SEMILEPTONIC DECAYS OF B MESONS

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In this section, we discuss some of the key questions related to semileptonic decays of B mesons: the inclusive semileptonic branching fraction, the determination of $|V_{cb}|$ and $|V_{ub}|$ from inclusive and exclusive measurements, recent progress in exclusive semileptonic decays, and form factor measurements and tests of heavy-quark effective theory. We emphasize the uncertainties that arise in extracting $|V_{cb}|$ and $|V_{ub}|$ from data. Further discussions of these and related issues are given in the references [1,2,3]. Before addressing the experimental progress, we review the formalism of form factors for both D and B exclusive semileptonic decays. Measurements of form factors in semileptonic D decays are included in the D meson Particle Listings.

Form-factor formalism for exclusive semileptonic decays

The amplitude for an exclusive semileptonic process can be constructed from the available four-vectors in the decay and from form factors, which are Lorentz invariant functions of q^2, the square of the mass of the virtual W. Because these functions describe the effect of strong interactions, nonperturbative techniques such as lattice QCD are needed to calculate them. Form factors are generally largest at the maximum value of q^2, where the daughter meson has the smallest recoil velocity and the overlap between the parent- and daughter-meson wave functions is largest. Studies of form factors in D semileptonic decays have focused on the modes $D \to \overline{K}\ell^+\nu_\ell$ and $D \to \overline{K}^*\ell^+\nu_\ell$, which dominate the inclusive semileptonic rate. In B decays, the analogous modes, $B \to \overline{D}\ell^+\nu_\ell$ and $B \to \overline{D}^*\ell^+\nu_\ell$, account for about two-thirds of the inclusive semileptonic rate. The decay $B \to \overline{D}\ell^+\nu_\ell$ has a large background from $B \to \overline{D}^*\ell^+\nu_\ell$, so in B decays, form factor measurements have focused mainly on $B \to \overline{D}^*\ell^+\nu_\ell$. In this section, we discuss the formalism used in form-factor measurements for the decays $P \to P'\ell\nu_\ell$, where P and P' are pseudoscalar mesons, and $P \to V\ell\nu_\ell$, where V is a vector meson.

The differential decay rate for $P(Q\overline{q}) \to P'(q'\overline{q})\,\ell\nu_\ell$ is

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{q'Q}|^2 k_{P'}^3 |f_+(q^2)|^2}{24\pi^3} \ . \tag{1}$$

Here G_F is the Fermi decay constant, $V_{q'Q}$ is the relevant CKM matrix element, $k_{P'}$ is the momentum of P' in the rest frame of the parent meson, and $f_+(q^2)$ is a vector form factor. (Eq. (1) assumes massless charged leptons, which is almost exact for electrons and a very good approximation for muons, but it is not correct for τ's.) The dominant q^2 dependence comes from the p-wave factor $k_{P'}^3$, which can be written in terms of q^2 and the particle masses. This factor increases the rate at low q^2, which is opposite to the q^2 dependence of the form factor f_+.

The exclusive decay rate for $P \to V\ell\nu_\ell$ can be expressed in terms of three q^2-dependent helicity amplitudes, $H_\pm(q^2)$ and $H_0(q^2)$, where the subscripts indicate the helicity of either the virtual W or the vector meson. The rate is given by

$$\begin{aligned}
\frac{d\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos_V\, d\chi} = & \frac{3G_F^2 |V_{q'Q}|^2\, k_V\, q^2}{8(4\pi)^4 M^2} \\
& \Big\{ [(1+\eta\cos\theta_\ell)^2 |H_+(q^2)|^2 + (1-\eta\cos\theta_\ell)^2 |H_-(q^2)|^2] \sin^2\theta_V \\
& + 4\sin^2\theta_\ell \cos^2\theta_V |H_0(q^2)|^2 \\
& - 2\sin^2\theta_\ell \sin^2\theta_V \cos(2\chi) H_+(q^2) H_-(q^2) \\
& - 4\eta \sin\theta_\ell (1+\eta\cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi\, H_+(q^2) H_0(q^2) \\
& + 4\eta \sin\theta_\ell (1-\eta\cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi\, H_-(q^2) H_0(q^2) \Big\} \ .
\end{aligned} \tag{2}$$

Here M is the mass of the parent meson, k_V is the momentum of the vector meson and is a function of q^2, and the factor η=+1 ($\eta = -1$) applies to B (D) decays. The angles θ_ℓ, θ_V, and χ are defined in Fig. 1. The helicity amplitudes $H_\pm$ and H_0 can be expressed in terms of two axial-vector form factors, $A_1(q^2)$ and $A_2(q^2)$, and a vector form factor $V(q^2)$:

$$\begin{aligned}
H_\pm(q^2) = & (M+m) A_1(q^2) \mp \frac{2M\, k_V}{(M+m)} V(q^2) \\
H_0(q^2) = & \frac{1}{2m\sqrt{q^2}} \Big[(M^2 - m^2 - q^2)(M+m) A_1(q^2) \\
& - \frac{4M^2\, k_V^2}{(M+m)} A_2(q^2) \Big] \ ,
\end{aligned} \tag{3}$$

where m is the mass of the daughter meson. The form factors f_+, A_1, A_2, and V are dimensionless.

