B^0–$\overline{B}^0$ MIXING AND CP VIOLATION IN B DECAY

(by H. Quinn, SLAC)

The neutral B meson system is like the neutral kaon system, in that two CP-conjugate states exist. For early work on CP violation in the B system see Ref. 1. The mass eigenstates are not CP eigenstates, but are mixtures of the two CP-conjugate quark states, the mixing being due to box diagrams, shown in Figure 1. The two mass eigenstates can be written

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle \ ,$$
$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle \ . \qquad (1)$$

Here H and L stand for Heavy and Light, respectively.

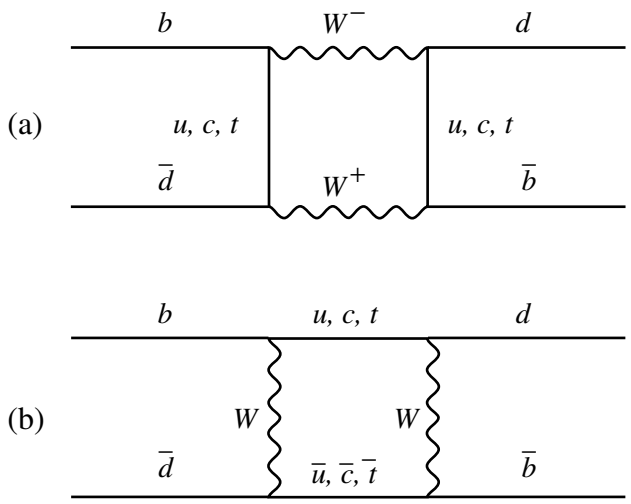

Figure 1: Mixing Diagrams.

Whereas in the kaon case the lifetimes of the two eigenstates are significantly different and the difference in masses between them is small, in the B system it is the mass differences that dominate the physics, and the two states have nearly equal predicted widths (and thus lifetimes).

$$\Gamma = (\Gamma_H + \Gamma_L)/2 \ , \ \Delta\Gamma = \Gamma_H - \Gamma_L \ . \qquad (2)$$

The difference between the widths of the two eigenstates is produced by the contributions from channels to which both B^0

and $\overline{B}^0$ can decay. These have branching ratios of $\mathcal{O}(10^{-3})$ [2]. Furthermore there are contributions of both signs to the difference, so there is no reason that the net effect should be much larger than the individual terms. Conservatively, one expects $\Delta\Gamma/\Gamma \leq 10^{-2}$. Experimentally no effect of a difference in lifetimes has been observed. In what follows, we neglect any effects from $\Delta\Gamma$, except where explicitly stated. We define also

$$M \equiv (M_H + M_L)/2 \ , \quad \Delta M \equiv M_H - M_L \ . \tag{3}$$

The proper time evolution of an initially ($t = 0$) pure B^0 or $\overline{B}^0$ is given by

$$\begin{aligned}
&|B^0_{\rm phys}(t)\rangle = \\
&\quad \exp(-\Gamma t/2)\exp(-iMt) \\
&\qquad \times \{\cos(\Delta M t/2)|B^0\rangle + i(q/p)\sin(\Delta M t/2)|\overline{B}^0\rangle\} \ , \\
&|\overline{B}^0_{\rm phys}(t)\rangle = \\
&\quad \exp(-\Gamma t/2)\exp(-iMt) \\
&\qquad \times \{i(p/q)\sin(\Delta M t/2)|B^0\rangle + \cos(\Delta M t/2)|\overline{B}^0\rangle\} \ .
\end{aligned} \tag{4}$$

The probability that an initial B^0 ($\overline{B}^0$) decays as a $\overline{B}^0$ (B^0) is thus

$$P(t) = \tfrac{1}{2}e^{-\Gamma t}(1 - \cos(\Delta M t)) \tag{5}$$

where we have used $|p/q| = 1$ which is true when we neglect the effects of $\Delta\Gamma$. Time-dependent mixing measurements are now being done; earlier experiments measured only the time-integrated mixing, which is parameterized by a parameter χ_d for B_d (*i.e.*, B^0) and χ_s for B_s (*i.e.*, B^0_s). The quantity χ measures the total probability that a created B^0 decays as a $\overline{B}^0$; it is given by

$$\chi_q = \int_0^\infty P_q(t)dt = \frac{x_q^2}{2(1 + x_q^2)} \ , \tag{6}$$

where $q = d, s$ and $x_q = \frac{\Delta M_q}{\Gamma_q}$, the ratio of the $B^0_q - \overline{B}^0_q$ oscillation frequency to the decay rate. The value of x_d is about 0.7, not very different from the similar quantity for the K^0

which is 0.48. The value of x_s is expected to be much larger, so that the quantity χ_s will be close to its upper limit of 0.5. This means that one cannot determine x_s accurately by measuring χ_s. It will require excellent time resolution to resolve the time-dependent mixing of the B_s^0 system, and thereby determine $\Delta M_{B_s^0}$ [3].

