CP VIOLATION IN K_L^0 DECAY

(by L. Wolfenstein, Carnegie-Mellon University and
T. Trippe, LBNL)

Experimentally Measured Parameters

CP violation has been observed in the semi-leptonic decays $K_L^0 \to \pi^{\mp}\ell^{\pm}\nu$ and in the nonleptonic decay $K_L^0 \to 2\pi$. The experimental numbers that have been measured are [1]

$$\delta = \frac{\Gamma(K_L^0 \to \pi^-\ell^+\nu) - \Gamma(K_L^0 \to \pi^+\ell^-\nu)}{\Gamma(K_L^0 \to \pi^-\ell^+\nu) + \Gamma(K_L^0 \to \pi^+\ell^-\nu)} \tag{1a}$$

$$\begin{aligned} \eta_{+-} &= A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-) \\ &= |\eta_{+-}|\, e^{i\phi_{+-}} \end{aligned} \tag{1b}$$

$$\begin{aligned} \eta_{00} &= A(K_L^0 \to \pi^0\pi^0)/A(K_S^0 \to \pi^0\pi^0) \\ &= |\eta_{00}|\, e^{i\phi_{00}} \ . \end{aligned} \tag{1c}$$

Thus there are five real numbers, three magnitudes, and two phases. We list $\delta(\mu)$ for $K_L^0 \to \pi\mu\nu$ and $\delta(e)$ for $K_L^0 \to \pi e\nu$ separately and a weighted average δ. Experimentally for the $K_L^0 \to \pi^0\pi^0$ decay the quantities directly measured (and also of greatest theoretical interest) are $|\eta_{00}/\eta_{+-}|$ and $\phi_{00} - \phi_{+-}$.

Analysis Based on CPT Invariance [2]

CP violation can occur either in the $K^0 - \overline{K}^0$ mixing or in the decay amplitudes. Assuming CPT invariance, the CP violation in the mixing is described by a single parameter ϵ:

$$\begin{aligned} |\, K_L^0\rangle = &\Big[(1+\epsilon)\,|\,K^0\,\rangle - (1-\epsilon)\,|\,\overline{K}^0\,\rangle\Big] \\ &/\left[2(1+|\,\epsilon\,|^2)\right]^{1/2} \end{aligned} \tag{2a}$$

$$\begin{aligned} |\, K_S^0\rangle = &\Big[(1+\epsilon)\,|\,K^0\,\rangle + (1-\epsilon)\,|\,\overline{K}^0\,\rangle\Big] \\ &/\left[2(1+|\,\epsilon\,|^2)\right]^{1/2} \ . \end{aligned} \tag{2b}$$

The decay amplitudes are written

$$\langle\, I=0\,|\,T\,|\,K^0\rangle = e^{i\delta_0}A_0 \tag{3a}$$

$$\langle\, I=2\,|\,T\,|\,K^0\rangle = e^{i\delta_2}A_2 \tag{3b}$$

where δ_I are the $\pi\pi$ scattering phase shifts at the K^0 mass and I is the isospin of the final state. CP violation is measured by $(\mathrm{Im}\ A_I/\mathrm{Re}\ A_I)$. One can then write

$$\eta_{+-} = \epsilon + \epsilon' \tag{4a}$$

$$\eta_{00} = \epsilon - 2\epsilon' \tag{4b}$$

where

$$\epsilon' = \frac{i}{\sqrt{2}}\, e^{i(\delta_2-\delta_0)}\, \frac{\mathrm{Re}\ A_2}{\mathrm{Re}\ A_0} \left[\frac{\mathrm{Im}\ A_2}{\mathrm{Re}\ A_2} - \frac{\mathrm{Im}\ A_0}{\mathrm{Re}\ A_0}\right] \tag{5}$$

neglecting small corrections of order ϵ' times $\mathrm{Re}(A_2/A_0)$. Only two of the three quantities $\epsilon, (\mathrm{Im}\ A_I/\mathrm{Re}\ A_I)$ are meaningful because of the ambiguity in defining the phase of K^0. The standard phase convention due to Wu and Yang [3] sets $\mathrm{Im}\ A_0 = 0$. A nonzero value of ϵ' would provide definite evidence for CP violation in the decay amplitudes independent of phase convention.