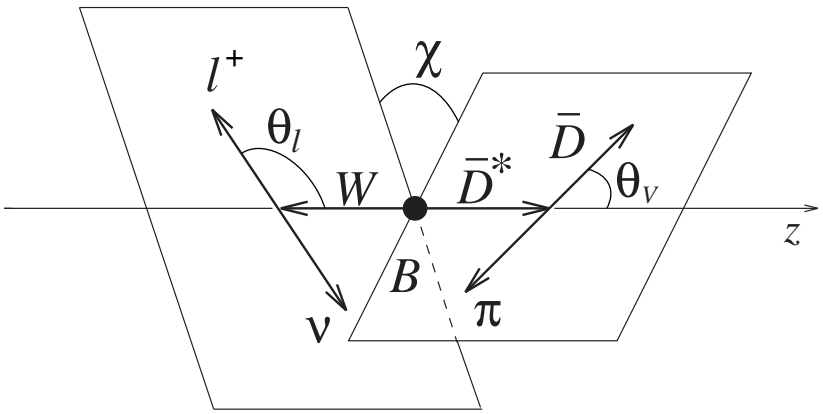

Figure 1: Definition of the angles θ_ℓ, θ_V, and χ. The decay $B \to \overline{D}^* \ell^+ \nu_\ell$ is used as an example. The polar angles θ_ℓ and θ_V are defined in the rest frames of the virtual W and the $\overline{D}^*$, respectively, and χ is the azimuthal angle between the projections of the lepton and the $\overline{D}$ momentum vectors in the plane perpendicular to z.

The $V-A$ coupling results in a larger amplitude to produce a negative-helicity vector meson in $c\overline{q}$ or $b\overline{q}$ decay than one of positive helicity: $|H_-| > |H_+|$. This difference produces a forward-backward asymmetry for the charged lepton in the virtual-W rest frame, since the net angular momentum along the decay axis of the initial heavy meson must be zero. For $D(c\overline{q})$ decays, a positively charged (right-handed) lepton is produced in association with a left-handed daughter s or d quark, resulting in a softer energy spectrum for the charged lepton than for the neutrino after boosting the lepton energy into the D rest frame. (A similar argument shows that the shape of the spectrum is the same for a $\overline{D}$ decay.) For $\overline{B}(b\overline{q})$ decays, a negatively charged (left-handed) lepton is produced in association with a left-handed daughter quark, giving a harder energy spectrum for the charged lepton than for the neutrino in the B rest frame. In $P \to P'\ell\nu_\ell$ decays, there is no asymmetry, since the P' meson can only have helicity zero. Thus, the effect of $V-A$ is to soften the inclusive lepton

spectrum in D decays and to harden it in B decays. It is useful to define rates for decays into specific helicity states:

$$\Gamma_i = \frac{G_F^2 |V_{q'Q}|^2}{96\pi^3} \int dq^2 \; k_V \frac{q^2}{M^2} |H_i(q^2)|^2 \; . \tag{4}$$

Experiments extract various ratios of these rates, including Γ_+/Γ_-, $\Gamma_L/\Gamma_T = \Gamma_0/(\Gamma_- + \Gamma_+)$, the lepton forward-backward asymmetry $A_{FB} = (3\eta/4)(\Gamma_- - \Gamma_+)/\Gamma$, and the polarization parameter $\alpha = 2\Gamma_0/(\Gamma_+ + \Gamma_-) - 1$. Because the form factor A_1 appears in all three helicity amplitudes, the ratios of form factors V/A_1 and A_2/A_1, can be obtained by fitting the measured shape of the distribution of variables q^2, θ_ℓ, θ_V, and χ. The actual values of the form factors can be extracted from these form-factor ratios and the measured total decay rate.