In the B^0–$\overline{B}^0$ mixing section of the B^0 Particle Listings, we list the χ_d measurements, most of which come from $\Upsilon(4S)$ data, and the Δm_{B^0} measurements, which come from Z data. We average these sections separately, but then include the results from both sections in "OUR EVALUATION" of χ_s and $\Delta M_{B_s^0}$. We convert both of these sets of measurements and list them in the x_d section. The x_d values obtained from Δm_{B^0} measurements have a common systematic error due to the error on τ_{B^0}. The averaging takes this common systematic error into account.

In the B_s^0–$\overline{B}_s^0$ mixing section of the B_s^0 Particle Listings, we give measurements of χ_B, the mixing parameter for a high-energy admixture of b-hadrons

$$\chi_B = f_d \frac{\mathcal{B}_d}{\langle \mathcal{B} \rangle} \chi_d + f_s \frac{\mathcal{B}_s}{\langle \mathcal{B} \rangle} \chi_s \ . \tag{7}$$

Here f_d and f_s are the fractions of b hadrons that are produced as B^0 and B_s^0 mesons respectively, and $\mathcal{B}_d$, $\mathcal{B}_s$, and $\langle \mathcal{B} \rangle$ are branching fractions for B_d, B_s, and the b-hadron admixture respectively decaying to the observed mode. If we assume that $\chi_s = 0.5$ and $\mathcal{B}_d/\langle \mathcal{B} \rangle = \mathcal{B}_s/\langle \mathcal{B} \rangle = 1$, Eq. (7) can be used to determine f_s as discussed in the note on "Production and Decay of b-Flavored Hadrons."

CP violation in B decays—Standard Model predictions: There are three symmetries of the strong interactions that are not conserved in weak processes. These are the symmetries C, charge conjugation, which relates particle to antiparticle, P, parity, which relates a left-handed particle to a similar right-handed one, and T, time-reversal invariance, which relates a process or state to the time-reversed process or state. In all field theories the product of these three operations, CPT, is an exact symmetry of the equations of motion. All weak decays

violate P and C, and a very small part of the weak decays also violate the product CP (and thus T). In the Standard Model this CP violation occurs because there is a single phase that remains in the Cabibbo-Kobayashi-Maskawa (CKM) matrix after all possible field redefinitions that can remove such phases have been made. In a minimal two-generation Standard Model no such phase occurs. The presence of CP-violating effects in K decays was interpreted by Kobayashi and Maskawa in 1973 to suggest a third quark generation. Other extensions beyond the minimal Standard Model, such as theories with additional Higgs multiplets, give further ways to introduce CP violation into the theory. Hence it is of great interest to study whether the pattern of CP-violating effects that can be observed in B decays follows the predictions of the minimal Standard Model, or instead requires the introduction of beyond Standard Model effects. In what follows we first discuss the predictions of the minimal Standard Model. Cosmologists attempting to understand the process by which the matter-antimatter asymmetry of the universe arose suggest that additional sources of CP violation may be needed to give the observed baryon to photon ratio of the universe [4]. Many models which go beyond the Standard Model indeed introduce such possibilities; a few of these are discussed in the final section of this review.

The CKM matrix is the matrix of weak couplings in the three generation Standard Model, expressed in the basis of quark mass eigenstates. This matrix, which must be unitary if the three generations are the complete theory, is discussed in some detail in a separate article in this *Review*. Here we need only remind ourselves of some notation that is commonly used in this context. The matrix can be written

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{8}$$

$$\simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \ .$$

The second expression here is a parameterization due to Wolfenstein [5] with $\lambda = \sin(\theta_{\rm Cabibbo})$, which is frequently used in discussing CP-violating effects. It is given here up to terms of order λ^3, since higher order terms in λ are negligible in most situations. For a way to include higher order terms see Ref. 6. The unitarity triangle is a simple geometrical representation of a relationship which results from the unitarity of the three-generation CKM matrix V:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ , \tag{9}$$

(In fact there are nine such relationships given by the unitarity of the CKM matrix, but only three are independent conditions and of those the other two will be more difficult to test because they have one term that is of order λ^2 relative to the others.) The three complex quantities $V_{id}V_{ib}^*$ form a triangle in the complex plane. The three angles of this triangle are labeled

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \ , \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \ ,$$
$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \ . \tag{10}$$

In terms of the Wolfenstein parameters we can also write

$$\tan(\alpha) = \frac{\eta}{(\eta^2 - \rho(1-\rho))} \ , \quad \tan(\beta) = \frac{\eta}{(1-\rho)} \ ,$$
$$\tan(\gamma) = \frac{\eta}{\rho} \ . \tag{11}$$

Notice that the sign as well the magnitude of these angles is meaningful and can be measured.

Figure 2 shows the unitarity triangle, as it is usually drawn, rescaled by $V_{cd}V_{cb}^*$. This makes the base of the triangle real and of unit length and the apex of the triangle is then the point(ρ, η) in the complex plane. A major aim of CP-violation studies of B decays is to make enough independent measurements of the sides and angles that this triangle is overdetermined and thus to check the validity of the Standard Model. Already a number of constraints can be made on the basis of present data on x_d, V_{ub}/V_{cb}, and ϵ in K decays. These constraints have been

discussed in many places in the literature; for a recent summary see Ref. 7. Their exact form depends on the mass of the top quark and on the range of values allowed for the B_K parameter in K decays and the parameter combination $B_B f_B^2$ in B decays.