By applying CPT invariance and unitarity it is possible to relate δ to ϵ and to determine the phases of ϵ. If one assumes the $\Delta S = \Delta Q$ rule (see below note on the "$\Delta S = \Delta Q$ rule in K^0 Decay") the expression for δ becomes

$$\delta = 2\mathrm{Re}\ \epsilon/(1 + \mid \epsilon \mid^2) \approx 2\mathrm{Re}\ \epsilon\ . \tag{6}$$

This quantity is independent of phase convention and is seen from Eq. (2) to equal $\langle K_L^0 \mid K_S^0\rangle$. The phase of ϵ is given by

$$\phi(\epsilon) \approx \tan^{-1} \frac{(2\Delta m \tau_s)}{\hbar} = 43.59 \pm 0.15^\circ \tag{7a}$$

while Eq. (5) gives

$$\phi(\epsilon') = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^\circ\ . \tag{7b}$$

The approximation in Eq. ($7a$) depends on the neglect of CP violation in decays other than $K^0 \to 2\pi$ and is expected to be good to a few tenths of a degree. Eq. ($7a$) is evaluated using the values of the $K_L^0 - K_S^0$ mass difference $\Delta m = (0.5333 \pm 0.0027) \times 10^{10}\hbar\mathrm{s}^{-1}$ and the K_S^0 mean life $\tau_\mathrm{s} = (0.8926 \pm 0.0012) \times 10^{-10}\mathrm{s}$ from the current edition. The value of the $\pi\pi$ phase shifts is taken from the fit given by Chell and Olsson [4]. The most important point for the analysis is that $\cos[\phi(\epsilon') - \phi(\epsilon)] \simeq 1$. The consequence of this analysis is that only two real quantities need be measured, the magnitude of ϵ and the value of (ϵ'/ϵ) including its sign. The measured quantity $|\eta_{00}/\eta_{+-}|^2$ which is very close to unity, is given to a good approximation by

$$\begin{aligned} |\eta_{00}/\eta_{+-}|^2 &\approx 1 - 6\mathrm{Re}\ (\epsilon'/\epsilon) \\ &= 1 - 6(\epsilon'/\epsilon)\cos\left[\phi(\epsilon') - \phi(\epsilon)\right]\ . \end{aligned} \tag{8}$$

Since the cos in Eq. (8) is expected theoretically to be very close to unity it is customary to say that $|\eta_{00}/\eta_{+-}|^2$ determines ϵ'/ϵ.

It is possible to use the values of ϕ_{+-} and $\phi_{00} - \phi_{+-}$ to set limits on CPT violation. [See Tests of Conservation Laws.]

Models

In the superweak model [5] CP violation is restricted to the mass mixing so that to a high degree of accuracy one expects $\epsilon' = 0$. The phase $\phi(\epsilon)$ is given in this model exactly by Eq. ($7a$) so that this has sometimes been referred to as the superweak phase; however, as noted above, all CPT invariant models give Eq. ($7a$) as a very good approximation. In the Standard Model CP violation is entirely due to the phase in the Cabibbo-Kobayashi-Maskawa mixing matrix [6](q.v.). Since CP violation occurs in first order in decay amplitudes and in second order in mass-matrix mixing, one expects a significant nonzero value of ϵ'. The calculation is uncertain partly because m_t and V_{td} are not well known and primarily because of the difficulty of estimating hadronic matrix elements [7]. The theoretical results for ϵ'/ϵ in the Standard Model are generally in the range from 10^{-4} to 3×10^{-3}.

****

THE REMAINDER OF THIS REVIEW HAS NOT BEEN UPDATED SINCE THE 1994 EDITION BUT WILL BE UPDATED BEFORE THE 1996 EDITION.

****

Fitting procedures

We list measurements of $|\eta_{+-}|$, $|\eta_{00}|$, and $|\eta_{00}/\eta_{+-}|$. Independent information on $|\eta_{+-}|$ and $|\eta_{00}|$ can be obtained from measurements of the K_L^0 and K_S^0 lifetimes (τ) and branching ratios (B) to $\pi\pi$, using the relations

$$|\eta_{+-}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau(K_L^0)} \; \frac{\tau(K_S^0)}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}\right]^{1/2} \;, \qquad (9a)$$

$$|\eta_{00}| = \left[\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\tau(K_L^0)} \; \frac{\tau(K_S^0)}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)}\right]^{1/2} \;. \qquad (9b)$$

We approximate a global fit to these independent sources by first performing two independent fits: 1) BRFIT, a fit to the K_L^0 branching ratios, rates, and mean life, and 2) ETAFIT, a fit to the $|\eta_{+-}|$, $|\eta_{00}|$, and $|\eta_{+-}/\eta_{00}|$ measurements. The results from fit 1, along with the K_S^0 values from this edition are used to compute values of $|\eta_{+-}|$ and $|\eta_{00}|$ which are included as measurements in the $|\eta_{00}|$ and $|\eta_{+-}|$ sections with a document ID of BRFIT 94. Thus the fit values of $|\eta_{+-}|$ and $|\eta_{00}|$ given in this edition include both the direct measurements and the results from the branching ratio fit.

