THE Z BOSON

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Precision measurements at the Z-boson resonance using electron-positron colliding beams began in 1989 at the SLC and at LEP. During 1989–95, the four CERN experiments have made high-statistics studies of the Z. The availability of longitudinally polarized electron beams at the SLC since 1993 has enabled a precision determination of the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ that is competitive with the CERN results on this parameter.

The Z-boson properties reported in this section may broadly be categorized as:

- The standard 'lineshape' parameters of the Z consisting of its mass, M_Z, its total width, Γ_Z, and its partial decay widths, Γ(hadrons) and $\Gamma(\ell\overline{\ell})$ where $\ell = e, \mu, \tau, \nu$;
- The b- and c-quark-related partial widths and charge asymmetries which require special techniques;
- Determination of rare Z decay modes and the search for modes that violate known conservation laws.

For the lineshape-related Z properties there are no new published LEP results after those included in the 1994 edition of this compilation. The reason for this is the identification in mid 1995 of a new systematic effect which shifts the LEP energy by a few MeV. This is due to a drift of the dipole field in the LEP magnets caused by parasitic currents generated by electrically powered trains in the Geneva area. The LEP Energy Working Group is studying the implications of this for the Z-lineshape properties which would be obtained after analysis of the high statistics 1993–95 data. The main consequence of this effect is expected to be in the determination of the Z mass.

Details on Z-parameter determination and the study of $Z \to b\overline{b}, c\overline{c}$ at LEP and SLC are given in this note.

The standard 'lineshape' parameters of the Z are determined with increasing precision from an analysis of the production cross sections of these final states in e^+e^- collisions. The $Z \to \nu\overline{\nu}(\gamma)$ state is identified directly by detecting single photon production and indirectly by subtracting the visible partial widths from the total width. Inclusion in this analysis of the forward-backward asymmetry of charged leptons, $A_{FB}^{(0,\ell)}$, of the τ polarization, $P(\tau)$, and its forward-backward asymmetry, $P(\tau)^{fb}$, enables the separate determination of the effective vector ($\overline{g}_V$) and axial vector ($\overline{g}_A$) couplings of the Z to these leptons and the ratio ($\overline{g}_V/\overline{g}_A$) which is related to the effective electroweak mixing angle $\sin^2\overline{\theta}_W$ (see the Standard Model review).

Determination of the b- and c-quark-related partial widths and charge asymmetries involves tagging the b and c quarks. Traditionally this was done by requiring the presence of a prompt lepton in the event with high momentum and high transverse momentum (with respect to the accompanying jet). Precision vertex measurement with silicon detectors has enabled one to do impact parameter and lifetime tagging. Neural-network techniques have also been used to classify events as b or non-b on a statistical basis using event-shape variables. Finally, the presence of a charmed meson (D/D^*) has been used to tag heavy quarks.

***Z*-parameter determination**

LEP is run at a few energy points on and around the Z mass constituting an energy 'scan.' The shape of the cross-section variation around the Z peak can be described by a Breit-Wigner *ansatz* with an energy-dependent total width [1]. The **three** main properties of this distribution, viz., the **position** of the peak, the **width** of the distribution, and the **height** of the peak, determine respectively the values of M_Z, Γ_Z, and $\Gamma(e^+e^-)\times\Gamma(f\overline{f})$, where $\Gamma(e^+e^-)$ and $\Gamma(f\overline{f})$ are the electron and fermion partial widths of the Z. The quantitative determination of these parameters is done by writing analytic expressions for these cross sections in terms of the parameters and fitting the calculated cross sections to the measured ones by varying these parameters, taking properly into account all the errors. Single-photon exchange (σ_γ^0) and γ-Z interference ($\sigma_{\gamma Z}^0$) are included, and the large ($\sim$25 %) initial-state radiation (ISR) effects are taken into account by convoluting the analytic expressions over a 'Radiator Function [1,2]' $H(s,s')$. Thus for the process $e^+e^- \to f\overline{f}$:

$$\sigma_f(s) = \int H(s,s')\ \sigma_f^0(s')\ ds' \tag{1}$$

$$\sigma_f^0(s) = \sigma_Z^0 + \sigma_\gamma^0 + \sigma_{\gamma Z}^0 \tag{2}$$

$$\sigma_Z^0 = \frac{12\pi}{M_Z^2}\ \frac{\Gamma(e^+e^-)\Gamma(f\overline{f})}{\Gamma_Z^2}\ \frac{s\ \Gamma_Z^2}{(s-M_Z^2)^2\ +\ s^2\Gamma_Z^2/M_Z^2} \tag{3}$$

$$\sigma_\gamma^0 = \frac{4\pi\alpha^2(s)}{3s}\ Q_f^2 N_c^f \tag{4}$$

$$\sigma_{\gamma Z}^0 = -\frac{2\sqrt{2}\alpha(s)}{3}\ (Q_f G_F N_c^f g_{Ve} g_{Vf}) \times \frac{(s-M_Z^2)M_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \tag{5}$$

where Q_f is the charge of the fermion, $N_c^f = 3(1)$ for quark (lepton) and g_{Vf} is the neutral vector coupling of the Z to the fermion-antifermion pair $f\overline{f}$.

Since $\sigma_{\gamma Z}^0$ is expected to be much less than σ_Z^0, the LEP collaborations have generally calculated the interference term in the framework of the Standard Model using the best known values of g_V. This fixing of $\sigma_{\gamma Z}^0$ leads to a tighter constraint on M_Z and consequently a smaller error on its fitted value.

Defining

$$A_f = 2\frac{g_{Vf} \cdot g_{Af}}{(g_{Vf}^2 + g_{Af}^2)} \tag{6}$$

where g_{Af} is the neutral axial-vector coupling of the Z to $f\overline{f}$, the lowest-order expressions for the various lepton-related asymmetries on the Z pole are [3] $A_{FB}^{(0,\ell)} = (3/4)A_e A_f$, $P(\tau) = -A_\tau$, $P(\tau)^{fb} = -(3/4)A_e$, $A_{LR} = A_e$. The full analysis takes into account the energy dependence of the asymmetries. Experimentally A_{LR} is defined as $(\sigma_L - \sigma_R)/(\sigma_L + \sigma_R)$ where $\sigma_{L(R)}$ are the $e^+e^- \to Z$ production cross sections with left- (right)-handed electrons.

In terms of g_A and g_V, the partial decay width of the Z to $f\overline{f}$ can be written as

$$\Gamma(f\overline{f}) = \frac{G_F M_Z^3}{6\sqrt{2}\pi}\,(g_{Vf}^2 + g_{Af}^2)\;N_c^f(1+\delta_{\rm QED})(1+\delta_{\rm QCD}) \quad (7)$$

where $\delta_{\rm QED} = 3\alpha Q_f^2/4\pi$ accounts for final-state photonic corrections and $\delta_{\rm QCD} = 0$ for leptons and $\delta_{\rm QCD} = (\alpha_s/\pi) + 1.409(\alpha_s/\pi)^2 - 12.77(\alpha_s/\pi)^3$ for quarks, α_s being the strong coupling constant at $\mu = M_Z$.

In the above framework, the QED radiative corrections have been explicitly taken into account by convoluting over the ISR and allowing the electromagnetic coupling constant to run [4]: $\alpha(s) = \alpha/(1-\Delta\alpha)$. On the other hand, weak radiative corrections that depend upon the assumptions of the electroweak theory and on the values of the unknown $M_{\rm top}$ and $M_{\rm Higgs}$ are accounted for by **absorbing them into the couplings**, which are then called the *effective* couplings $\overline{g}_V$ and $\overline{g}_A$ (or alternatively the effective parameters of the $\star$ scheme of Kennedy and Lynn [5]).