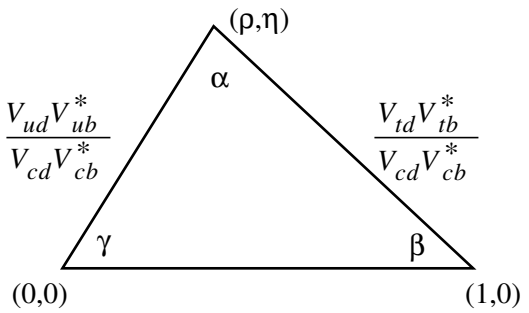

Figure 2: The (rescaled) Unitarity Triangle.

The CKM phases, that is the phases in decay amplitudes which arise because of the phase in the CKM matrix, are often called weak phases, in contrast to the phases which arise from final state rescattering effects, which are referred to as strong phases. When one compares the amplitude for decay to a CP eigenstate to that for the related CP conjugate process, the weak phase ϕ_i of each contribution changes sign, while the strong phase δ_i is unchanged:

$$\mathcal{A} = \Sigma_i \mathcal{A}_i e^{i(\delta_i+\phi_i)} \ , \ \overline{\mathcal{A}} = \Sigma_i \mathcal{A}_i e^{i(\delta_i-\phi_i)} \ . \tag{12}$$

Direct CP violation is a difference in the direct decay rate between $B \to f$ and $\overline{B} \to \overline{f}$ without any contribution from mixing effects. This requires $|\mathcal{A}| \neq |\overline{\mathcal{A}}|$, which occurs only if there is more than one term in the sum Eq. (12), and only if the two terms have both different weak phases and different strong phases. A nonzero result for Re ϵ' in K decay is a direct CP-violation effect. Direct CP violation can occur both in charged channels and in neutral channels.

In the Standard Model direct CP violation occurs when there are two sets of diagrams with different weak phases that contribute to the same decay. There are two major classes of diagrams that contribute to weak decays, tree diagrams and

penguin diagrams, examples of which are shown in Figure 3. Tree diagrams are those in which the W does not reconnect to the quark line from which it was emitted. Penguin diagrams are loop diagrams in which the W is reabsorbed on the same quark line, producing a net change of flavor, and a gluon, photon or Z is emitted from the loop. There may be several different tree diagrams for a given process, namely emission from the heavy quark line accompanied by W decay, W exchange between the initial valence quarks, and/or valence quark-antiquark annihilation to produce the W. However all such contributions which enter a given transition do so with the same CKM (weak) phase. Thus, in the Standard Model, direct CP violation occurs because of interference between tree diagrams and penguin diagrams when these have different weak phases, or, in channels where there are no tree contributions, it can also arise because of different weak phases of different penguin contributions. This latter can be a significant effect for $b \to s\bar{s}d$ decays, as is discussed below.

To calculate the size of expected CP-violation effects one begins from the relevant quark decay diagrams. In general weak-decay amplitudes for b quarks can be divided into two factors: a CKM factor given by the CKM-matrix elements that enter at each W vertex, and a Feynman amplitude from evaluating the diagram. In addition to the suppression from being loop diagrams, penguin diagrams for B decay require the emission of a hard gluon (or photon) from the loop to account for the mass difference between the b quark and the s or d quark produced when the W is reabsorbed. The Feynman amplitude of the penguin diagram is thus suppressed relative to tree diagrams by a factor of order $\alpha_s(m_b)/4\pi$. It is difficult to make firm predictions based on this argument for the strength of the CP-violating effects in exclusive charged B-decay channels because the relationship between the free-quark decay diagrams and the exclusive meson-decay amplitudes is model dependent. Furthermore one cannot reliably predict the strong phases that contribute to the asymmetry.

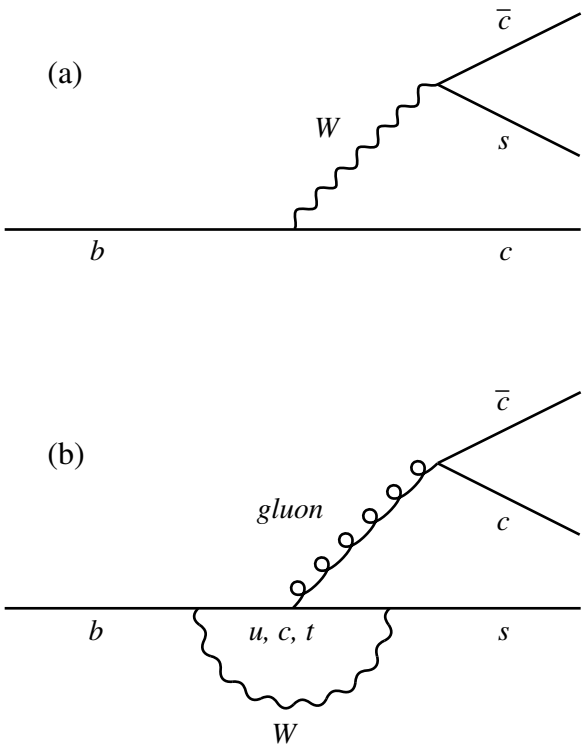

Figure 3: Quark level processes for $b \to c\bar{c}s$: (a) Tree diagram; (b) Penguin diagram. In the case of elelectroweak penguin contributions, the *gluon* is replaced by a Z or a γ.