The process is reversed in order to include the direct $|\,\eta\,|$ measurements in the branching ratio fit. The results from fit 2 above (before including BRFIT 94 values) are used along with the K_L^0 and K_S^0 mean lives and the $K_S^0 \to \pi\pi$ branching fractions to compute the K_L^0 branching ratios $\Gamma(K_L^0 \to \pi^+\pi^-)/\Gamma(\mathrm{total})$ and $\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_L^0 \to \pi^+\pi^-)$. These branching ratio values are included as measurements in the branching ratio section with a document ID of ETAFIT 94. Thus the K_L^0 branching ratio fit values in this edition include the results of direct measurements of $|\eta_{+-}|$, $|\eta_{00}|$, and $|\eta_{00}/\eta_{+-}|$. Details of these fits are given in the 1990 edition of this *Review* [8].

Since the last edition, results on ϵ'/ϵ have been published by CERN NA31 (BARR 93D) and Fermilab E731 (GIBBONS 93B). These results dominate our fit to ϵ'/ϵ, $|\eta_{+-}|$, $|\eta_{00}|$, and $\mathrm{B}(K_L \to \pi\pi)$. The resulting value $\epsilon'/\epsilon = (1.5 \pm 0.8) \times 10^{-3}$

(S = 1.8) has a large scale factor S included in the error, indicating a continuing disagreement in the input data. The CERN NA31 result continues to indicate the presence of direct CP violation with a result which is more than three σ above zero while the Fermilab E731 result is consistent with zero.

The Fermilab E731 experiment also has published recent results on ϕ_{+-}, ϕ_{00}–ϕ_{+-}, and Δm. GIBBONS 93 reports $\Delta m = (0.5286 \pm 0.0028) \times 10^{10}$ $\hbar$ s^{-1} assuming $\phi_{+-} = \phi_{00} = \phi_{\rm SW}$. GIBBONS 93C reports $\Delta m = (0.5257 \pm 0.0049) \times 10^{10}$ $\hbar$ s^{-1} when a joint fit with ϕ_{+-} is done. We use this value in our average rather than the GIBBONS 93 value to avoid the $\phi_{\rm SW}$ constraint. This is so that when we reevaluate ϕ_{+-} experiments using our average Δm, we will avoid biasing ϕ_{+-} toward $\phi_{\rm SW}$ The resulting average $\Delta m = (0.5333 \pm 0.0027) \times 10^{10}$ $\hbar$ s^{-1} (S = 1.2) is 0.0018×10^{10} $\hbar$ s^{-1} lower than our 1992 average as a result of the lower E731 value. GIBBONS 93 reports $\phi_{+-} = 42.2 \pm 1.4°$ for their Δm and τ_s. We use their Δm and τ_s dependence quoted in NAKADA 93. We revaluate all ϕ_{+-} measurements for the current Δm and τ_s PDG averages. The resulting value $\phi_{+-} = 44.3 \pm 0.8°$ is two old standard deviations below our 1992 average and is now consistent with the "superweak" phase given in Eq. (7a).

Footnotes and References

* The S values in parentheses are scale factors by which the errors have been increased to account for discrepancies in the data.

1. K. Kleinknecht in CP Violation (ed. C. Jarlskog), World Scientific, (1989), p. 41.
2. V. Barmin, *et al.*, Nucl. Phys. **B247**, 293 (1984);
   L. Wolfenstein,Ann. Rev. Nucl. Sci. **36**, 137 (1986).
3. T.T. Wu and C.N. Yang, Phys. Rev. Lett. **13**, 380 (1964).
4. E. Chell and M.G. Olsson, Phys. Rev. **D48**, 4076 (1993).
5. L. Wolfenstein, Phys. Rev. Lett. **13**, 562 (1964).
6. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 659 (1973).
7. J.F. Donoghue *et al.*, Phys. Reports **131**, 320 (1986).
8. J.J. Hernandez *et al.*, Phys. Lett. **B239**, 1 (1990).