The inclusive semileptonic branching fraction

The b-hadron inclusive semileptonic branching fraction, $\mathcal{B}_{SL}$, has been measured both at the $\Upsilon(4S)$, where the b hadrons are a mixture of B^+ and B^0 mesons, and at the Z, where B_s mesons and b baryons are produced as well. Here $\mathcal{B}_{SL}$ is the branching fraction to either electrons or muons, not their sum. Semileptonic decays to τ leptons are suppressed by phase space and have been observed, within large errors, at the expected level [4,5]. Measurements of $\mathcal{B}_{SL}$ are given in Table 1 for single-lepton measurements at the $\Upsilon(4S)$, in Table 2 for the LEP experiments, and in Table 3 for dilepton measurements at the $\Upsilon(4S)$.

Table 1: Measurements of the inclusive semileptonic branching fraction (%), $\mathcal{B}_{SL} = \mathrm{B}(B \to X\ell^+\nu_\ell)$, averaged over the B mesons produced at the $\Upsilon(4S)$ (B^+ and B^0). These results are based on analyses of the inclusive single-lepton spectrum. Results are given separately for each of the models used to extract $\mathcal{B}_{SL}$. In the ARGUS measurement, the first error combines both statistical and systematic uncertainties; the second error in the ARGUS ACCMM value is due to the extra free parameters present in this model. The fit of the CLEO data using the unmodified ISGW model is poor, so the results from that fit are less reliable. The table also gives the CLEO inclusive branching fraction to charm final states ($X_{\overline{c}}$) only, which is extracted from the same fit. (Sources of error in these measurements are discussed in the text.)

Expt. (1ℓ method)	ACCMM	ISGW	ISGW**
ARGUS [6]	$10.2 \pm 0.5 \pm 0.2$	9.8 ± 0.5	
CRYSTAL BALL [7]	$12.0 \pm 0.5 \pm 0.7$	$11.9 \pm 0.4 \pm 0.7$	
CUSB-II [8]	$10.0 \pm 0.4 \pm 0.3$	$10.0 \pm 0.4 \pm 0.3$	
CLEO-I [9]	$10.5 \pm 0.2 \pm 0.4$	$9.9 \pm 0.1 \pm 0.4$	$11.2 \pm 0.3 \pm 0.4$
CLEO-II (prelim.) [10]	$10.65 \pm 0.05 \pm 0.33$	$10.42 \pm 0.05 \pm 0.33$	$10.98 \pm 0.10 \pm 0.33$
Average	10.51 ± 0.21	10.22 ± 0.20	11.05 ± 0.28
CLEO-II (prelim.) [10] $B \to X_{\overline{c}}\ell^+\nu_\ell$	$10.48 \pm 0.07 \pm 0.33$	$10.41 \pm 0.07 \pm 0.33$	$10.87 \pm 0.10 \pm 0.33$

Table 2: Measurements from LEP experiments of the inclusive b-hadron semileptonic branching fraction, $\mathrm{B}(X_{\overline{b}} \to X\ell^+\nu_\ell)$, where X_b is a hadron containing a b-quark. At the Z, the population of b hadrons includes not only B^0 and B^+ mesons, but also a small fraction of B_s mesons and b baryons. The three errors are statistical, systematic, and the uncertainty due to model dependence.

Expt.	Ref.	$\mathrm{B}(X_{\overline{b}} \to X\ell^+\nu_\ell)\%$
ALEPH	[11]	$11.39 \pm 0.33 \pm 0.33 \pm 0.26$
DELPHI	[12]	$11.06 \pm 0.39 \pm 0.12 \pm 0.19$
L3 (prelim.)	[13]	$11.73 \pm 0.48 \pm 0.28 \pm 0.31$
OPAL	[14]	$10.5 \pm 0.6 \pm 0.4 \pm 0.3$
LEP Avg.		11.2 ± 0.4

The challenge for inclusive measurements is to determine what part of the observed lepton momentum spectrum is due to leptons from b-hadron decay (primary leptons) and what part is due to leptons from charm decay (secondary leptons) or to other sources (misidentified hadrons, photon conversions,

Table 3: Measurements from CLEO and ARGUS experiments of the inclusive B-meson semileptonic branching fraction, using the dilepton method, which reduces the model dependence of the result.

Expt.	Ref.	$\mathcal{B}(B \to X\ell^+\nu_\ell)\%$
ARGUS	[15]	$9.6 \pm 0.5 \pm 0.4$
CLEO II	[16]	$10.49 \pm 0.17 \pm 0.43$
Average		10.19 ± 0.37

$J/\psi(1S)$ decays, *etc.*). The standard technique is to fit the observed lepton momentum spectrum to a sum of the shapes expected for primary and secondary decays, after subtracting out backgrounds from other sources. Thus, a large part of the effort (and uncertainty) in the analysis is in the determination of these shapes.