There are additional CP-violating effects in neutral B decays which arise from interference between the two paths to a given final state f

$$B \to f \text{ or } B \to \overline{B} \to f$$

This CP violation in the interference between the mixed and unmixed decay paths is sometimes called CP violation due to interference between mixing and decay. It is similar to the effect measured by the parameter Im ϵ in K decay. The interference between the two contributions can produce rate differences between the decay and its CP conjugate. These effects are of particular interest because they do not depend upon strong phases and hence the measured asymmetries can be directly related to the CKM phases. In some channels there can be direct CP violation in addition to this effect. In such channels the relationship between the measured asymmetry and the

CKM parameters is more complicated. We will briefly discuss techniques to separate such contributions later in this review.

A third type of CP violation, referred to as indirect CP violation, or CP violation in the mixing, would arise from any difference in the widths $\Delta\Gamma$ of the two mass eigenstates, or more precisely from complex mixing effects that would also give a nonvanishing lifetime difference for the two B mass eigenstates. Such effects are expected to be tiny in the B_d system. For B_s a small difference in the widths is possible, due to the fact that a number of the simplest two-body channels contribute only to a single CP. The difference in widths could be as much as 20% of the total width in the B_s system [8]. In the particular case of semileptonic decays there are no penguin diagram contributions, and hence, in the approximations used throughout the discussion above, the CP-conjugate decay rates are equal. An indirect CP-violating asymmetry would be seen as an charge asymmetry in the same-sign dilepton events produced via mixing from an incoherent state that initially contains a $B^0\overline{B}^0$ pair. This asymmetry vanishes with $\Delta\Gamma$; it is expected to be no larger than 1% in B_d decays. [9].

A simple way to distinguish the three types of CP violation is to note that direct CP violation occurs when $|\overline{\mathcal{A}}/\mathcal{A}| \neq 1$, indirect CP violation requires $|q/p| \neq 1$, but CP violation due the interference between direct decay and decay after mixing can occur when both quantities have unit absolute value; it requires only that their product have a nonzero weak phase [10].

Neutral B decays to CP eigenstates: The decays of neutral B's into CP eigenstates is of particular interest because many of these decays allow clean theoretical interpretation in terms of the parameters of the Standard Model [11]. We denote such a state by f_{CP}, for example $f_{CP} = J/\psi(1S)K_S$ or $f_{CP} = \pi\pi$, and define the amplitudes

$$\mathcal{A}_{f_{CP}} \equiv \left\langle f_{CP}|B^0\right\rangle, \quad \overline{\mathcal{A}}_{f_{CP}} \equiv \left\langle f_{CP}|\overline{B}^0\right\rangle \quad . \tag{13}$$

For convenience let us introduce the quantity $r_{f_{CP}}$

$$r_{f_{CP}} \equiv \frac{q}{p}\,\frac{\overline{\mathcal{A}_{f_{CP}}}}{\mathcal{A}_{f_{CP}}} \quad . \tag{14}$$

In the limit of no CP violation, $r_{f_{CP}} = \pm 1$, where the sign is given by the CP eigenvalue of the particular state f_{CP}. (Note that in the literature the quantity $r_{f_{CP}}$ is frequently denoted by λ, but we have chosen to avoid this notation as it introduces a confusion with the $\lambda = \sin(\theta_{\rm Cabibbo})$ in the Wolfenstein parameterization of the CKM matrix.)

The time-dependent rates for initially pure B^0 or $\overline{B}^0$ states to decay into a final state f_{CP} at time t is then given by:

$$\begin{aligned}
&\left\langle f_{CP}|B^0_{\rm phys}(t)\right\rangle = \\
&\quad \mathcal{A}_{f_{CP}} \exp(-\Gamma t/2)\exp(-iMt) \\
&\qquad \times \left[\cos(\Delta M t/2) + i r_{f_{CP}} \sin(\Delta M t/2)\right] \ , \\
&\left\langle f_{CP}|\overline{B}^0_{\rm phys}(t)\right\rangle = \\
&\quad \mathcal{A}_{f_{CP}} \exp(-\Gamma t/2)\exp(-iMt)(p/q) \\
&\qquad \times \left[i \sin(\Delta M t/2) + r_{f_{CP}} \cos(\Delta M t/2)\right] \ . \qquad (15)
\end{aligned}$$