S-matrix approach to the Z

While practically all experimental analyses of LEP/SLC data have followed the 'Breit-Wigner' approach described above, an alternative S-matrix-based analysis is also possible. The Z, like all unstable particles, is associated with a complex pole in the S matrix. The pole position is process independent and gauge invariant. The mass, $\overline{M}_Z$, and width, $\overline{\Gamma}_Z$, can be defined in terms of the pole in the energy plane via [6]

$$\overline{s} = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z \quad (8)$$

leading to the relations

$$\overline{M}_Z = M_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \quad (9)$$

$$\approx M_Z\ - 34\ \text{MeV} \quad (10)$$

$$\overline{\Gamma}_Z = \Gamma_Z/\sqrt{1+\Gamma_Z^2/M_Z^2} \quad (11)$$

$$\approx \Gamma_Z\ - 0.9\ \text{MeV} \quad (12)$$

Some authors [7] choose to define the Z mass and width via

$$\overline{s} = (\overline{M}_Z - \frac{i}{2}\overline{\Gamma}_Z)^2 \tag{13}$$

which yields $\overline{M}_Z \approx M_Z - 26$ MeV, $\overline{\Gamma}_Z \approx \Gamma_Z - 1.2$ MeV.

The L3 collaboration at LEP (ACCIARRI 96B) have analyzed their data using the S-matrix approach as defined in Eq. (8), in addition to the conventional one. They observe a downward shift in the Z mass as expected.

Handling the large-angle e^+e^- final state

Unlike other $f\overline{f}$ decay final states of the Z, the e^+e^- final state has a contribution not only from the s-channel but also from the t-channel and s-t interference. The full amplitude is not amenable to fast calculation, which is essential if one has to carry out minimization fits within reasonable computer time. The usual procedure is to calculate the non-s channel part of the cross section separately using the Standard Model program ALIBABA [8] using the measured value of M_{top}, and the 'central' value of M_{Higgs} (300 GeV) and add it to the s-channel cross section calculated as for other channels. This leads to two additional sources of error in the analysis: firstly, the theoretical calculation in ALIBABA itself is known to be accurate to $\sim 0.5\%$, and secondly, there is uncertainty due to the error on M_{top} and the unknown value of M_{Higgs} (60–1000 GeV). These additional errors are propagated into the analysis by including them in the systematic error on the e^+e^- final state.

Errors due to uncertainty in LEP energy determination [9]

The systematic errors related to the LEP energy measurement can be classified as:

- The absolute energy scale error;
- Energy-point-to-energy-point errors due to the non-linear response of the magnets to the exciting currents;
- Energy-point-to-energy-point errors due to possible higher-order effects in the relationship between the dipole field and beam energy;
- Energy reproducibility errors due to various unknown uncertainties in temperatures, tidal effects, corrector settings, RF status *etc.* Since one groups together data taken at 'nominally same' energies in different fills, it can be assumed that these errors are uncorrelated and are reduced by $\sqrt{\overline{N}_{\rm fill}}$ where $\overline{N}_{\rm fill}$ is the (luminosity weighted) effective number of fills at a particular energy point.

At each energy point the last two errors can be summed into one point-to-point error.

Choice of fit parameters

The LEP collaborations have chosen the following primary set of parameters for fitting: M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, R(lepton), $A_{FB}^{(0,\ell)}$, where $R(\text{lepton}) = \Gamma(\text{hadrons})/\Gamma(\text{lepton})$, $\sigma^0_{\rm hadron} = 12\pi\Gamma(e^+e^-)\Gamma(\text{hadrons})/M_Z^2\Gamma_Z^2$. With a knowledge of these fitted parameters and their covariance matrix, any other parameter can be derived. The main advantage of these parameters is that they form the **least correlated** set of parameters, so that it becomes easy to combine results from the different LEP experiments.

Thus, the most general fit carried out to cross section and asymmetry data determines the **nine parameters**: M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, $R(e)$, $R(\mu)$, $R(\tau)$, $A_{FB}^{(0,e)}$, $A_{FB}^{(0,\mu)}$, $A_{FB}^{(0,\tau)}$. Assumption of lepton universality leads to a **five-parameter fit** determining M_Z, Γ_Z, $\sigma^0_{\rm hadron}$, R(lepton), $A_{FB}^{(0,\ell)}$. The use of **only** cross-section data leads to six- or four-parameter fits if lepton universality is or is not assumed, *i.e.*, $A_{FB}^{(0,\ell)}$ values are not determined.

In order to determine the best values of the effective vector and axial vector couplings of the charged leptons to the Z, the above mentioned nine- and five-parameter fits are carried out with added constraints from the measured values of A_τ and A_e obtained from τ polarization studies at LEP and the determination of A_{LR} at SLC.