Experiments at the $\Upsilon(4S)$ (ARGUS and CLEO) use theoretical models to describe the primary lepton spectrum. Some of these models have free parameters that are determined from the fit. The ACCMM model [17], for example, is based on an inclusive calculation of b-quark decay, and it has parameters corresponding to the c-quark mass and the Fermi momentum of the spectator quark, among others. A commonly used exclusive model is ISGW [18], in which the dominant contributions to the primary spectrum are from $B \to \overline{D}\ell^+\nu_\ell$ and $B \to \overline{D}^*\ell^+\nu_\ell$, with some $B \to \overline{D}^{**}\ell^+\nu_\ell$. Here, D^{**} refers to a mixture of p-wave and radially excited charm mesons. CLEO finds [9,10] that the amount of $B \to \overline{D}^{**}\ell^+\nu_\ell$ in this model is too low to adequately describe the lepton momentum spectrum, so a modified version of the ISGW model, ISGW**, has been created. In ISGW**, the D^{**} fraction is allowed to vary, but the D^*-to-D ratio is fixed at the value (2.3) predicted by ISGW. The fit to the CLEO data using ISGW** is significantly better than that using ISGW.

The shape used to describe the secondary lepton spectrum in these fits, although somewhat more complicated to obtain, is based on data. The DELCO charm-decay lepton spectrum [19] is fit to a theoretical model (ACCMM) and then boosted according to the inclusive D-meson momentum spectrum measured

at the $\Upsilon(4S)$. Future measurements should be able to use a charm lepton spectrum obtained by summing the spectra for the known exclusive charm semileptonic modes, which account for most of the inclusive rate.

LEP experiments (ALEPH, DELPHI, L3, and OPAL) measure $\mathcal{B}_{SL}$ by fitting the spectra of p and p_T (the momentum transverse to the jet axis) in single lepton and dilepton events. The shape of the primary spectrum is taken from CLEO or ARGUS, so that model-related uncertainties in these experiments are propagated into the LEP results.

The extraction of the B semileptonic branching fraction from the momentum spectrum of single leptons, therefore, relies on models. In CLEO, which currently has the largest data sample, the spread of values obtained using different models is comparable to the experimental errors. The dominant experimental errors are due to tracking and lepton identification uncertainties.

The ARGUS collaboration [15] has introduced a second method, using dilepton events, that substantially reduces the need for models. One lepton (the "tagging lepton") is required to have high momentum and is thus nearly always primary. The analysis then examines the momentum spectrum of the second lepton in the event. By requiring that both leptons be in the same hemisphere, events in which the two leptons come from the decay chain of a single B meson (produced nearly at rest at the $\Upsilon(4S)$) are effectively removed. Thus, (1) the tagging lepton is primary, and (2) the leptons are from different B mesons. Then, unless mixing occurs, a lepton whose charge is opposite to that of the tagging lepton must be primary, while one with the same charge as the tagging lepton must be secondary. One corrects for mixing by using the known mixing probability. The relative charges of the two leptons, therefore, can be used to separate the primary and secondary spectra of the second lepton. There is a lower momentum cutoff due to experimental acceptance, however, and a small extrapolation, based on models, is required to obtain the total semileptonic rate. The ARGUS and CLEO measurements based on this technique are listed in Table 3. This method also very much

reduces the sensitivity to any possible non-$B\overline{B}$ decays of the $\Upsilon(4S)$, which are assumed to be negligible in the single-lepton method.

The values of $\mathcal{B}_{SL}$ given in Table 1, Table 2, Table 3 are lower than most of the theoretical predictions, which give [20] $\mathcal{B}_{SL} \geq 12.5\%$. Such calculations, however, are difficult partly because they must determine the total hadronic rate, which has uncertainties associated with both perturbative and non-perturbative QCD effects. In particular, enhancements to the $b \to c\overline{c}s$ rate have been discussed as a possible explanation for the low value of $\mathcal{B}_{SL}$. For example, a recent analysis by Bagan, *et al.* [21] predicts values of $\mathcal{B}_{SL}$ in the range 11% to 12% (with large uncertainties), as well as a somewhat larger average number, n_c, of charm quarks per decay than is found by experiment [1]. The problem of $\mathcal{B}_{SL}$ is perhaps best rephrased as the joint problem of understanding $\mathcal{B}_{SL}$ and n_c.