Thus

$$\begin{aligned}
&\Gamma(B^0_{\rm phys}(t) \to f_{CP}) = \\
&\quad |\mathcal{A}_{f_{CP}}|^2 e^{-\Gamma t}\Bigg[\frac{1+|r_{f_{CP}}|^2}{2} + \frac{1-|r_{f_{CP}}|^2}{2} \\
&\qquad \times \cos(\Delta M t) - {\rm Im}\, r_{f_{CP}} \sin(\Delta M t)\Bigg], \\
&\Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP}) = \\
&\quad |\mathcal{A}_{f_{CP}}|^2 e^{-\Gamma t}\Bigg[\frac{1+|r_{f_{CP}}|^2}{2} - \frac{1-|r_{f_{CP}}|^2}{2} \\
&\qquad \times \cos(\Delta M t) + {\rm Im}\, r_{f_{CP}} \sin(\Delta M t)\Bigg] \ . \qquad (16)
\end{aligned}$$

The time-dependent CP asymmetry is

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) - \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})}{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\rm phys}(t) \to f_{CP})} \qquad (17)$$

and thus

$$a_{f_{CP}}(t) = \frac{(1-|r_{f_{CP}}|^2)\cos(\Delta M t) - 2{\rm Im}\,(r_{f_{CP}})\sin(\Delta M t)}{1+|r_{f_{CP}}|^2} \qquad (18)$$

When the small difference in width of the two B_d states is ignored we can write

$$(q/p)_{B_d} = \frac{(V_{tb}^* V_{td})}{(V_{tb} V_{td}^*)} \ , \tag{19}$$

and thus

$$q/p = e^{-2i\phi_M} \ , \tag{20}$$

where $2\phi_M$ denotes the CKM phase of the B–$\overline{B}$ mixing diagram. Further, when there is no direct CP violation in a channel, that is when all amplitudes that contribute have the same CKM decay-phase, ϕ_D, then $|\mathcal{A}_{f_{CP}}/\overline{\mathcal{A}_{f_{CP}}}| = 1$. In that case $r_{f_{CP}}$ depends on CKM-matrix parameters only, without hadronic uncertainties, and can be written $r_{f_{CP}} = \pm e^{-2i(\phi_D+\phi_M)}$. Then Eq. (18) simplifies to

$$\begin{aligned} a_{f_{CP}}(t) &= \mp \mathrm{Im}\,(r_{f_{CP}}) \sin(\Delta M t) \\ &= \pm \sin(2(\phi_M + \phi_D)) \sin(\Delta M t) \ . \end{aligned} \tag{21}$$

where the overall sign is given by the CP eigenvalue, ± 1, of the final state f_{CP}. The mixing phase ϕ_M and the decay phase ϕ_D are each convention dependent, that is their value can be changed by redefining the phases of some of the quark fields. However Im $r_{f_{CP}}$ depends on convention-independent combinations of CKM parameters only, and thus from Eq. (21) one can directly relate the measured CP-violating asymmetry to the phase of particular combination of CKM-matrix elements in the Standard Model.

In an e^+e^- B collider running at the $\Upsilon(4S)$ resonance, the initial B system is produced in a definite CP-eigenstate state which evolves coherently and thus remains $B^0\overline{B}^0$ until such time as one of the particles decays. The time evolution of the second particle to decay thus begins at the time of the first decay. Events where one B decays to a flavor-tagging mode while the other decays to a CP-study mode can be used to reconstruct the dependence of the asymmetry on the time between the tagging decay and the CP-study mode decay. The tagging decay may be later, in which case the event is assigned a negative time.

Note that the measurement of time dependence is essential at such a machine since, in the interesting cases where Eq. (21) applies, the time-integrated CP asymmetry vanishes.

Hadron machines on the other hand produce uncorrelated B and $\overline{B}$ mesons. In that case the time in the above equations is the time between production and decay, which is always positive, so time-integrated asymmetries do not vanish. Both the tagging particle and the particle decaying to the CP-study mode evolve through mixing, beginning from the time of production. Such machines produce many more B's than will an e^+e^- B factory but the necessity of triggering selections to isolate B events reduces the effective signal somewhat. In addition there are significant backgrounds to contend with in purely hadronic channels, so those channels with leptonic signatures are more readily studied in this environment. The results from the two types of machines will have many complementary features.

Extracting CKM parameters from measured asymmetries: In order to relate the measured asymmetries to the CKM-matrix parameters one looks at the CKM elements that appear in the relevant decay amplitudes and in the mixing diagrams. If the final state of the decay includes a K_S, an additional contribution from the K-mixing phase must be included in relating the measured asymmetry to the CKM parameters.

Table 1 gives the CKM factors for the various b-quark decay channels. For penguin diagrams the table gives the CKM factor of the dominant contributions. Unitarity of the CKM matrix is used to re-express the three different up-type quark loop contributions as a sum of two terms, one of which dominates the contribution. In the case of $b \to d$ processes the subdominant term is suppressed by a term which vanishes with the difference between charm and up quark masses. In the case of $b \to s$ decays the subdominant penguin amplitudes are suppressed by two powers of λ relative to the dominant term given here.