Combining results from the LEP and SLC experiments [10]

Each LEP experiment provides the values of the parameters mentioned above together with the full covariance matrix. The statistical and experimental systematic errors are assumed to be uncorrelated among the four experiments. The sources of **common** systematic errors are i) the LEP energy uncertainties, and ii) the effect of theoretical uncertainty in calculating the small-angle Bhabha cross section for luminosity determination and in estimating the non-s channel contribution to the large-angle Bhabha cross section. Using this information, a full covariance matrix, V, of all the input parameters is constructed and a combined parameter set is obtained by minimizing $\chi^2 = \Delta^T V^{-1} \Delta$, where Δ is the vector of residuals of the combined parameter set to the results of individual experiments.

Non-LEP measurement of a Z parameter, (*e.g.*, $\Gamma(e^+e^-)$ from SLD) is included in the overall fit by calculating its value using the fit parameters and constraining it to the measurement.

Study of $Z \to b\overline{b}$ and $Z \to c\overline{c}$

In the sector of c- and b-physics the LEP experiments have measured the ratios of partial widths $R_b = \Gamma(Z \to b\overline{b})/\Gamma(Z \to \text{hadrons})$ and $R_c = \Gamma(Z \to c\overline{c})/\Gamma(Z \to \text{hadrons})$ and the forward-backward (charge) asymmetries $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$. Several of the analyses have also determined other quantities, in particular the semileptonic branching ratios $\mathrm{B}(b \to \ell)$ and $\mathrm{B}(b \to c \to \ell^+)$ and the average $B^0\overline{B}^0$ mixing parameter $\overline{\chi}$. The latter measurements do not concern properties of the Z boson and hence they are not covered in this section. However, they are correlated with the electroweak parameters, and since the mixture of b-hadrons is different from the one at the $\Upsilon(4S)$, their values might differ from those measured at the $\Upsilon(4S)$.

All the above quantities are correlated to each other since:

- Several analyses (for example the lepton fits) determine more than one parameter simultaneously;
- Some of the electroweak parameters depend explicitly on the values of other parameters (for example R_b depends on R_c);
- Common tagging and analysis techniques produce common systematic uncertainties.

The LEP Electroweak Heavy Flavour Working Group has then developed [11] a procedure for combining the measurements taking into account known sources of correlation. The combining procedure determines seven parameters: the four parameters of interest in the electroweak sector, R_b, R_c, $A_{FB}^{b\overline{b}}$, and $A_{FB}^{c\overline{c}}$ and, in addition, $\mathrm{B}(b \to \ell)$, $\mathrm{B}(b \to c \to \ell^+)$ and $\overline{\chi}$, to take into account their correlations with the electroweak parameters. Before the fit both the peak and off-peak asymmetries are translated to $\sqrt{s} = 91.26$ GeV using the predicted dependence from ZFITTER [2].

Summary of the measurements and of the various kinds of analysis

The measurements of R_b and R_c fall into two classes. In the first, named single-tag measurement, a method for selecting b and c events is applied and the number of tagged events is counted. The second technique, named double-tag measurement, is based on the following principle: if the number of events with a single hemisphere tagged is N_t and with both hemispheres tagged is N_{tt}, then given a total number of N_{had} hadronic Z decays one has:

$$\frac{N_t}{2N_{\mathrm{had}}} = \varepsilon_b R_b + \varepsilon_c R_c + \varepsilon_{uds}(1 - R_b - R_c) \tag{14}$$

$$\frac{N_{tt}}{N_{\mathrm{had}}} = \mathcal{C}_b \varepsilon_b^2 R_b + \varepsilon_c^2 R_c + \varepsilon_{uds}^2 (1 - R_b - R_c) \tag{15}$$

where ε_b, ε_c, and ε_{uds} are the tagging efficiencies per hemisphere for b, c, and light quark events, and $\mathcal{C}_b \approx 1$ accounts for the fact that the tagging efficiencies between the hemispheres are correlated. Neglecting the c and uds background and the hemisphere correlation, these equations give:

$$\varepsilon_b = 2N_{tt}/N_t \tag{16}$$

$$R_b = N_t^2/(4N_{tt}N_{\mathrm{had}}) \tag{17}$$

The double-tagging method has thus the great advantage that the tagging efficiency is directly derived from the data, reducing the systematic error of the measurement. The backgrounds, dominated by $c\overline{c}$ events, obviously complicate this simple picture, and their level must still be inferred by other means. The rate of charm background in these analyses depends explicitly on the value of R_c. The correlations in the tagging efficiencies between the hemispheres (due for instance to correlations in momentum between the b-hadrons in the two hemispheres) are small but nevertheless lead to further systematic uncertainties.