The semileptonic branching fraction can be used to calculate $|V_{cb}|$ using $\mathcal{B}_{SL} = \gamma_{\rm thy}|V_{cb}|^2\tau_B$, where $\gamma_{\rm thy}$ is a constant predicted by theory and τ_B is the appropriate B-hadron lifetime. Whereas the model dependence in the determination of $\mathcal{B}_{SL}$ is associated with the predicted shapes of momentum spectra, the extraction of $|V_{cb}|$ is also sensitive to the uncertainty in $\gamma_{\rm thy}$. It is difficult to assign errors to rate predictions based on quark-model calculations. Quite often, a nominal theoretical error of 20% in the rate is assumed, leading to a 10% theoretical error on $|V_{cb}|$. For example, the value of $|V_{cb}|$ using the ACCMM model and the average of all experiments performing a single-lepton analysis at the $\Upsilon(4S)$ is $|V_{cb}| = 0.041 \pm 0.001$ (expt) ± 0.004 (thy) (assuming the value $\tau_B = 1.55 \pm 0.06$ ps used in Sec. 11, "The Cabibbo-Kobayashi-Maskawa Mixing Matrix"). However, some of the calculations based on heavy-quark expansions suggest that this theoretical uncertainty may be overestimated by a factor of two. HQET-based calculations have been presented by Shifman *et al.* [22], Luke and Savage [23], and Ball *et al.* [24]; this subject is controversial and is reviewed by Richman and Burchat [1], who assign a theoretical uncertainty of ± 0.003.

The lepton endpoint region and determination of $|V_{ub}|$

The determination of $|V_{ub}|$ is one of the most important and challenging measurements in B physics. For $|V_{ub}/V_{cb}| \approx 0.1$, the rate for $B \to X_{\overline{u}}\ell^+\nu_\ell$, where $X_{\overline{u}}$ is a charmless hadronic system, is expected to be only about 1% of the inclusive semileptonic rate. By working in the region at and beyond the lepton-momentum-spectrum endpoint for $B \to X_{\overline{c}}\ell^+\nu_\ell$ processes, however, one gains enormously in sensitivity to $B \to X_{\overline{u}}\ell^+\nu_\ell$ decays.

Although the advantages of working in this endpoint region ($2.3 < p_\ell < 2.6$ GeV/c) are decisive, there are also disadvantages. A major difficulty is the need to convert the measured rate for this tiny portion of phase space into a value of $|V_{ub}|$. This calculation can be performed using either inclusive or exclusive models, but both have substantial uncertainties in predicting the rate in the endpoint region. Inclusive models are expected to be fairly reliable, if one considers a large enough part of phase space, but they may not be reliable in the endpoint region, which some theorists argue [18] is dominated by a small set of exclusive channels ($B \to \rho\ell^+\nu_\ell$, $B^+ \to \omega\ell^+\nu_\ell$, and $B \to \pi\ell^+\nu_\ell$). Alternatively, exclusive models can be used to predict the sum of contributions of individual modes in this region. However, large uncertainties exist in the calculations of the rates for exclusive modes, and some of the observed rate may be due to nonresonant final states [25]. The exclusive calculations here are more difficult than those for $B \to X_{\overline{c}}\ell^+\nu_\ell$: because the u-quark mass is small, the kinematic configuration in which the final-state hadron has zero recoil velocity does not provide a reliable place to normalize the form factors, as it does in $b \to c$ decays. Furthermore, the range of recoil velocities available to the light final-state mesons in a $B \to X_{\overline{u}}\ell^+\nu_\ell$ transition is much larger than for the charm mesons in a $B \to X_{\overline{c}}\ell^+\nu_\ell$ decay. One therefore expects a much larger variation in the form factors. As a result, measurements of $|V_{ub}|$ are currently quite model dependent, and there is substantial variation among values obtained using different models.

The analysis of the endpoint region, although an "inclusive" measurement, is quite different from the measurement of $\mathcal{B}_{SL}$

described in the previous section: at the $\Upsilon(4S)$, nonresonant (continuum) processes produce high-momentum leptons that constitute an enormous background (relative to a $B \to X_{\overline{u}} \ell^+ \nu_\ell$ signal) unless suppressed by kinematic cuts. The signal efficiency of these cuts is model dependent, unlike the very loose cuts used in the analysis of the inclusive lepton spectrum. In particular, the efficiency depends on the q^2 distribution of the signal events, so the value obtained for the rate in the endpoint region depends on the shape assumed for this distribution. The most important sensitivity to models, however, arises when one converts the rate to $|V_{ub}|$.

Table 4 lists the measurements of $|V_{ub}/V_{cb}|$ from CLEO and ARGUS. The CLEO-II studies, which are based on about five times as much data as either the original ARGUS or the CLEO-I analyses, yield values of $|V_{ub}/V_{cb}|$ significantly lower than the earlier measurements.

