d–$\overline{B}_d$ system is very well measured (with an accuracy of $\sim 3.5\%$) but, despite an impressive theoretical effort, the hadronic uncertainty still limits the precision of the extracted estimate of $|V_{td}|$. The mass difference in the B_s–$\overline{B}_s$ system is much larger and still unmeasured. However, the current experimental lower limit on Δm_s already provides, together with Δm_d, a significant constraint on the CKM matrix within the Standard Model. No strong experimental evidence exists yet for the rather large decay width difference expected in the B_s–$\overline{B}_s$ system. It is interesting to recall that the ratio $\Delta\Gamma_s/\Delta m_s$ does not depend on CKM matrix elements in the Standard Model (see Eq. (22)), and that a measurement of either Δm_s or $\Delta\Gamma_s$ could be turned into a Standard Model prediction of the other one.

The LEP and SLD experiments have still not finalized all their B_s oscillation analyses, but a measurement of Δm_s from data collected at the Z pole becomes unlikely. In the near future, the most promising prospects for B_s mixing are from Run II at the Tevatron, where both Δm_s and $\Delta\Gamma_s$ are expected to be measured; CDF will be able to observe B_s oscillations for values of Δm_s up to $\sim 40\ \text{ps}^{-1}$ [53], well above the current Standard Model prediction.

CP violation in B mixing, which has not been seen yet, as well as the phases involved in B mixing, will be further investigated with the large statistics that will become available both at the B factories and at the Tevatron.

B mixing may not have delivered all its secrets yet, because it is one of the phenomena where new physics might very well reveal itself (for example new particles involved in the box diagrams). Theoretical calculations in lattice QCD are becoming more reliable and further progress in reducing hadronic uncertainties is expected. In the long term, a stringent check of the consistency, within the Standard Model, of the B_d and B_s mixing measurements with all other measured observables in B physics (including CP asymmetries in B decays) will be possible, allowing to place limits on new physics or, better, discover new physics.

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