The measurements in the b- and c-sector can be grouped in the following categories:

- Lepton fits which use hadronic events with one or more leptons in the final state. Each analysis usually gives several electroweak parameters chosen among: R_b, R_c, $A_{FB}^{b\overline{b}}$, $A_{FB}^{c\overline{c}}$, $\mathrm{B}(b \to \ell)$, $\mathrm{B}(b \to c \to \ell^+)$ and $\overline{\chi}$. The output parameters are then correlated. The dominant sources of systematics are due to lepton identification, to other semileptonic branching ratios and to the modelling of the semileptonic decay;
- Event shape tag for R_b (both single and double-tagging have been used);
- Lifetime (and lepton) double-tagging measurements of R_b. These are the most precise measurements of R_b and obviously dominate the combined result. The main sources of systematics come from the assumed properties of the $c\overline{c}$ events and from estimating the hemisphere b-tagging efficiency correlation;

- Measurements of $A_{FB}^{b\bar{b}}$ using lifetime tagged events with a measurement of the jet charge. Their contribution to the combined result has roughly the same weight as the lepton fits;
- Analyses with $D/D^{*\pm}$ to measure R_c. These measurements separate charmed hadrons coming from $b\bar{b}$ and $c\bar{c}$ decays on a statistical basis; thus R_c depends on properties of $b\bar{b}$ events but not on the value of R_b;
- Analyses with $D^{*\pm}$ to measure simultaneously $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$.

Averaging procedure

All the measurements are provided by the LEP Collaborations in the form of tables with a detailed breakdown of the systematic errors of each measurement and its dependence on other electroweak parameters.

The average proceeds via the following steps:

- Define and propagate a consistent set of external inputs such as branching ratios, hadron lifetimes, fragmentation models *etc.* All the measurements are also consistently checked to ensure that all use a common set of assumptions (for instance for the lifetime/jet-charge measurements of asymmetries, where the QCD effects are already included as an inherent part of the analysis, a QCD correction is subtracted before combining);
- Form the full (statistical and systematic) covariance matrix of the measurements. The systematic correlations between different analyses are calculated from the detailed error breakdown in the measurement tables. The correlations relating several measurements made by the same analysis are also considered;
- Take into account any explicit dependence of a measurement on the other electroweak parameters. As an example of this dependence we illustrate the case of the double-tag measurement of R_b,

where c-quarks constitute the main background. The normalization of the charm contribution is not fixed by the data and the measurement of R_b depends on the assumed value of R_c, which can be written as:

$$R_b = R_b^{\mathrm{meas}} + a(R_c)\frac{(R_c - R_c^{\mathrm{used}})}{R_c}\ , \qquad (18)$$

where R_b^{meas} is the result of the analysis which assumed a value of $R_c = R_c^{\mathrm{used}}$ and $a(R_c)$ is the constant which gives the dependence on R_c. It is worth noting that the combining procedure shows that the only significant correlation between any of the resulting electroweak parameters turns out to be just between R_b and R_c. With the data contained in the present Listing the correlation coefficient between these two variables amounts to -0.39;

- Perform a χ^2 minimization with respect to the combined electroweak parameters.

After the fit the average peak asymmetries $A_{FB}^{c\overline{c}}$ and $A_{FB}^{b\overline{b}}$ are corrected for the energy shift and for QED, QCD, γ exchange, and γZ interference effects to obtain the corresponding pole asymmetries $A_{FB}^{0,c}$ and $A_{FB}^{0,b}$. A small correction is also applied to both R_b and R_c to account for the contribution of γ exchange.

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