Table 4: Measurements of $|V_{ub}/V_{cb}|$ using the inclusive rate in the endpoint region. The ARGUS and CLEO-I results are each based on about $200,000$ $b\overline{b}$ events, and the CLEO-II results are based on about $955,000$ $b\overline{b}$ events.

Model	ARGUS [26]	CLEO-I [27]	CLEO-II [28]
ACCMM	0.11 ± 0.012	0.09 ± 0.01	0.076 ± 0.008
ISGW	0.20 ± 0.023	0.15 ± 0.02	0.101 ± 0.010

Progress on exclusive semileptonic decays

In the past year, substantial progress has been made in understanding the exclusive semileptonic decays of B mesons. In particular, the observation of $B \to \pi \ell^+ \nu_\ell$ and $B \to \rho \ell^+ \nu_\ell$ by CLEO represents a milestone for these studies. In this section, we give an overview of the exclusive modes, focusing primarily on recent progress.

Measurements of exclusive semileptonic decays of B mesons are less precise and less complete than those for D mesons. Unlike D decays, where $D \to K \ell^+ \nu_\ell$ and $D \to K^* \ell^+ \nu_\ell$ come close to saturating the Cabibbo-favored rate, the analogous

modes $B \to \overline{D}\ell^+\nu_\ell$ and $B \to \overline{D}^*\ell^+\nu_\ell$ account for only 60% to 70% of the rate for $b \to c\ell\nu_\ell$. The only well-measured semileptonic decay is $B \to \overline{D}^*\ell^+\nu_\ell$, which has the largest branching fraction of all B decays, accounting for about 45% of the semileptonic rate. Recently, there has been some progress in improving the measurement for $B \to \overline{D}\ell^+\nu_\ell$ [29], which has a large background due to feed down from $B \to \overline{D}^*\ell^+\nu_\ell$. Another contrast with D semileptonic decays is that the ratio of $B \to \overline{D}^*\ell^+\nu_\ell$ to $B \to \overline{D}\ell^+\nu_\ell$ is about 2.3, whereas in D decays the analogous vector-to-pseudoscalar ratio is about 0.6. LEP experiments [30,31] have made significant progress in observing the decays $B \to \overline{D}_1\ell^+\nu_\ell$ and $B \to \overline{D}_2^*\ell^+\nu_\ell$, where the use of high-precision vertex detectors has proved to be a powerful technique for reducing combinatorial background.

The most significant recent development has been the observation [32] of exclusive charmless semileptonic decays, which should eventually lead to better determinations of $|V_{ub}|$. For these decays, unlike $B \to X_{\overline{c}}\ell^+\nu_\ell$, model predictions [18] indicate that the rate should be distributed over many exclusive channels, with no dominant modes. The decays with the largest expected branching fractions are $B^0 \to \rho^-\ell^+\nu_\ell$, $B^+ \to \rho^0\ell^+\nu_\ell$, and $B^+ \to \omega\ell^+\nu_\ell$, which in the quark model are predicted to occur in the ratio 2:1:1. The branching fractions for $B^0 \to \pi^-\ell^+\nu_\ell$ and $B^+ \to \pi^0\ell^+\nu_\ell$ are expected to be roughly one-third to one-half of those for the corresponding decays to ρ mesons. For reasons that are largely independent of models, the lepton spectra for the decays to vector mesons are expected to be harder than those for the decays to pseudoscalars. CLEO has now obtained preliminary branching fractions for $B \to \pi\ell^+\nu_\ell$ and $B \to \rho\ell^+\nu_\ell$, which are listed in Table 5. These branching fractions are consistent with predictions based on models, assuming the value of $|V_{ub}|$ obtained from the analysis of the lepton spectrum endpoint.

Measurement of $|V_{cb}|$ from $B \to \overline{D}^*\ell^+\nu_\ell$

Three types of measurements can be used to determine $|V_{cb}|$: (1) the inclusive semileptonic rate (discussed above), (2) the rates for $B \to \overline{D}\ell^+\nu_\ell$ or $B \to \overline{D}^*\ell^+\nu_\ell$, and (3) the partial rate for $B \to \overline{D}^*\ell^+\nu_\ell$ for the region of phase space in which the

Table 5: Preliminary CLEO II branching fractions for exclusive B semileptonic decays to final states without charm. These results assume efficiencies obtained from the ISGW model [18]; if the WSB model [33] is used, the branching fractions are somewhat higher. The first error is statistical and the second systematic. The third error on the $B^0 \to \rho^- \ell^+ \nu_\ell$ branching fraction is due to a possible non-resonant $\pi\pi$ contribution.