The columns labeled "Sample B_d Modes" and "Sample B_s Modes" list some of the simplest CP-study modes for each case. (These are either CP eigenstates, or modes from

which CP-eigenstate contributions can be isolated, for example by angular analysis.) The columns labeled "Angle" show the particular combination of CKM phases $\phi_M + \phi_D$ that is measured by the CP-violating asymmetry in these decays, given as an angle of the unitarity triangle. For most channels the measured asymmetry in a time-dependent measurement is $\pm\sin(2(\phi_M + \phi_D))\sin(\Delta M t)$. For a time-integrated measurement (uncorrelated production) the asymmetry is $\pm(x_q/(1 + x_q^2))\sin(2(\phi_M + \phi_D))$. The sign is given by the CP eigenvalue of the particular final state studied. (The exception to these statements is the channel DK discussed below.)

In obtaining the results given in the table several simplifying approximations have been used. Terms of higher order in $\lambda = \sin(\theta_{\rm Cabibbo})$ have been dropped. Penguin diagrams that occur at the same order of λ as the corresponding tree diagrams are neglected in stating the relationship of the asymmetry to angles in the unitarity triangle. The comments below the table state where these assumptions are used. Even with these assumptions there are cases where the tree and penguin diagrams are expected to give comparable contributions with different CKM phases. For these decays, as with other direct CP-violating processes, there is no simple relationship between the measured asymmetry and a CKM phase, and thus no entry in the "Angle" columns in Table 1.

The mode $D^0 K^*(892)$ is listed even though it is not a CP eigenstate because it has been shown that an analysis of this mode can be used to extract the angle γ [12]. The same type of analysis can also be applied to charged B decays [13]. However the relationship between the decay asymmetry and the angle is not as simple as Eq. (21) in this case. The result will require accurate measurements of a number of branching ratios.

In the case of the $b \to u\overline{u}d + d\overline{d}d$, the penguin contributions occur at the same order in λ as the tree diagrams and are thus are expected to be small compared to them because of the $\alpha(m_b)/\pi$ suppression factor. The result given in Table 1 makes this approximation. If however this expectation proves false, so that the contributions are comparable, one still may be able to extract a measurement of $\sin(2\alpha)$ from the $\pi^+\pi^-$ asymmetry.

This is achieved by measuring the rates in several isospin-related channels and using a multiparameter fit to separate tree and penguin contributions to the amplitudes [14]. The impact of electroweak penguins, which will not be removed by this analysis [15] is quite small in this channel. [16] The isospin analysis will require measuring the decay rate for channel $\pi^0\pi^0$, which will be a challenge. For the $\rho\pi$ decays, if penguins are not negligible, the restrictions due to isospin can again be used to make a multiparameter fit to the ρ-regions of the Dalitz plot for $\pi^+\pi^-\pi^0$ distribution [17]. The interference between different ρ-charge channels is significant and may provide sufficient information to allow the separation of tree and penguin effects and thus extraction of the parameter α. Such analyses at the very least can be used to test whether the penguin contributions are indeed small enough to be neglected in the determination of α.

In the case $b \to s\overline{s}d$ there are no tree graph contributions. The phase of the dominant penguin contribution is such that, combined with mixing effects, it gives a zero asymmetry for B_d decays and an asymmetry proportional to β for B_s decays. However, Gérard and Hou [18] have pointed out that the sub-dominant penguin terms, proportional to $V_{ub}V_{ud}^*$ can give significant direct CP-violation asymmetries for such channels. Fleischer [19] has estimated that this asymmetry is possibly as large as 50%. While the sub-dominant term in this case would vanish if the masses of the up quark and the charm quark were equal, these estimates, based on the actual quark mass values and operator matrix elements estimated using models, cannot be excluded. Thus, contrary to some comments in the literature, observation of CP-violating asymmetries in channels such as $B_d \to \phi\pi^0$ or $K^0\overline{K}^0$ would not necessarily require beyond-Standard-Model effects to explain them. The B_s decays $b \to c\overline{c}s$ and $b \to s\overline{s}s$ are not affected by this argument. In the first of these the tree terms dominate and the dominant penguin contributions have the same weak phase as those, so the doubly Cabibbo-suppressed sub-dominant penguin contributions are truly negligible. Even in the second case, where there are no tree contributions, the sub-dominant penguin terms are again

doubly Cabibbo-suppressed ($V_{ub}V_{us}^*$ compared to $V_{cb}V_{cs}^*$) and thus the possible Standard Model asymmetry is less than a few percent.

There are some common decay channels of the B^0 and $\overline{B}^0$ which are not CP eigenstates. For example the channel $J/\psi(1S)K^*(892)$ where the $K^*(892) \to K_S\pi^0$, the final state is not a CP eigenstate because both even and odd relative angular momenta between the $J/\psi(1S)$ and the $K^*(892)$ are allowed. If there is sufficient data one can use angular analysis to separate the different CP final states and measure the asymmetry in each [21]. The same applies in many quasi-two-body decays, such as other vector-vector channels, or those with higher-spin particles in final states. The branching ratio to these channels can be significantly larger than the CP-eigenstate (vector-scalar or scalar-scalar) channels with the same quark content. Such angular analyses may therefore be important in achieving accurate values for the parameters α and β.