Mode	Ref.	Branching fraction
$B^0 \to \rho^- \ell^+ \nu_\ell$	[32]	$(2.28 \pm 0.36 \pm 0.59^{+0.00}_{-0.46}) \times 10^{-4}$
$B^0 \to \pi^- \ell^+ \nu_\ell$	[32]	$(1.34 \pm 0.35 \pm 0.28) \times 10^{-4}$

D^* recoils slowly. The $B \to \overline{D}^* \ell^+ \nu_\ell$ mode is especially well suited to measuring $|V_{cb}|$. With HQET, the rate for this process can be accurately predicted (as a function of $|V_{cb}|^2$) for the kinematic configuration in which the D^* is produced at rest (with the lepton and neutrino back to back in the B rest frame). This configuration occurs when $q^2 = q^2_{\rm max} = (m_B - m_{D^*})^2$. The light constituents of the initial B meson are then essentially undisturbed by the $B \to X_{\overline{c}} \ell^+ \nu_\ell$ transition, at least in the limit where the b and c quark masses are taken to be very large compared with $\Lambda_{\rm QCD}$.

In this heavy-quark symmetry (HQS) limit, all of the $B \to \overline{D}^* \ell^+ \nu_\ell$ form factors are related to a single form factor, the Isgur-Wise function, which depends only on the relative four-velocities of the initial and final hadrons: $\xi = \xi(v_B \cdot v_{D^*})$. Note that $v_B \cdot v_{D^*} = \gamma_{D^*}$, where the relativistic factor γ_{D^*} is measured in the B-meson rest frame. The quantity $v \cdot v'$, where v is the initial and v' is the final meson four-velocity, is often called w or y in the literature. It is linearly related to q^2 by

$$w = v \cdot v' = \frac{M^2 + m^2 - q^2}{2Mm} \ , \tag{5}$$

where M and m are the masses of the parent and daughter mesons.

At zero recoil ($w = 1$ or $q^2 = q^2_{\rm max}$), the normalization of ξ is known in the HQS limit, $\xi(1) = 1$, which means that the decay rate in this configuration can be accurately predicted as a function of $|V_{cb}|^2$. Corrections to this picture arise from

hard-gluon corrections and because the masses of the b and c quarks are not truly infinite. However, the HQS-limit prediction for $B \to \overline{D}^* \ell^+ \nu_\ell$ is partly protected by Luke's theorem [34], which states that at zero recoil there are no leading order $(1/m_Q)$ nonperturbative corrections at $w = 1$, where m_Q is the mass of a heavy quark (c or b). As a consequence, the leading nonperturbative corrections to the zero-recoil decay-rate prediction arise at order $1/m_Q^2$. The calculation of these corrections, which introduces some model dependence, has been the subject of many investigations. The results are generally expressed in terms of a function $\mathcal{F}(w)$ that can be derived from the form factors and which determines the differential rate through [35]

$$\frac{1}{\tau_B} \cdot \frac{d\mathrm{B}(B^0 \to D^{*-} \ell^+ \nu_\ell)}{dw} = \frac{G_F^2}{48\pi^3} m_{D^*}^3 \, (m_B - m_{D^*})^2 \times \sqrt{w^2-1}(w+1)^2 \left[1 + \frac{4w}{w+1} \frac{1 - 2wr + r^2}{(1-r)^2}\right] \times |V_{cb}|^2 \mathcal{F}(w)^2, \quad (6)$$

where $r = m_{D^*}/m_B$. The rate at zero recoil is determined by $\mathcal{F}(1)$, which is predicted to be in the range 0.89 to 0.96 [36,37,22]. These predictions can be used to determine $|V_{cb}|$ from the measured rate at zero recoil. Strictly speaking, there is no phase space for this configuration, so in practice one has to measure the rate in a small region near $w = 1$.

Current experiments have difficulty in measuring the rate in this region due to limited statistics. The rate at zero recoil is therefore obtained by measuring the rate as a function of w and then extrapolating to $w = 1$. This procedure introduces some model dependence because HQET does not predict the w dependence of the form factors, which involves nonperturbative QCD physics. (Several groups are using lattice QCD and QCD sum rules to predict the w dependence and are beginning to obtain interesting results [38,3].) Because the w range is small, however, the form factors are expected to have only modest variation, which is approximately linear. CLEO, ARGUS, ALEPH, and DELPHI use a variety of functional forms to parametrize the w dependence. The different extrapolations to

$w = 1$ lead to a range of values for $|V_{cb}|$. This method leads to $|V_{cb}| = 0.041 \pm 0.003 \pm 0.002$, as discussed in Sec. 11, "The Cabibbo-Kobayashi-Maskawi Mixing Matrix." Thus, the inclusive and zero-recoil methods agree well. It should be noted that $|V_{cb}|$ can also be obtained from the total rates for $B \to \overline{D}\ell^+\nu_\ell$ and $B \to \overline{D}^*\ell^+\nu_\ell$. These values tend to be somewhat lower [1]. However, the theoretical predictions for the full decay rate are expected to be less reliable than those for the zero-recoil configuration of $B \to \overline{D}^*\ell^+\nu_\ell$, since the form factors must be known over the full q^2 range.