Additional ways to extract CKM parameters by relationships between rates for channels such as $\pi\pi$, πK that can be extracted using SU(3) invariance have received considerable recent attention in the literature. [20] While these relationships will be interesting to investigate, the uncertainties introduced by SU(3) corrections may be significant. The review by Buras cited above gives a good summary of these ideas.

Beyond-Standard-Model effects: The predictions given above are all for the Standard Model. Models beyond the Standard Model may introduce additional contributions to the mixing amplitudes and thereby destroy the relationships given here; in addition they may introduce further direct CP violation.

One model often used as a "straw man" in evaluating the potential of experimental tests of Standard Model predictions is the superweak model, which was one of the earliest proposals for the mechanism of CP violation; in fact it predates the Standard Model [22]. In the modernized version of this model it is assumed that the CKM matrix is real and that all CP-violating effects arise from a contribution to the mixing that comes from beyond the Standard Model. In this case

all the CP-eigenstate channels for B decay would have the same CP-violating asymmetry (up to a sign which differs for CP-odd and CP-even channels) [23]. This applies even to those channels predicted to have zero asymmetry in the Standard Model, as well as those for which the Standard Model prediction is complicated by the competition between tree and penguin contributions. Observation of significantly different asymmetries in any two neutral B decay CP-eigenstate channels would rule out such a model. In addition the observation of any asymmetry in a charged B decay or in a neutral B decay to a flavor-tagging final state would be evidence for direct CP violation [24] and would exclude the superweak model.

Many other models for the physics beyond the Standard Model have been discussed in the literature [25]. The most common additional CP-violating effect is a new contribution to the mixing process, due for example to charged Higgs contributions. The appearance of such contributions in K mixing is already severely restricted by the neutral-K mass difference. However this does not rule out additional contributions to B mixing that would destroy the relationship between the mixing phase ϕ_M and the CKM-matrix elements. This in turn would lead to violations of the predictions given in Table 1 which are based on this relationship. Models with additional (exotic or fourth generation) quarks would remove the constraints of the three-generation mixing matrix and hence lead to the failure of Eq. (9) and would allow new contributions to $B^0 - \overline{B}^0$ mixing and hence lead to the failure of Eq. (19) [26]. *Any* observed deviations from the relationships predicted by the Standard Model will provide a window on the nature of physics beyond the Standard Model.

While the discussion above stresses those channels in which there is a simple relationship between an observed asymmetry and the parameters of the CKM matrix in the Standard Model, this does not mean that other channels are entirely without interest. To date CP violation has only been observed in the neutral K system. Any observation of CP violation in B decays would be exciting. The Standard Model prediction is that direct CP-violating asymmetries are likely to be at most a few percent,

so large effects in these channels would suggest beyond Standard Model effects. On the other hand, even within the Standard Model the asymmetries due to the interference between decays with and without mixing in the neutral B system can be quite large; current constraints do not rule out cases where Im $(r_{f_{CP}})$ is 1. It is likely that study of the many common decay channels of the B^0 and the $\overline{B}^0$ will greatly expand our understanding of the sources of CP violation.

References

1. A.B. Carter and A.I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. **D23**, 1567 (1981); I. I. Bigi and A. I. Sanda Nucl. Phys. **B193**, 85 (1981) and Nucl. Phys. **B281**, 41 (1987).
2. I.I. Bigi, V.A. Khoze, N.G. Uraltsev, and A.I. Sanda, in *CP Violation*, ed. C. Jarlskog (World Scientific, Singapore, 1989), p. 175.
3. See the references at the end of the B^0 Particle Listings..
4. For a review of this subject see lectures given by M. Dine at the 1992 Theoretical Advanced Study Institute, UC Santa Cruz preprint Preprint SCIPP-92-46.
5. L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1984).
6. A. Buras, M.E. Lautenbacher and G. Ostermaier Phys. Rev. **D50**, 3433 (1994).
7. A. Buras, Proceedings of "Beauty 95" meeting, Oxford, hep-ph/9509329.
8. R. Aleksan, A. Le Yaouanc, L. Oliver, O. Pene and J.C. Raynall, Phys. Lett. **B316**, 567 (1993).
9. T. Altomari, L. Wolfenstein, and J.D. Bjorken, Phys. Rev. **D37**, 1860 (1988).
10. For a review of CP violation in B decays, Y. Nir and H.R. Quinn, Ann. Rev. Nucl. and Part. Sci. **42**, 211 (1992) and references contained therein.
11. I. Dunietz and J.L. Rosner, Phys. Rev. **D34**, 1404 (1986).
12. I. Dunietz, Phys. Lett. **B270**, 75 (1991).
13. M. Gronau and D. Wyler, Phys. Lett. **B265**, 172 (1991).
14. M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
15. N.G. Deshpande and X.-G. He, Phys. Rev. Lett. **74**, 26 (1995).

16. M. Gronau, O.F. Hernandez, D. London and J.L. Rosner hep-ph/5904327(1995).
17. H.R. Quinn and A.E. Snyder, Phys. Rev. **D48**, 2139 (1993).
18. J.M. Gérard and W.S. Hou, Phys. Rev. **D43**, 2909 (1991) and Phys. Lett. **B253**, 478 (1991);
H. Simma, G. Eilam, and D. Wyler, Nucl. Phys. **B352**, 367 (1991).
19. R. Fleischer, Phys. Lett. **B341**, 205 (1994).
20. See for example M. Gronau, J.L. Rosner and D. London, Phys. Rev. Lett. **73**, 21 (1994); M. Gronau, O.F. Hernandez, D. London and J.L. Rosner Phys. Rev. **D50**, 4529 (1994); and M. Gronau and J.L. Rosner, Phys. Rev. Lett. **76**, 1200 (1996).
21. J.R. Dell'Aquila and C.A. Nelson, Phys. Rev. **D33**, 101 (1986);
B. Kayser *et al.*, Phys. Lett. **B237**, 3339 (1990);
I. Dunietz *et al.*, Phys. Rev. **D43**, 2193 (1991).
22. L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).
23. For a discussion see J.M. Soares and L. Wolfenstein, Phys. Rev. **D46**, 256 (1992).
24. M. Bander, D. Silverman and A. Soni, Phys. Rev. Lett. **43**, 242 (1979).
25. For a summary of the predictions of such models see C.O. Dib, D. London, and Y. Nir, Int. J. Mod. Phys. **A6**, 1253 (1991).
26. See for example Y. Nir and D. Silverman, Nucl. Phys. **B345**, 301 (1990);
Y. Nir and D. Silverman, Phys. Rev. **D42**, 1477 (1990).

Table 1: B decay modes for CP studies.

Quark process	Tree CKM	Leading-penguin CKM	Sample B_d modes	B_d angle	Sample B_s modes	B_s angle	Notes
$b \to c\bar{c}s$	$V_{cb}V_{cs}^* = A\lambda^2$	$V_{cb}V_{cs}^* = A\lambda^2$	$J/\psi(1S)K_S$	β	$J/\psi(1S)\eta', D_s\overline{D}_s$	0	(a)
$b \to s\bar{s}s$	0	$V_{cb}V_{cs}^* = A\lambda^2$	ϕK_S	β	$\phi\eta'$	0	(b)
$b \to u\bar{u}s$	$V_{ub}V_{us}^* = A\lambda^4(\rho - i\eta)$	$V_{cb}V_{cs}^* = A\lambda^2$	$K_S\pi^0, K_S\rho^0$	–	$\phi\pi^0, K^+K^-$	–	(c)
$b \to d\bar{d}s$	0	$V_{cb}V_{cs}^* = A\lambda^2$	$K_S\pi^0, K_S\rho^0$	–	$\phi\pi^0, K_SK_S$	–	
$b \to c\bar{c}d$	$V_{cb}V_{cd}^* = -A\lambda^3$	$V_{tb}V_{td}^* = A\lambda^3(1 - \rho + i\eta)$	D^+D^-, $J/\psi(1S)\pi^0$, $D^0\overline{D}^0(\dagger)$	β	$J/\psi(1S)K_S$	0	(d)
$b \to s\bar{s}d$	0	$V_{tb}V_{td}^* = A\lambda^3(1 - \rho + i\eta)$	$\phi\pi^0, K_SK_S$	–	ϕK_S	–	(b)
$b \to u\bar{u}d$	$V_{ub}V_{ud}^* = A\lambda^3(\rho - i\eta)$	$V_{tb}V_{td}^* = A\lambda^3(1 - \rho + i\eta)$	$\pi\pi, \pi\rho, \pi a_1$	α	π^0K_S, ρ^0K_S	γ	(e)
$b \to d\bar{d}d$	0						
$b \to c\bar{u}s$	$V_{cb}V_{us}^* = A\lambda^3$	0	$D_{CP}^0K^*(892)$	γ	$D_{CP}^0\phi$	–	$(f), (g)$
$b \to u\bar{c}s$	$V_{ub}V_{cs}^* = A\lambda^3(\rho - i\eta)$						
$b \to c\bar{u}d$	$V_{cb}V_{ud}^* = A\lambda^2$	0	$D_{CP}^0\pi^0, D_{CP}^0\rho^0$	β	$D_{CP}^0K_S$	0	(g)
$b \to u\bar{c}d$	$V_{ub}V_{cd}^* = A\lambda^4(\rho - i\eta)$						

(a) Tree and penguin contribute with same weak phase.

(b) Penguin only, rare decays.

(c) Tree and penguin compete. Isospin analysis may allow extraction of α, β, for B_d channels, γ, 0 for B_s, where these angles come from tree and penguin contributions respectively.

(d) Ignoring penguin relative to tree.

(e) Ignoring penguin relative to tree, or using isospin analysis.

(f) Self-tagging $K^*(892)$ decay modes can give γ when data from $B_d \to D_{CP}^0$, *i.e.* decays to CP eigenstates, and D^0- or $\overline{D}^0$-identified modes are combined. Similar results for charged $B \to DK$.

(g) Neglecting doubly Cabibbo-supressed $b \to u\bar{c}d$ contributions.

$(\dagger)$ $D^0\overline{D}^0$ from rescattering only, rate expected to be small.