Form factor measurements

Both CLEO and ARGUS have used measurements of kinematic distributions to obtain information on the form factors for $B \to \overline{D}^*\ell^+\nu_\ell$. In contrast to D-meson semileptonic decays, however, one expects the predictions of heavy quark effective theory to be applicable to $B \to X_{\overline{c}}\ell^+\nu_\ell$. Here, both the initial- and final-state quarks are heavy compared with the typical hadronic scale set by $\Lambda_{\rm QCD}$.

The differential decay rate for $\overline{B}^0 \to D^{*+}\ell^+\nu_\ell$, $D^{*+} \to D^0\pi^+$ can be expressed in terms of three form factors $A_1(q^2)$, $A_2(q^2)$, and $V(q^2)$, as discussed at the beginning of this article. In the heavy-quark symmetry limit, the form factors are related to the Isgur-Wise function ξ:

$$V(q^2) = A_2(q^2) = \frac{A_1(q^2)}{\left[1 - \frac{q^2}{(m_B+m_{D^*})^2}\right]} = \frac{(m_B + m_{D^*})}{2\sqrt{m_B m_{D^*}}}\,\xi(w)\ . \tag{7}$$

This limit motivates the choice of form factor ratios [3]

$$R_1 \equiv \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right]\frac{V(q^2)}{A_1(q^2)}$$

$$R_2 \equiv \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right]\frac{A_2(q^2)}{A_1(q^2)}\ , \tag{8}$$

which are predicted to be unity, independent of w, in the heavy-quark symmetry limit. For finite heavy-quark masses, R_1 and R_2 are expected to have a mild dependence on w. Note that

these quantities differ from the traditional form-factor ratios (A_2/A_1 and V/A_1) used to describe charm semileptonic decays, where heavy-quark symmetry is not expected to be a good approximation. To consider departures from the heavy-quark symmetry limit but still express the form factors in a manner that makes this limit transparent, one can use

$$A_1(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] h_{A_1}(w) \ ,$$

$$A_2(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} R_2(w)\, h_{A_1}(w) \ ,$$

$$V(q^2) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} R_1(w)\, h_{A_1}(w) \ , \qquad (9)$$

where $h_{A_1}(w) \to \xi(w)$ in the heavy-quark symmetry limit. Departures from this limit produce two effects: deviations of $R_1(1)$ and $R_2(1)$ from unity and a slight variation of R_1 and R_2 with w.

To measure the form factors, CLEO has performed [39] a four-dimensional fit, including correlations, to the kinematic variables q^2, $\cos\theta_\ell$, $\cos\theta_V$, and χ. (The angles are defined at the beginning of this article.) In this fit, R_1 and R_2 were assumed to be constant, while h_{A_1} was assumed to have a linear dependence on w, with slope $\rho^2_{A_1}$: $h_{A_1}(w) = h_{A_1}(1)(1 - \rho^2_{A_1}(w-1))$. The preliminary fit results are

$$R_1 = 1.18 \pm 0.30 \pm 0.12 \ ,$$

$$R_2 = 0.71 \pm 0.22 \pm 0.07 \ ,$$

$$\rho^2_{A_1} = 0.91 \pm 0.15 \pm 0.06 \ . \qquad (10)$$

The values of R_1 and R_2 are in good agreement with predictions based on HQET with corrections [3,40]. The results for R_1 and R_2 are rather insensitive both to the form assumed for $h_{A_1}(w)$ and to the mild w dependence of R_1 and R_2 suggested by these theoretical calculations. However, the value of $\rho^2_{A_1}$ is sensitive to the form of h_{A_1}. Functions with curvature, such as exponentials or pole forms, give somewhat larger values of $\rho^2_{A_1}$.

Earlier results obtained by CLEO [41,42] and ARGUS [43] obtained A_{FB}, the lepton forward-backward asymmetry, and

A_{pol}, the D^* polarization, each integrated over phase space. These results are consistent with the new result discussed above [1] but have lower precision.

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